Introduction Lindenmayer Systems

Presented by: FanFan Huang

Outline

- What is a L-System
- Types of L systems
- What is a Language (in terms of L systems)
- Drawing Fractals
- Trees

What is a Lindenmayer system

- System having the structure $G = \langle \Sigma , h , \omega \rangle$
- Σ an alphabet
  - an atomic set of symbols example: {a,b,c}, {1,2,3}
  - Σ^* a set of all words over an alphabet Σ
- h a set of homomorphic production rules
  - Of form $h : \Sigma \rightarrow \Sigma^*$
  - Example $\Sigma = \{a\}, h(a) = a^2$
- ω an axiom (an initial word)
  - Example $\omega = ab$

D0L

- Simplest type of system $G = \langle \Sigma, h, \omega \rangle$
- D: Deterministic
- 0: Rewriting that takes place is context-free
- L: Lindenmayer System

- Example
  - G = ({a,b},{h(a)=a, h(b)=ab},ab)

Words and Languages

- The language is constructed by:
  - Denote h^i as the ith production rule
  - $h^2 = h \circ h$
  - $L(G) = \{\omega, h(\omega), h^2(\omega), h^3(\omega), \ldots\}$
  - $L(G) = \{h^i(\omega) | i \geq 0\}, where\, h^0(\omega) = \omega$

- *E(G)* is the word sequence generated by the (same as the language at the ith step)

Language Equivalence

- We say that languages are equivalent that is $L(G_1)=L(G_2)$ iff the branching structures generated by them are isomorphic.
- Does not imply the sequences are equivalent.
- Example:

$$G_1=\langle\{a,b\},\{a\rightarrow b^{2,} b\rightarrow a\},b\rangle$$
$$G_2=\langle\{a,b\},\{a\rightarrow b,b\rightarrow a^2\},a\rangle$$
$$L(G_1)=L(G_2)$$

0L

- All the rules of D0L $G = \langle \Sigma, h, \omega \rangle$
- Remove D for deterministic
- Difference is in production rules

- Example:

$$G = (\{a\}, \{a \to a, a \to a^2\}, a)$$

E0L

- Form: $G = \langle \Sigma, h, S, \Delta \rangle$
- Known as the Extension to 0L
- Allows for use of symbols not in the final form
- S Axiom (may contain aux symbols)
- Δ Target alphabet (no aux symbols)

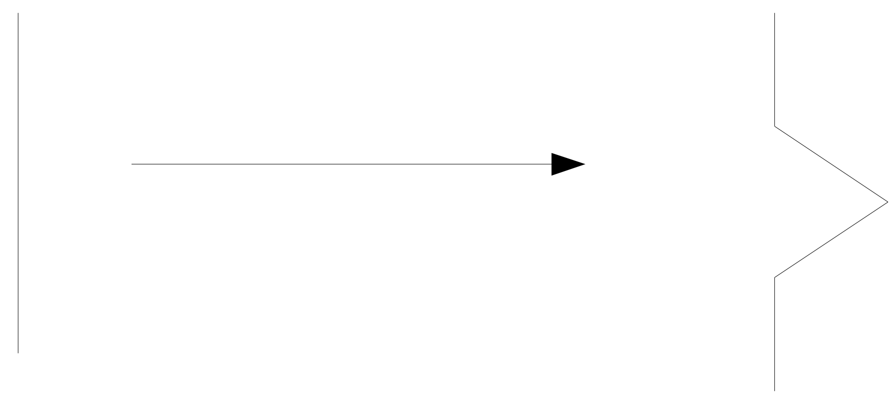
Lindenmayer Systems

- Also referred to as L systems
- D0L
- 0L
- E0L
- C0L
- DT0L
- EDT0L
- *1L
- .... (there are many)

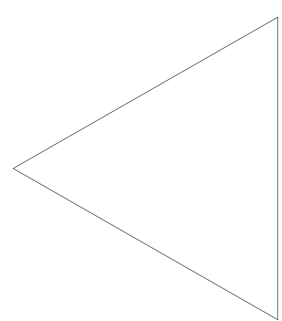
Tree drawing and fractals

- So far we've considered term rewriting as symbols, consider the following:

h:

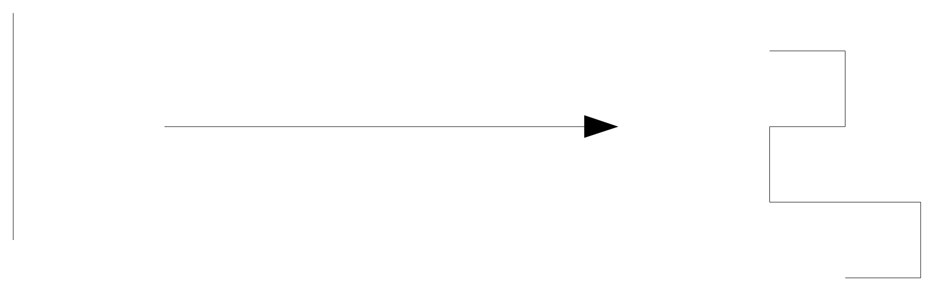


- Axiom:

