Assignment 4

Do the following problems and exercises from the book. Note that the ordering reflects the order in which the relevant material is being covered by the course. Always *justify* your answers.

1. Do exercises 4.1.31, 4.1.32
   Recommended exercise: 4.1.30

2. Do exercises 4.2.31, 4.2.41, 4.2.43
   Recommended exercise: 4.2.42
   Hints:

   4.2.42 *First show that If the DAG has exactly one vertex v with outdegree 0, then it is reachable from every other vertex.*

   4.2.43 *Note that it is enough to find a* strongly connected component *which is reachable from all nodes (since, in that case, all nodes in such a component are reachable from all vertices). Then the question becomes "find a strongly connected component that is reachable from all other strongly connected components".*

3. Do exercises 4.3.20, 4.3.32
   Recommended exercises: 4.3.4, 4.3.8
   Hints:

   4.3.8 *Suppose the max weight edge of a cycle $e = (u, v)$ is contained in an MST. Then if we delete it, u and v belong to different subtrees $T, V \setminus T$. But there is a path (from the cycle) from u to v in the original graph with cheaper edges; what's the relation between this path and the cut $(T, V \setminus T)$?*

4. Do exercises 4.4.25, 4.4.33, 4.4.47
   Recommended exercises: 4.4.22, 4.4.40
   Hints:

   4.4.33 *You can use exercise 4.4.22.*

   4.4.40 *If for some nodes u,v there is another path (not the MST one) with a longest edge shorter than the longest edge of the MST path. Then do the two paths together agree with exercise 4.3.8?*

   4.4.47 *If an edge (u, v) is relaxed in the Vth pass, then what is the number of edges in the shortest path from s to v? Can it be a simple path then?*