CS and SWF ENG 4-6TD3: MidTerm

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THIS EXAMINATION PAPER INCLUDES **2** PAGES AND **4** QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.
DURATION OF THE EXAM: 2 hours

Special instructions:
You might use a letter-size sheet with your hand-written notes.
The use of the standard McMaster (Casio FX-991) calculator is allowed.

Questions:

1. Consider the functions

$$\begin{aligned} f(x_1,x_2) &= \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 - x_1x_2 \\ h(x_1,x_2,x_3) &= 4x_2^2 + 9x_3^2 - x_1^2 \end{aligned}$$

(a) Prove that $f(x)$ is convex, but not strictly convex.

(b) Demonstrate that the function $h(x)$ is not convex but the set $\{x \geq 0 | h(x) \leq 0\}$ is convex.

(c) Show that the set $\{x | h(x) \leq 0\}$ is a convex cone.

(d) Examine if the point $x = (0,0,0)^T$ is a local/global minimum of $h(x)$.

(e) *(i)* Give the gradient and the Hessian of $f(x_1,x_2)$.
(ii) Give second-order Taylor series expansion of $f(x_1,x_2)$ at $(1,2)^T$.

12p

2. Determine the limit point and the rate of convergence of the following sequences:

(a) $\alpha_k = 1 - \frac{1}{3^{8^k}}, \qquad k = 1,2,\cdots$

(b) $x^k = (1 + \frac{1}{2^k}, \frac{4}{k})^T, \qquad k = 1,2,\cdots$

6p

3. Consider the function $f(x_1,x_2) = e^{-4x_1} + (x_1-2)^2 + (x_2-1)^2$.

(a) Let $x^0 = (0,0)^T$. Apply one step of the gradient (steepest descent) method. Determine the Goldstein-Armijo step when $\mu_1 = \frac{5}{19}$, $\mu_2 = \frac{15}{19}$, $\rho = \frac{1}{2}$.

(b) Let $x^0 = (0,0)^T$. Apply a full Newton step and give x^1.

(c) Let $x^0 = (0,0)^T$. Calculate the Trust-Region search direction with the initial value $\alpha = 3$. Will this step be accepted in the Trust Region Algorithm or α should be changed.

(d) Give two reasons why it is advantageous to apply the Trust Region method compared to Newton's method.

10p

SEE THE OTHER SIDE FOR QUESTION 4!

4. Consider the Conjugate Gradient method:

(a) Let $f(x) : \mathbb{R}^n \to \mathbb{R}$ be a function and H be an $n \times n$ symmetric matrix. When is the the direction $s = H\nabla f(x)$ is a descent direction of the function $f(x)$?

(b) Consider the vectors

$u = (1, 2, 3)^T$ and $v = (2, 3, 3)^T$ and the matrix $A = \begin{pmatrix} \gamma & -1 & 2\gamma \\ -1 & \gamma & -4 \\ 2\gamma & -4 & 4\gamma \end{pmatrix}$.

For what γ value are the vectors u and v conjugate w.r.t. the matrix A.

(c) Consider the quadratic function

$$q(x_1, x_2) = x_1^2 + 2x_2^2 + x_1 x_2 - x_1 - 2x_2$$

Apply the conjugate gradient algorithm of Fletcher and Reeves to find the minimum of this function starting from the point $x^0 = (0, 0)^T$.

(d) Explain the Wolfe line-search rule in Quasi-Newton methods. Why we need to use the Wolfe line-search rule in the DFP and BFGS Quasi-Newton method?

12p

THE END