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CAS 701 Fall 2007 Exercise Sheet 1

# CAS 701 — Logic and Discrete Mathematics in Software Engineering

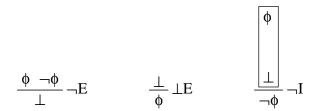
4 October 2007

### **1** Natural Deduction Proofs

For each of the following sequents, produce a formal proof on paper, and transscribe your proof into the language of the "Logic Daemon" proof checker to check your proofs.

Be aware of the distinction between **basic rules** and derived rules, and produce basic-rule-proofs for any derived rules you use.

The following basic rules have not yet been shown in class (" $\perp$ ", read also "bottom", stands for "Falsum"):



In the Logic Daemon, there is no " $\perp$ ", and you have to use rule RAA.

(a) 
$$\vdash p \rightarrow (q \rightarrow p)$$
  
(b)  $\vdash (p \rightarrow q) \rightarrow ((q \rightarrow r) \rightarrow (p \rightarrow r))$   
(c)  $p \lor q, \neg q \lor r \vdash p \lor r$   
(d)  $p \land \neg p \vdash \neg (r \rightarrow q) \lor (r \rightarrow q)$   
(e)  $p \rightarrow q, s \rightarrow t \vdash p \land s \rightarrow q \lor t$   
(f)  $\neg (\neg p \lor q) \vdash p$   
(g)  $\vdash \neg p \rightarrow (p \rightarrow (p \rightarrow q))$   
(h)  $\neg (p \rightarrow q) \vdash q \rightarrow p$   
(i)  $\vdash ((p \rightarrow q) \land (q \rightarrow p)) \rightarrow ((p \lor q) \rightarrow (p \land q))$   
(j)  $\vdash p \lor q \leftrightarrow \neg (\neg p \land \neg q)$   
(k)  $\vdash (p \lor q) \leftrightarrow ((p \rightarrow q) \rightarrow q)$ 

Note: Questions 2-4 use semantics that has not yet been presented in class. Please find the standard definitions, for example on the Wikipedia page under the heading "Soundness and completeness of the rules"; where Wikipedia says "S semantically entails \$\$" I use the notation " $S \models \phi$ ".

## 2 Invalidity

For each of the following sequents, show that it is not valid by providing a valuation for which all the antecedents evaluate to True, but the consequent evaluates to False.

(a) 
$$\neg m \lor (q \to p) \vdash \neg p \land q$$

(b) 
$$\neg r \rightarrow (p \lor q), r \land \neg q \vdash r \rightarrow q$$
  
(c)  $p \rightarrow (q \rightarrow r) \vdash p \rightarrow (r \rightarrow q)$ 

(c) 
$$p \to (q \to r) \vdash p \to (r \to q)$$

- (d)  $\neg p, p \lor q \vdash \neg q$
- (e)  $p \to (\neg q \lor r), \neg r \vdash \neg q \to \neg p$
- (f)  $p \to (q \lor r) \vdash (p \to q) \land (p \to r)$

### **3** Semantic Equivalence

Page 54, Def. 1.40:  $\phi \equiv \psi$  (read:  $\phi$  is semantically equivalent to  $\psi$ ) iff  $\phi \models \psi$  and  $\psi \models \phi$ . (a) Show the following semantic equivalences in two principally different ways:

- $\phi \lor (\phi \land \eta) \equiv \phi$
- $\phi \land (\phi \lor \eta) \equiv \phi$
- $\phi \lor (\psi \land \eta) \equiv (\phi \lor \psi) \land (\phi \lor \eta)$
- (b) Show also the following semantic equivalences:
  - $\phi \land (\psi \lor \eta) \equiv (\phi \land \psi) \lor (\phi \land \eta)$
  - $\neg(\phi \lor \eta) \equiv \neg\phi \land \neg\eta$
  - $\neg(\phi \land \eta) \equiv \neg \phi \lor \neg \eta$

#### **4** Normal Forms

For each of the following formulae, calculate the indicated normal forms.

- (a) CNF(NNF(IMPL\_FREE( $\neg (p \rightarrow (\neg (q \land (\neg p \rightarrow q)))))))$
- (b)  $CNF(NNF(IMPL\_FREE(((p \rightarrow \neg q) \rightarrow \neg p) \rightarrow q)))$
- (c)  $\mathsf{DNF}(\mathsf{NNF}(\mathsf{IMPL\_FREE}((p \to q) \to (q \to p))))$