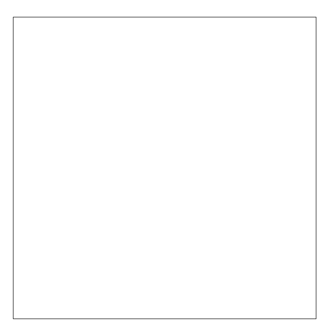


Koch curve

h:

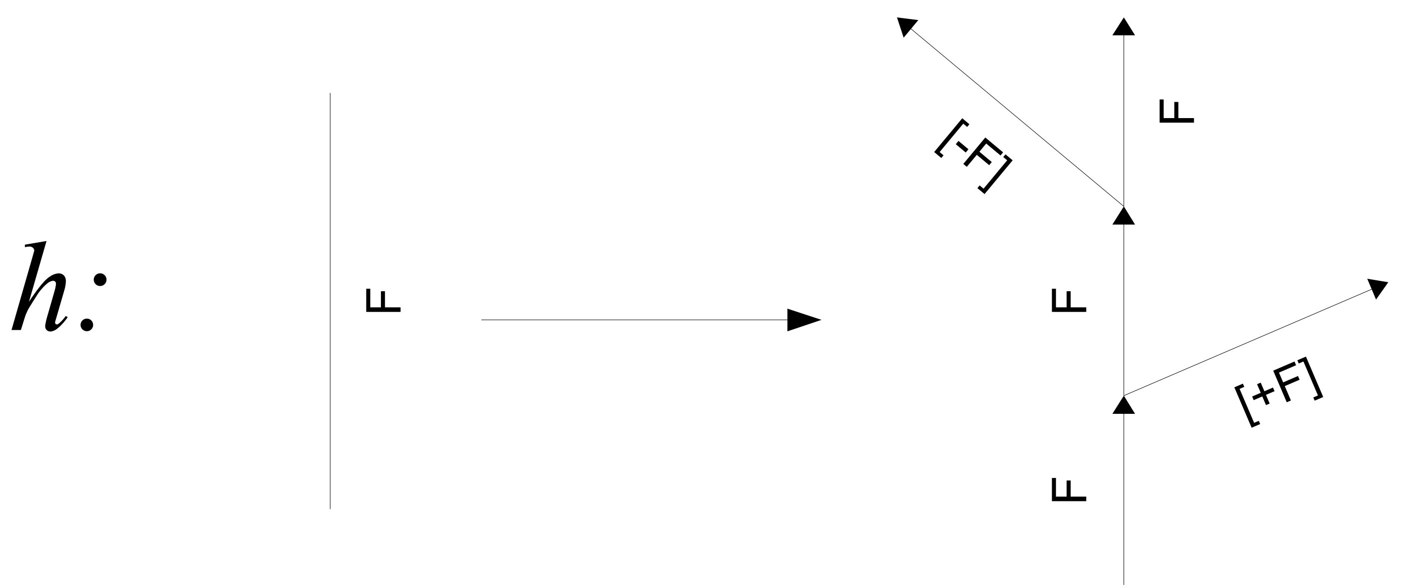


Axiom:

