

Computer Algebra vs Computer Analysis

Jacques Carette
McMaster University

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Overview

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 - some well-known
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- Only a glimpse of solutions, when known, will be given
- Systems and UI issues will also be ignored

Definition: Computer Algebra

Abstract domains: rings, fields, semi groups, categories, etc but
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Algebraic algorithms: arithmetic for all the above objects, term-rewriting, etc

- Gröbner bases
- Matrix factorizations
- Elimination in non-commutative Ore algebras
- Differential Algebra

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Key idea 1: **finite representations** of functions.

Key idea 2: functions are **not** necessarily computable!

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- Accurateness of the translation not questioned
- Parametric problems are always assumed “generic”
- These are not bugs! They are non-trivial theoretical issues.

Hands-on!

Specific examples of problems, using Maple.

But these are common to all Computer Algebra systems.