McMaster University
Department of Computing and Software
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SE 3E03, Fall 2002
Assignment 6

Design and Selection of Programming Languages

3rd October 2002

Problem 22

Review lessons 1–11 from the "Two Dozen Short Lessons in Haskell" by Rex Page — make sure you can handle all review questions and that you can confidently produce correct Haskell scripts and interact with Hugs.

Then work through lessons 12–14 and 21–22.

Problem 23 (Cardinalities)

Assuming that the sets U, V, and W are finite with $|U| = u$ and $|V| = v$ and $|W| = w$, compute the cardinalities of the following sets ($U \to V$ denotes the set of all **total** functions from U to V):

a) $U \to (V \times W)$ b) $(U \times V) \to W$ c) $U \to (V \to W)$ d) $(U \to V) \to W$

Problem 24 (List Comprehension)

Using list comprehension define `makeMatrix :: (a -> b -> c) -> [a] -> [b] -> [[c]]` as a Haskell function for which the following holds (the Matrix has to be considered as a list of lists):

$$\texttt{makeMatrix f } [x_1, \ldots, x_m] \; [y_1, \ldots, y_n] = \begin{bmatrix} \texttt{[(f } x_1 \; y_1\texttt{)}, & \ldots & \texttt{, (f } x_1 \; y_n\texttt{)]} \\ \vdots & \ddots & \vdots \\ \texttt{[(f } x_m \; y_1\texttt{)}, & \ldots & \texttt{, (f } x_m \; y_n\texttt{)]} \end{bmatrix}$$

Think of interesting applications of this function and bring examples into the tutorial!

Problem 25 (Lambda-Calculus)

a) Reduce the following λ-terms to normal form:

$$(\lambda\, x\,.\, y\ ((\lambda\, z\,.\,(z\ x)\ x)\ (u\ x)\ (u\ w))) \qquad (\lambda\, z\,.\,(z\ x)\ x)\ (\lambda\, x\,.\, u\ x)$$
$$(\lambda\, z\,.\,(\lambda\, x\,.\,(z\ x)\ x))\ (\lambda\, x\,.\, u\ x) \qquad (\lambda\, z\,.\,(\lambda\, x\,.\,(z\ x)\ x))\ (u\ x)$$

b) Perform four β-reductions on: $(\lambda\, x\,.\,(x\ x)\ x)\ (\lambda\, x\,.\,(x\ x)\ x)$

c) Let M and N be arbitrary λ-terms. Reduce $(\lambda\, x\,.\, M\ x)\ N$, considering the different cases with respect to variable occurrence.

d) Remember: In Haskell we write "`\ x -> e`" for "$(\lambda\, x\,.\, e)$".

Evaluate "`(\ f -> 5 + f 3) (\ x -> 2 * x)`" in Hugs and understand the result!

What are the types of "`(\ f -> 5 + f 3)`" and "`(\ x -> 2 * x)`"?

e) Reduce the following λ-terms to normal form:

$$(\lambda\, f\,.\,(\lambda\, x\,.\, f\ (f\ x)))\ (\lambda\, f\,.\,(\lambda\, x\,.\, f\ (f\ (f\ x))))$$
$$(\lambda\, g\,.\,(\lambda\, h\,.\,(\lambda\, f\,.\,(\lambda\, x\,.\, g\ f\ (h\ f\ x)))))\ (\lambda\, f\,.\,(\lambda\, x\,.\, f\ (f\ x)))\ (\lambda\, f\,.\,(\lambda\, x\,.\, f\ (f\ (f\ x))))$$