Assignment 3

Due. Nov. 19, Monday, 11:30.

1. (12 marks) The following figures from the Census Bureau give the population of the United States:

Year	Population
1900	75,994,575
1910	91,972,266
1920	105,710,620
1930	122,775,046
1940	131,669,275
1950	150,697,361
1960	179,323,175
1970	203,235,298

   - Since there are eight points, there is a unique polynomial of degree 7 which interpolates the data. However, some of the ways of representing this polynomial are computationally more satisfactory than others. Here are four possibilities, each with t ranging over the interval $1900 \le t \le 1970$:

$$\sum_{j=0}^{7} a_j t^j,$$
$$\sum_{j=0}^{7} b_j (t-1900)^j,$$
$$\sum_{j=0}^{7} c_j (t-1935)^j,$$
$$\sum_{j=0}^{7} d_j \left(\frac{t-1935}{35}\right)^j.$$

     In each case, the coefficients are found by solving an 8-by-8 Vandermond system, but the matrices of various systems are quite different. Set up each of the four matrices, and find the estimate of its condition using Matlab/Octave function `cond()`. Then use Matlab/Octave operator "\" to find the coefficients. Check each of the representations to see how well it reproduces the original data.
   - Interpolate the data by a 7th-degree polynomial, using the best conditioned representation found above, and by the natural cubic spline using `ncspline.m`. Graph the resulting functions at one-year intervals over the period from 1900 to 1980. Find the 1980 census data. Which approach gives more accurate prediction?

2. (12 marks) Modify `QUADR` so that it returns `fcnt` as the total number of function evaluations and `minl` as the length of the smallest panel which it uses. Then write a MATLAB/Octave program `QUADS` replacing the rectangle rule with the Simpson's rule. Run both programs on a fairly hard problem such as $f(x) = \sqrt{x}$. Compare the numbers of function evaluations and the lengths of the smallest panels.