Assignment 5

Do the following problems and exercises from the book. Note that the ordering reflects the order in which the relevant material is being covered by the course. Exercises are spread throughout a chapter, and problems can be found at the end of each chapter.

Exercise 8.2

Hint: Note that P knows the input ahead of its interaction with V (note that x is an argument to *all* steps in (8.1)), so it can pick how to reply, according to x.

Exercise 8.3

Hint: Note that if we treat P's answer as the certificate for $3SAT$, the only way for P to fool us when $x \notin L$ is for the randomized reduction to fail $(1 = 3SAT(M(x)) \neq L(x))$.

Exercise 8.6

(a) Given exercise 8.3., it is enough to boost the success probability for $BP \cdot NP$. Show how to improve the failure probability of Definition 7.17 to less that $(1/3)^k$ with parallel repetition. (*Hint:* If we use k random r's $r_1, r_2, \ldots, r_k$ then what is the probability of failure? First think what is a failure. You want to have a probabilistic TM M' that takes $r_1, r_2, \ldots, r_k$ and runs $M(x)$ with each of $r_1, r_2, \ldots$, and checks $\exists u_i : 3SAT(M_{r_i}(x, u_i)) = 1,\ \forall i$ if $L(x) = 1$ or fails that check if $L(x) = 0$. In the latter case, M' must not be fooled by *some* reductions being wrong and give you some satisfiable $3SAT$ formulas $M_{r_i}(x)$, BUT there is a bad case where M' is fooled and $L(x) = 0$ passes the test!)

(b) Finish the exercise. Remember, everything must run in polynomial time! (*Hint:* Use the logic of Exercise 8.3.

Exercise 8.11

(*Hint:* If L has a MIP, then we can 'guess' the provers. Note that MIP is probabilistic but $NEXP$ is deterministic, so you need to derandomize...)