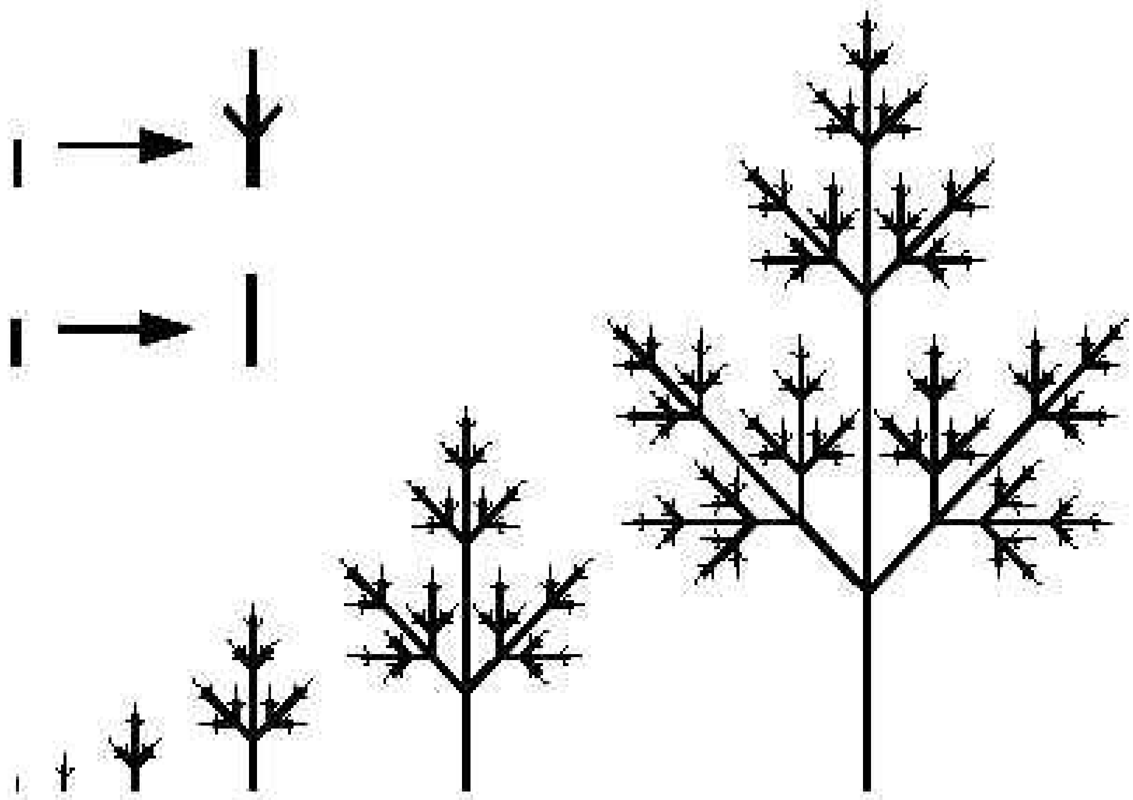


Growing trees

- Bracketed L-Systems
- Add brackets as a form of denoting branching structure.
- We can draw trees!



