Assignment 3

Do the following problems and exercises from the book. Note that the ordering reflects the order in which the relevant material is being covered by the course. Exercises are spread throughout a chapter, and problems can be found at the end of each chapter.

Exercise 5.2

Hint: Show how the computation of the ATM(x) that decides $L \in \cup_c \Sigma_i TIME(n^c)$ can be represented by a tree with nodes labeled $\exists$ or $\forall$. What is the label of the root? What does it mean for this tree the fact that the ATM has exactly $i-1$ alternations on its computation paths? What is the height of the tree? What does it mean for the tree to have an accepting computation of the ATM(x) when $x \in L$? (Things are symmetric for $\cup_c \Pi_i TIME(n^c)$.) Also show how Σ_i^p can be computed by an ATM that *'guesses'* certificates $u_1, u_2, \ldots, u_i$ using its transition function choices, and then runs $M(x, u_1, u_2, \ldots, u_i)$. (Similarly for Π_i^p).

(a) Show that $\Sigma_i^p \subseteq \cup_c \Sigma_i TIME(n^c)$. (*Hint:* Show how the ATM for Σ_i^p maps to the ATM for $\cup_c \Sigma_i TIME(n^c)$.)

(b) Show that $\cup_c \Sigma_i TIME(n^c) \subseteq \Sigma_i^p$.

Exercise 5.12

(*Hint:* Use induction on i, and $\overline{\Sigma_i^p} = \Pi_i^p = \Sigma_i^p = \overline{\Pi_i^p}$ in place of "...P is closed under complement..." in Theorem 5.4.)

Exercise 5.3

(a) Show that if $L \leq_P \bar{L}$, then $\bar{L} \leq_P L$.

(b) Show that $\Sigma_1^p = \Sigma_2^p$ (*Hint:* Use $3SAT \leq_p \overline{3SAT}$ and argue like Theorem 5.4.)

(c) Show that $\Pi_1^p = \Pi_2^p$ (*Hint:* Proceed like in part (b).)

(d) Use the previous exercise to finish exercise 5.3. (*Hint:* Recall that $\Sigma_i^p \subseteq \Pi_{i+1}^p \subseteq \Sigma_{i+2}^p$.)

Exercise 5.7

Show only the $APSPACE \subseteq EXP$ direction. For an extra challenge, you can try to prove the other direction as well.