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SFWR ENG 3E03
Exercise Sheet 5
Solution Hints

Design and Selection of Programming Languages

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Exercise 5.1 — Haskell Evaluation (36% of Midterm 1, 2004)

Assume the following Haskell definitions to be given:

```
succ n = n+1                         -- reduce in one step, e.g.: succ 5 → 6

take :: Int -> [a] -> [a]
take 0 _      =  []
take _ []     =  []
take n (x:xs) =  x : take (n-1) xs

feed h q y =  q : feed h (q + y) (h y)
```

Simulate Haskell evaluation for the following expression (write down the sequence of intermediate expressions):

$\boxed{\texttt{take 3 (feed succ 0 1)}}$

Note: You may introduce **abbreviations for repeated subexpressions**, or use **repetition marks for material that is unchanged from the previous line**. In particular, ***write*** "*s*" ***instead of*** "*succ*"!

Solution Hints

13 steps, 1 contractible arith

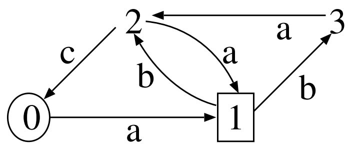
take 3 (*feed succ* 0 1)
= *take* 3 (0 : *feed succ* (0 + 1) (*succ* 1))
= 0 : *take* (3−1) (*feed succ* (0 + 1) (*succ* 1))
= 0 : *take* 2 (*feed succ* (0 + 1) (*succ* 1))
= 0 : *take* 2 ((0 + 1) : *feed succ* ((0 + 1) + *succ* 1) (*succ* (*succ* 1)))
= 0 : (0 + 1) : *take* (2−1) (*feed succ* ((0 + 1) + *succ* 1) (*succ* (*succ* 1)))
= 0 : 1 : *take* (2−1) (*feed succ* (1 + *succ* 1) (*succ* (*succ* 1)))
= 0 : 1 : *take* 1 (*feed succ* (1 + *succ* 1) (*succ* (*succ* 1)))
= 0 : 1 : *take* 1 ((1 + *succ* 1) : *feed* (+) *succ* ((1 + *succ* 1) + *succ* (*succ* 1)) (*succ* (*succ* (*succ* 1))))
= 0 : 1 : (1 + *succ* 1) : *take* (1–1) (*feed succ* ((1 + *succ* 1) + *succ* (*succ* 1)) (*succ* (*succ* (*succ* 1))))
= 0 : 1 : (1 + 2) : *take* (1–1) (*feed succ* ((1 + 2) + *succ* (*succ* 1)) (*succ* (*succ* 2)))
= 0 : 1 : 3 : *take* (1–1) (*feed succ* (3 + *succ* (*succ* 1)) (*succ* (*succ* 2)))
= 0 : 1 : 3 : *take* 0 (*feed succ* (3 + *succ* (*succ* 1)) (*succ* (*succ* 2)))
= 0 : 1 : 3 : [ ]

3% per necessary step:
- 1% for reducing the right redex
- 2% for performing the reduction correctly
- -1% for not writing down

Exercise 5.2 — Finite-State Machines (25% of Midterm 1, 2004)

Let the following type synonyms be given, as in the presentation in the first lecture:

```
type State = Int
type Symbol = Char
type TransRel = [(State, Symbol, State)]
type FSM = (State, TransRel, [State])
```



(a) Define *fsm1* :: *FSM* such that it represents the finite-state machine drawn above (with start state circled and end states in boxes):

(b) Define the Haskell function *isDet* :: *FSM* → *Bool* such that *isDet fsm* evaluates to the Boolean value indicating whether the finite-state machine *fsm* is deterministic or not.

For example, *isDet fsm1* = **False** since there are two *b*-edges from state 1 to different nodes.

Hint: Define auxiliary functions! For example:
- Calculate all start nodes of transitions in a *TransRel*.
- Given a state, calculate all edges leaving that state in a *TransRel*.
- Given a *Symbol* and a *TransRel*, find all target nodes of edges with that symbol.
- Given a *State* and a *TransRel*, find out whether any edges from that state violate determinacy.

