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SE 3E03, Fall 2002
Assignment 1

Design and Selection of Programming Languages

5th September 2002

Review: Discrete Mathematics and Oberon

Problem 1 (Set Cardinality)

Calculate the cardinalities of the following sets:

a) $\{1\}$	e) $\{1,2,1\}$	i) $\{\}$	m) $\{0,\varnothing\}$
b) $\{3\}$	f) $\{1,\{2\},1\}$	j) $\{\{\}\}$	n) $\{\varnothing,\{0\}\}$
c) $\{c\}$	g) $\{1,2,\{1\}\}$	k) $\{\{\},\varnothing\}$	o) $\{\{\{\},\{0\}\}\}$
d) $\{c,d\}$	h) $\{1,2,\{1,2\}\}$	l) $\{\varnothing,\{\varnothing\}\}$	p) $\{\{\{\},\{0\}\},\{\{2-2\},\varnothing\}\}$

Which of these sets contain some *non-empty* set both as subset and as element?

Problem 2 (Set Comprehension)

List the elements of each of the following sets:

a) $\{x : \mathbb{N}_1 \mid x^2 < 20 \bullet x^3\}$

c) $\{x, y : \mathbb{N}_1 \mid 5 \leq x + y \leq 6 \bullet x * y\}$

d) $\{s : \mathbb{P}\{1,2,3\} \mid \#s \geq 2\}$

b) $\{x, y : \mathbb{N}_1 \mid x^2 + y^2 < 20\}$

e) $\{s : \mathbb{P}\,\mathbb{P}\{1,2\} \mid \#s > \#\bigcup s\}$

Problem 3 (Relations)

Let the sets $X = \{1,2,3\}$, $Y = \{4,5\}$ und $Z = \{6,7,8,9\}$ and the following relations be given:

$$
\begin{array}{lcl}
R : X \leftrightarrow Y & \text{with} & R = \{(1,4),(2,4),(2,5),(3,5)\} \\
S : X \leftrightarrow Z & \text{with} & S = \{(1,6),(1,7),(3,7),(3,9)\} \\
T : Z \leftrightarrow Y & \text{with} & T = \{(7,4),(9,4),(9,5)\} \\
U : Y \leftrightarrow X & \text{with} & U = \{(4,3),(5,1)\}
\end{array}
$$

In addition, we consider the subsets $A = \{1,2\}$, $B = \{4\}$, and $C = \{6,7\}$. List the elements of each of the following sets:

a) $A \times Y$	e) $T^{\smile}$	i) $(R \setminus (\mathsf{dom}\, S \times Y)) \cup U^{\smile}$	m) $X \rightarrowtail\!\!\!\!\twoheadrightarrow Y$
b) $\mathrm{id}\, X$	f) $S ; T$	j) $U \times A$	n) $(\mathbb{P}\, R) \cap (X \rightarrow Y)$
c) $\mathsf{ran}\, S$	g) $R ; T^{\smile} \cap S$	k) $A \leftrightarrow B$	o) $(\mathbb{P}\, T) \cap (Z \twoheadrightarrow Y)$
d) $\mathsf{dom}\,(\mathrm{id}\, Z)$	h) $U \cap (Y \times A)$	l) $A \rightarrowtail C$	p) $\#(\mathbb{P}(X \leftrightarrow Z))$

Problem 4 (Relations)

For each of the following statements, check whether it is true, and if it is false, give a counterexample:

a) A transitive and symmetric relation is reflexive, too.

b) The composition of two orders cannot be an equivalence.

c) Intersecting an order with an equivalence yields an order, again.

d) The composition of an injective mapping with a surjective mapping is injective, again.

e) The composition of a transitive relation with its converse is again transitive.

f) The composition of an asymmetric relation with its converse is again asymmetric.

g) If an injective function $F : A \rightarrowtail\!\!\!\!\!\!+\;\; B$ is contained in a surjective mapping $G : A \twoheadrightarrow B$, then G is bijective.

Problem 5 (Formal Languages)

The concatenation operation $\frown$ for sequences can be generalized to a *concatenation operation for formal languages*.

Starting with two formal languages L and M over the alphabet Σ, the concatenation of L with M, written $L \cdot M$, is defined as that set of words over Σ that contains a word w if and only if there are a word $u \in L$ and a word $v \in M$ such that $w = u \frown v$.

a) Calculate:

1. $\{\langle 1 \rangle, \langle 1, 1 \rangle\} \cdot \{\langle 2 \rangle, \langle 2, 2 \rangle\}$
2. $\{\langle 1 \rangle, \langle 1, 1 \rangle\} \cdot \{\langle 2 \rangle, \langle 1, 2 \rangle\}$
3. $\{\langle 1 \rangle, \langle 1, 1 \rangle\} \cdot (\{\langle 2 \rangle, \langle 1, 2 \rangle\} \cup \{\langle 2 \rangle, \langle 2, 2 \rangle\})$
4. $\{\langle 1 \rangle, \langle 1, 1 \rangle\} \cdot \{\langle\, \rangle\}$
5. $\{\langle\, \rangle\} \cdot \{\langle 2 \rangle, \langle 1, 2 \rangle\}$
6. $\{\} \cdot \{\langle 2 \rangle, \langle 1, 2 \rangle\}$

b) Is the concatenation operation for formal languages associative?

c) Can you state a law for $L \cdot (M \cup N)$?

Problem 6 (Oberon-2 Execution)

For the following Oberon-2 program, simulate execution by drawing the dynamic call tree and recording the values of parameters and variables for every block entry and exit.

Which is the final result?

```
MODULE Scope1;
IMPORT Out;
VAR n : INTEGER;
PROCEDURE B(VAR x : INTEGER; z : INTEGER);
  VAR hv : INTEGER;
  BEGIN
    IF z = 0
    THEN x := 0
    ELSE hv := x;
         B(x, z-1);
         x := x+hv  END;
  END B;
```

```
PROCEDURE A(y,x: INTEGER; VAR result: INTEGER);
  BEGIN
    IF x = 0
    THEN result:=1;
    ELSE A(y, x-1, result);
         B(result, y)       END;
  END A;
BEGIN
  n := 0;
  A(2,1,n);
  Out.Int(n,0); Out.Ln
END Scope1.
```

Which functions are implemented by the procedures `A` and `B`? Produce precise specifications!