Monads

Oooh, scary!

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Outline of the talk

- Categories
- Functors
- Monads

Categories

- Objects (e.g., types)
  - $\mathrm{Int}, \mathrm{Bool}, \mathrm{Int} \times \mathrm{Int}, \ldots$
- Morphisms (e.g., functions)
  - $f : \mathrm{Char} \to \mathrm{Int}, \ldots$
- Composition
  - $\forall f : A \to B.\ \forall g : B \to C.\ \exists\, g \circ f : A \to C$
- Identity morphisms
  - $\forall A.\ \exists\, \mathrm{id}_A : A \to A$ such that
    - $\forall f : A \to B. \quad f \circ \mathrm{id}_A = f$
    - $\forall f : B \to A. \quad \mathrm{id}_A \circ f = f$

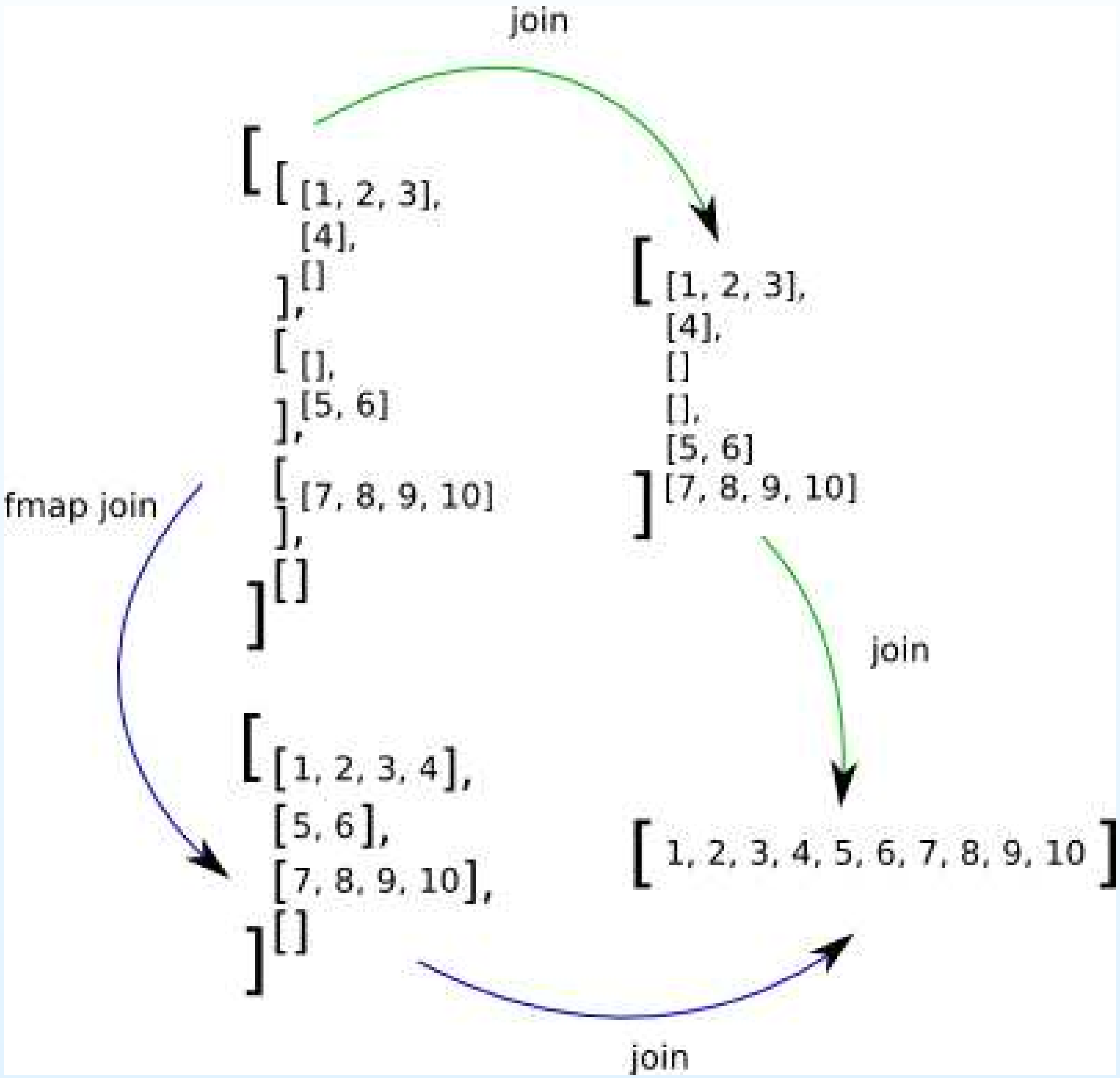
Functors

- A map $F : \mathbf{C} \to \mathbf{D}$ from category $\mathbf{C}$ to category $\mathbf{D}$
  - Let's assume $\mathbf{C} = \mathbf{D}$
- F takes $\mathbf{C}$-objects to $\mathbf{D}$-objects
  - $\text{data List } A = \text{Nil} \mid \text{Cons } A \ (\text{List } A)$
  - $\text{data Maybe } A = \text{Nothing} \mid \text{Just } A$
  - $\text{data Tree } A = \text{Tip } A \mid \text{Bin } (\text{Tree } A) \ (\text{Tree } A)$
- F takes $\mathbf{C}$-morphisms to $\mathbf{D}$-morphisms
  - $\text{map}_{\text{List}} : (A \to B) \to (\text{List } A \to \text{List } B)$
  - $\text{map}_{\text{Maybe}} : (A \to B) \to (\text{Maybe } A \to \text{Maybe } B)$
  - $\text{map}_{\text{Tree}} : (A \to B) \to (\text{Tree } A \to \text{Tree } B)$
- And follows some laws
  - $\forall A. \quad F \ (\text{id}_A) = \text{id}_{(F \ A)}$
  - $\forall f. \forall g. \quad (F \ f) \circ (F \ g) = F \ (f \circ g)$

Monads

- A functor T with some extra stuff
- A "unit" natural transformation (aka pure, return, $\mathrm{point}, \ldots$)
  - $\eta : \mathrm{Id} \dot{\to} T$ . . . What?!
  - $\forall A.\ \eta_A : A \to T\ A$ . . . not quite
  - $\eta : \forall A.\ A \to T\ A$
- And a "join" natural transformation (or "bind" $(\gg=)$ instead)
  - $\mu : T \circ T \dot{\to} T$
  - $\mathrm{concat} : \forall A.\ \mathrm{List}\ (\mathrm{List}\ A) \to \mathrm{List}\ A$
- Following some laws
  - $\mu_A \circ \eta_A = \mathrm{id}_A$
  - $\mu_A \circ T\ \eta_A = \mathrm{id}_A$
  - $\mu_A \circ T\ \mu_A = \mu_{T\,A} \circ \mu_A$

Monads


join
[[[1, 2, 3],
[4],
[]
],
[[],
[5, 6]
],
[[7, 8, 9, 10]
],
[]
]
[[1, 2, 3],
[4],
[]
[],
[5, 6]
[7, 8, 9, 10]]
fmap join
join
[[1, 2, 3, 4],
[5, 6],
[7, 8, 9, 10],
[]]
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
join

∼fin.