CS and SWF ENG 4-6TE3: MidTerm

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THIS EXAMINATION PAPER INCLUDES **2** PAGES AND **5** QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.
DURATION OF THE EXAM: 2 hours

Special instructions:
You might use a letter-size sheet with your hand-written notes.
The use of the standard McMaster (Casio FX-991) calculator is allowed.

Questions:

1. Consider the functions

$$\begin{aligned} f(x_1,x_2) &= -2x_1^2+3x_1^4+e^{4x_1}+e^{x_1-x_2} \\ h(x_1,x_2,x_3) &= 2(x_1+x_2+x_3)^2\log|x_1+x_2+x_3| \end{aligned}$$

   (a) (*i*) Give the gradient and the Hessian of $f(x_1,x_2)$.
   (*ii*) Give second-order Taylor series expansion of $f(x_1,x_2)$ at $(1,1)^T$.

   (b) Prove that $f(x)$ is strictly convex.

   (c) Demonstrate that the function $h(x)$ is not convex
   but the set $\{x|h(x)\le 0;\ x_1+x_2+x_3>0\}$ is convex.

   (d) Examine if the point $x=(0,e^{0.5},0)^T$ is a local/global minimum of $h(x)$.

12p

2. Determine the limit point and the rate of convergence of the following sequences:

   (a) $\alpha_k=\frac{1}{3^{8^k}}$, $\quad k=1,2,\cdots$

   (b) $x^k=(\frac{1}{k!},\frac{4}{k^2})^T$, $\quad k=1,2,\cdots$

6p

3. Consider the function $f(x_1,x_2)=e^{x_1+x_2-2}+(x_1-x_2)^2$.

   (a) Let $x^0=(1,1)^T$. Apply a full Newton step and give x^1.

   (b) Let $x^0=(1,1)^T$. Calculate the Trust-Region search direction with the initial value $\alpha=1$. Let choose $\mu=0.2$, $\eta=0.8$, $\gamma_1=0.5$, $\gamma_2=2.5$. Would you accept this step in the Trust Region Algorithm or α should be changed.

   (c) Compare the Steepest Descent, Newton's and the Trust Region methods w.r.t. applicability, computational cost and convergence.

8p

SEE THE OTHER SIDE FOR QUESTION 4 and 5!

4. Conjugate directions, QN-methods.

(a) Describe the Fletcher-Reeves conjugate gradient method applied to a convex quadratic function $q(x) : \mathbb{R}^n \to \mathbb{R}$.

(b) Prove that the first two generated search directions are conjugate

6p

5. Descent and Conjugate directions:

(a) Let $f(x) : \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable function and

$$H := I - \nabla f(x) \nabla f(x)^T$$

be a given $n \times n$ symmetric matrix.
When is the direction $s = H \nabla f(x)$ a descent direction of the function $f(x)$?

(b) Consider the matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 8 & -2 \\ 0 & -2 & 2 \end{pmatrix}$.
Give a conjugate basis in $\mathbb{R}^3$, i.e., give three conjugate vectors w.r.t. the matrix A.

(c) Explain why we need to apply the the Wolfe line-search rule in Quasi-Newton methods?

(d) Does Wolfe's line-search warrantee positive definiteness of the matrix H if Broyden's symmetric-rank-one update is used?

8p

THE END