Assignment 2

Do the following problems and exercises from the book. Note that the ordering reflects the order in which the relevant material is being covered by the course. Exercises are spread throughout a chapter, and problems can be found at the end of each chapter.

Exercise 3.2

(a) Show that NP is closed under polynomial-time reductions, i.e., if $L_1 \in NP$ and $L_2 \leq_P L_1$, then $L_2 \in NP$.

(b) Show that $SPACE(n)$ is *not* closed under polynomial-time reductions, i.e., there are $L_1 \in SPACE(n)$, $L_2 \notin SPACE(n)$ such that $L_2 \leq_P L_1$. (*Hint:* Take any $L_2 \in SPACE(n^2) \setminus SPACE(n)$ (why does such a language exist?), and do an easy (polynomial) input-padding reduction to $SPACE(n)$. What does this imply?)

(c) Conclude the exercise solution.

Exercise 3.5

Hint: Let $L \in DTIME(n^2) \setminus DTIME(n)$, that exists because of the Time Hierarchy Theorem. Then start by the definition of a "time-constructible function" to argue that function $f(n) = n + L(n)$ is not time constructible ($L(n)$ is the 1/0 answer of $n \in / \notin L$).

Exercise 4.4

Hint: Use the fact that $PATH$ is NL-complete.

Exercise 4.7

(a) We have defined a NDTM as a TM with an extra tape that is *read-only* and *read-once*, i.e., its head moves only from left to right. Let's call our read-once NDTM $NDTM_{r-o}$. Show that $L \in NP$ if and only if L is accepted by a polynomial-time $NDTM_{r-o}$. (*Hint:* Look carefully at how the NDTM of the book uses its certificate in the proof of Thm. 2.6, and show how to simulate it.)

(b) Show that if L is accepted by a NDTM M that uses only $O(\log n)$ working-tape space and only $p(n)$ certificate-tape space for some polynomial p, then $L \in NP$. (*Hint:* What is the running time of this NDTM?)

(c) Show that if $L \in NP$, then L is accepted by a NDTM that uses only $O(\log n)$ working-tape space and only $p(n)$ certificate-tape space for some polynomial p. (*Hint:* Can you move the computation done by a polynomial-time NDTM that decides L in part (a) *into the certificate* and then use only $O(\log n)$ working space just to check its correctness?)

Exercise 4.12