Other functions may be useful, too. **Document your functions!**

Solution Hints

```
type State = Int
type Symbol = Char
type TransRel = [(State, Symbol, State)]

type FSM = (State, TransRel, [State])

fsm1 :: FSM             -- 6%
fsm1 = (0, tr1, [1])
 where
  tr1 =
   [(0,'a',1)
   ,(1,'b',2)
   ,(1,'b',3)
   ,(2,'a',1)
   ,(2,'c',0)
   ,(3,'a',2)
   ]

edgeStarts tr = [ s | (s, c, t) ← tr ]   -- 3%
```

$outEdges\ tr\ s = [(c,t) \mid (s',c,t) \leftarrow tr,\ s' \equiv s\ ]$ -- 3%

$isUnique\ es\ (c,t) = all\ (t \equiv)\ [\ t' \mid (c',t') \leftarrow es,\ c' \equiv c\ ]$ -- 5%

$isDetState\ tr\ s = all\ (isUnique\ es)\ es$ -- 4%
 where $es = outEdges\ tr\ s$

$isDet\ (s0,\ tr,\ fin) = all\ (isDetState\ tr)\ (edgeStarts\ tr)$ -- 4%

Exercise 5.3 — Haskell Typing (19% of Midterm 1, 2004)

Provide **detailed derivations** of the Haskell types of the following functions:

```
swibble x y = [ ( x , y ) , ( x ++ "'", y + 1 ) ]
swoon g h = [ g ( (1 + ) . h ) ]
```

Solution Hints

Type classes have not been taught yet, only mentioned: Numeric types can be defaulted to *Integer* or *Int*.

$swibble :: (Num\ n) \Rightarrow String \rightarrow n \rightarrow [(String,\ n)]$

Assuming $1 :: Integer$, we must have $y :: Integer$ because of $y + 1$.

Since "'" $::$ *String*, we also have $x :: String$ because of x ++ "'" $:: String$.

Then $(x,y) :: (String,\ Integer)$, and the type of *swibble* follows easily.

$swoon :: (Num\ n) \Rightarrow ((a \rightarrow n) \rightarrow b) \rightarrow (a \rightarrow n) \rightarrow [b]$

Assuming $1 :: Integer$, we have $(1+) :: Integer \rightarrow Integer$, and because of the composition, we must have

$h :: a \rightarrow Integer$ for some type a.

Therefore, we have $((1+) \circ h) :: a \rightarrow Integer$, and may assume $g :: (a \rightarrow Integer) \rightarrow b$ for some type b.

Then we have $[\ g\ ((1+) \circ h)\ ] :: b$, and therefore

$swoon\ g :: (a \rightarrow Integer) \rightarrow b$

and

$swoon :: ((a \rightarrow Integer) \rightarrow b) \rightarrow (a \rightarrow Integer) \rightarrow b$.

Exercise 5.4 (***Skeleton file is on the course page***)

We define a type of transition functions that define state transitions triggered by *input*s and also producing *output*s:

type $Transition\ state\ input\ output = (state,\ input) \rightarrow (state,\ output)$

(a) Define a Haskell function

```
process :: Transition state input output → state → [input] → [output]
```

that calculates the list of outputs produced by a transition function given a starting state and a list of inputs.

Solution Hints

```
process tr s [] = []
process tr s (input : inputs) = let
   (s', output) = tr (s, input)
 in output : process tr s' inputs
```

Using *process* from (b) and prelude functions, the definition

```
runprocess :: Transition state String String → state → IO ()
runprocess tr s = do
  hSetBuffering stdout LineBuffering -- requires: "import System.IO" at beginning of module
  interact (unlines ∘ process tr s ∘ lines)
```

allows *runprocess* to turn a transition with *String* inputs and outputs into a runnable program.

Try: *runprocess id* 0

(b) Define a transition function

```
countEcho :: Transition Integer String String
```

that keeps a counter as its state and otherwise just reproduces the input prefixed withline numbers as output.

Try: *runprocess countEcho* 0

Solution Hints

```
countEcho (count, input) = (count', shows count' (' ' : input))
  where count' = succ count
```

(c) Define a transition function

```
trAdd :: Transition Integer String String
```

that uses the prelude functions *read* and *show* to add the *Integer* reading of the input to the accumulating state, and outputs that state as a string.

Try: *runprocess trAdd* 0

Solution Hints

```
trAdd (s, input) = (s', show s')
 where
   n = read input
   s' = s + n
```

(d) Define a transition function

```
polish :: Transition [Integer] String String
```

that implements a reverse Polish notation calculator by pushing number inputs on the stack, always outputing the top of the stack (if present), and interpreting +, −, ∗, / as taking their arguments from the stack and pushing the result back onto the stack.

Try: *runprocess polish* [ ]

Solution Hints

```
polish (n : m : ks, "+") = (k : ks, show k) where k = m + n
polish (n : m : ks, "-") = (k : ks, show k) where k = m - n
polish (n : m : ks, "*") = (k : ks, show k) where k = m * n
polish (n : m : ks, "/") = (k : ks, show k) where k = m `div` n
polish (ks        , input) = (k : ks, show k)  where k = read input
```