Further Examples



Further Examples

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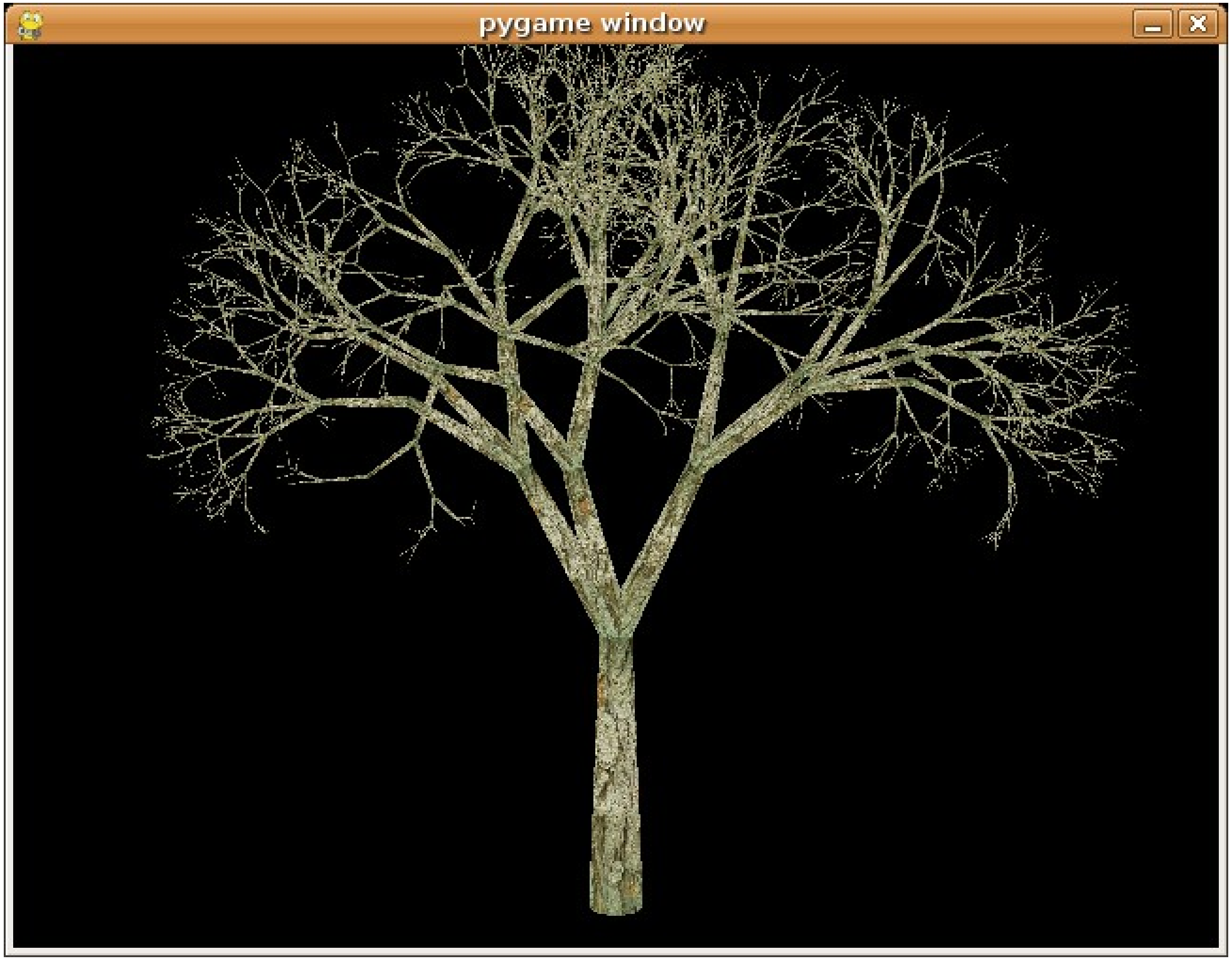
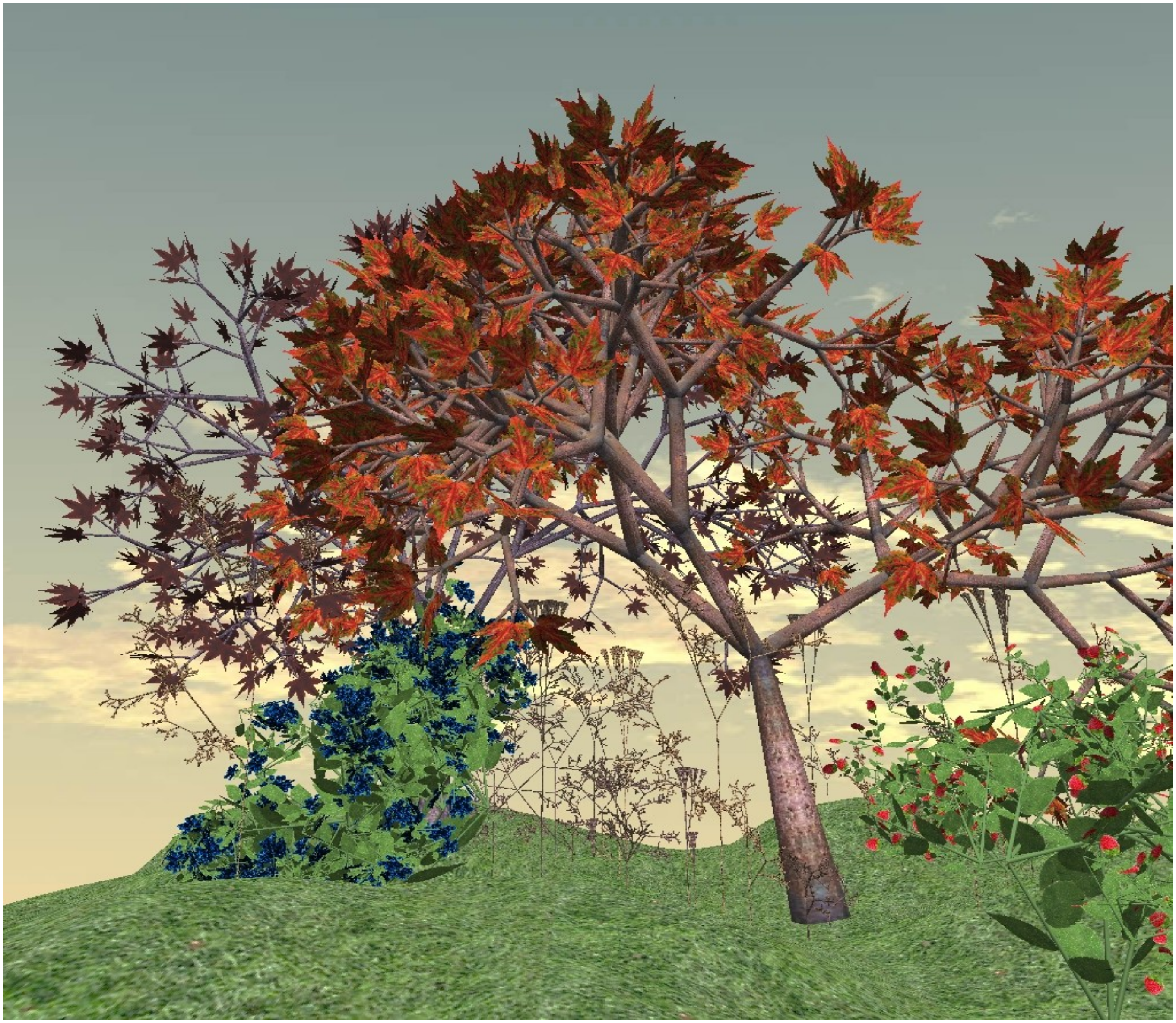


Further Examples



References

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