1. Consider the so-called Rosenbrock (banana) function:

\[100(x_2 - x_1^2)^2 + (1 - x_1)^2.\]

Is this function convex? Make surface and contour plots to find out. Use the definition of convexity or the equivalent condition in terms of derivatives to prove that it is convex or not convex.

2. Apply the Gradient Descent method to minimize the Rosenbrock banana function, using the initial guess,

\[x^0 = (-2, 1); \quad x^1 = (-1, 1); \quad x^2 = (-1, 2);\]

and making exact line searches, i.e. make a step size which gets as close to the minimum as possible. Perform 3 iterations.

3. Again with the band function, perform two steps of a Golden Section line-search starting at point \((-1, -1)\) with search direction \((1, 1)\) on step-length interval \([0, 1]\).

4. Which of the following are convex problems?

- \(\min x \cos(x)\)
- \(\min -\log(x) + x^2\)
- \(\min x \sqrt{x}\)
- \(\min -\log(x) - \log(10 - x)\)

5. In the general optimization strategy for unconstrained optimization, why do we need stopping criteria, and give three possible stopping criteria with a justification of the advantage of each.