Properties

1. The reason to introduce BESTT is to be able to make precise statements about systems.
2. Use formulas, predicates and functions to express desired (or undesired!) properties of some ”system”.
3. Note that ”system” could mean any of:
   - specifications
   - design
   - program
   - etc

   as well as combinations.
4. Now: simple examples to demonstrate BESTT and its use.

Consider the following declarations:

$\texttt{val ValveIsOpen : bool}$

$\texttt{val Pressure : int}$

$\texttt{val} \leq : \texttt{int} \rightarrow \texttt{int} \rightarrow \texttt{bool}$

We want to express that if `Pressure` is greater than 50, then a valve must be closed, otherwise it should be open. Using mathematical notation,

$$\texttt{ValveIsOpen(Pressure)} = \begin{cases} \texttt{true} & \texttt{Pressure} \leq 50 \\ \texttt{false} & \texttt{Pressure} > 50 \end{cases}$$

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$$(\texttt{pressure} \leq 50 \wedge \texttt{ValveIsOpen} = \texttt{true}) \vee$$
$$(\texttt{pressure} > 50 \wedge \texttt{ValveIsOpen} = \texttt{false})$$

Implication

if p then q

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Beware!

In english *If it is sunny then we go to the beach* often is regarded to also mean that ”if it is not sunny, then we do not go to the beach”.

This is **not** a logical implication.

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- A states that it rains.
- $A \vee \neg A$ states that it rains or it does not rain. (true?)
- $A \rightarrow D$ states that if it rains then we stay home.
- $B \rightarrow \neg C$ states that if we are going for a walk, then we are not reading.

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Used to make statements about all values of a specific type. For the purposes of this course, one can think of types as *representing* sets.

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- All lions are fierce. $\forall x : C.P(x) \to Q(x)$
- Some fierce creatures do not drink coffee. $\exists x : C.Q(x) \wedge \neg R(x)$.
- Some lions do not drink coffee. $\exists x : C.P(x) \wedge \neg R(x)$. Cannot be written as $\exists x : C.P(x) \to \neg R(x)$, as it is always true for all non-lions, even if all lions drink coffee!