Sample solution to 6L03 graduate student exercise 3G

Definition: X_E^y means any expression whose value before executing the assignment statement y:=E is equal to the value of X afterward. More formally, X_E^y is any expression satisfying the requirement that

$$X_E^y.d = X.((y:=E).d) \tag{1}$$

for every data environment d in the domain of the assignment statement y:=E. **[end of definition]**

X may be any expression, not necessarily a Boolean expression. Of interest below is an expression which is an array variable.

Theorem: $[x(E1)]_{E3}^{x(E2)} = ($if $[E1]_{E3}^{x(E2)} = E2$ then E3 else $x([E1]_{E3}^{x(E2)})$ endif$)$

where x is the name of an array and E1, E2 and E3 are any expressions.

Proof (overview): It will be shown that the expression on the right side of the equation above satisfies the definition above applied to $[x(E1)]_{E3}^{x(E2)}$, i.e. that

$$(\text{if } [E1]_{E3}^{x(E2)} = E2 \text{ then E3 else } x([E1]_{E3}^{x(E2)}) \text{ endif}).d0 = (x(E1)).d1$$

for every data environment d0 in the domain of the assignment statement x(E2):=E3 and where

$$d1 = (x(E2):=E3).d0$$

Note that it follows from the definition of the assignment statement that

$$x(E2.d0).d1 = E3.d0 \tag{2}$$

and

$$x(i).d1 = x(i).d0 \text{ for all } i \neq E2.d0. \tag{3}$$

Proof: For every data environment d0 in the domain of the assignment statement x(E2):=E3,

$$(\text{if } [E1]_{E3}^{x(E2)} = E2 \text{ then E3 else } x([E1]_{E3}^{x(E2)}) \text{ endif}).d0$$

=   [Function application (.) distributes to all arguments.]

$$(\text{if } [E1]_{E3}^{x(E2)}.d0 = E2.d0 \text{ then E3.d0 else } x([E1]_{E3}^{x(E2)}).d0 \text{ endif})$$

=   [definition of the value of an array variable in a data environment]

$(\text{if } [E1]_{E3}^{x(E2)}.d0 = E2.d0 \text{ then } E3.d0 \text{ else } x([E1]_{E3}^{x(E2)}.d0).d0 \text{ endif})$

=   [equations 1 and 2 above]

$(\text{if } E1.d1 = E2.d0 \text{ then } x(E2.d0).d1 \text{ else } x(E1.d1).d0 \text{ endif})$

=   [equality in the then term, equation 3 above for the else term]

$(\text{if } E1.d1 = E2.d0 \text{ then } x(E1.d1).d1 \text{ else } x(E1.d1).d1 \text{ endif})$

=   [The then and else terms are the same.]

$x(E1.d1).d1$

=   [definition of the value of an array variable in a data environment]

$x(E1).d1$ ∎