Computer Science 1FC3

Lab 3 – Set Theory

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The purpose of this lab is to introduce the set theory maple commands.

SETS

A *set* is an unordered collection of objects that we usually denote by capital letters. The objects in a set are called its *elements*, or *members*, which are *contained* by the set. The notation $\{a, b, c, d, e\}$ represents the set of the five elements a, b, c, d and e. We use the symbols $\in$ and $\notin$ to indicate membership of an element in a set, for example $a \in \{a, b, c, d, e\}$ and $27 \notin \{a, b, c, d, e\}$. In maple we can do the following:

```
]>A:={a,b,c,d,e};
                              A:={a,b,c,d,e}
]>a in A;
                              a∈{a,b,c,d,e}
]>evalb(a in A);
                                  true
]>B:={seq(2*i,i=1..10);
                        B:={2,4,6,8,10,12,14,16,18,20}
]>evalb(1 in B);
                                  false
]>ES:={};
                                 ES:={}
```

Notice that $\{a, b, c, d, e\} = \{e, d, c, b, a\} = \{a, d, e, c, b\}$ in fact all permutations are equal. We say that two sets are equal if and only if they have the same elements, that is, $(A = B) \overset{def}{\leftrightarrow} \forall x(x \in A \leftrightarrow x \in B)$.

The ES in the above code stands for, *empty set*, which is the set with no elements. This set is denoted by $\varnothing$.

SET OPERATIONS

The set A is said to be a *subset* of B, denoted $A \subseteq B$, if and only if every element of A is also an element of B, that is, $A \subseteq B \overset{def}{\leftrightarrow} \forall x(x \in A \rightarrow x \in B)$. So, for example, $\{a, b\} \subseteq \{a, b, c, d, e\}$ but $\{a, b, f\} \not\subseteq \{a, b, c, d, e\}$. In Maple:

```
]>{a,b,c} subset A;
                                  true

]>{3,4,5} subset B;
                                  false
```

The *union* of A and B, denoted $A \cup B$ is the set that contains those elements that are either in A or B, or both. That is $A \cup B = \{x \mid x \in A \vee x \in B\}$ or alternatively $x \in A \cup B \overset{def}{\leftrightarrow} x \in A \vee x \in B$. For example $\{a, b\} \cup \{a, c, d\} = \{a, b, c, d\}$. In maple:

```
]>{a,b} union {a,c,d}
                                {a,b,c,d}
]>{a,b} union {1,2,3}
                                {a,b,1,2,3}
```

The *intersection* of A and B, denoted $A \cap B$ is the set containing those elements that are both in A and B. That is $A \cap B = \{x \mid x \in A \wedge x \in B\}$ or alternatively $x \in A \cap B \overset{def}{\leftrightarrow} \forall x(x \in A \wedge x \in B)$. For example $\{a, b\} \cap \{a, c, d\} = \{a\}$. In maple:

```
]>{a,b} intersect {a,c,d}
                                {a}
]>{a,b} intersect {1,2,3}
                                { }
```

The *set difference* of A and B, denoted $A \setminus B$ is the set containing those elements that are in A but not in B. That is $A \setminus B = \{x \mid x \in A \wedge x \notin B\}$ or alternatively $x \in A \setminus B \overset{def}{\leftrightarrow} \forall x(x \in A \wedge x \notin B)$. For example $\{a, b, c, d\} \setminus \{a, c, d\} = \{b\}$. In maple:

```
]>{a,b,c,d} minus {a,c,d}
                                {b}
]>{a,b} minus {a,b}
                                {}
```

POWER SET

Given a set A, the *power set* of S is the set of all subsets of the set S. The power set of *S* is denoted by *P(S)*. The set theory notation for power set is $P(S) = \{A \mid A \subseteq S\}$ or $X \in P(S) \overset{def}{\leftrightarrow} X \subseteq S$. In order to find the power set in maple it will be necessary to import the library that has that command. To import a library we use the `with` command as follows:

```
]>with(combinat,powersets):
]>powerset({1,2});
                         {{},{1},{1,2},{2}}
```

PROBLEMS

Question 1:

We say `A` is a *strict subset* of `B`, $A \subset B$, if $A \subseteq B$ and $A \neq B$.

a. Complete the definition

$$A \subset B \overset{def}{\leftrightarrow}$$

b. Complete the maple function

```
]>strictSubset:=(A,B)->evalb(                    );
```

c. Test your results

```
]>strictSubset({1,2},{1,2,3});
```

what it should be: what your function gives:

```
]>strictSubset({1,2,3},{1,2,3});
```

what it should be: what your function gives:

Question 2:

Complete the maple function

```
]>setEqual:=(A,B)->evalb(                    );
```

that tests the equality of two sets. *Hint* use `subset` twice.

Question 3:

The *symmetric difference* of `A` and `B`, denoted by $A \otimes B$ is the set containing those elements in either `A` or `B` **but not in both**.

a. Complete the maple function

```
]>symDiff:=(A->B)
```

b. Test your results

```
]>symDiff({1,3,5,7},{2,4,5,6});
```

what it should be: what your function gives:

```
]>symDiff({1,2,3,5,6,7},{2,4,5,6});
```

what it should be: what your function gives:

Question 4:

Let U be a universal set. The *complement* of a set `A`, denoted by $\overline{A}$, is the set $U \setminus A$.

a. Complete the maple function

```
]>comp:=(A,B)->
```

for

```
]>U:={seq(i,i=1..50);
```

use `comp` find the complement of the even numbers in this universe.

Question 5:

Under what conditions on `A` and `B` do the following statements hold?

a. $A \subseteq A \cup B$

b. $A \subseteq A \cap B$

c. $\varnothing \subseteq A$

d. $A \otimes B \subseteq A$

e. $P(A) \subseteq P(B)$

e. $A \subset A \cup B$