COMP SCI and SFWR ENG 4-6TE3: MidTerm

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THIS EXAMINATION PAPER INCLUDES **2** PAGES AND **5** QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.
DURATION OF THE EXAM: 2 hours
Special instructions:
You might use a letter-size sheet with your hand-written notes.
The use of the standard McMaster (Casio FX-991) calculator is allowed.

Questions:

1. Consider the function

$$f(x_1, x_2) \quad = \quad e^{((x_1+x_2)^2)} + (x_1 + 2x_2)^3$$

   (a) (i) Give the gradient and the Hessian of $f(x_1, x_2)$.
   (ii) Give the second-order Taylor series expansion of $f(x_1, x_2)$ at the point $x^0 = (2, -1)^T$.

   (b) Prove that the function $f(x)$ is not convex.

   (c) Make a step with Newton's method from the point $x^0 = (-1, 1)^T$ when you minimize the function $f(x)$.

   (d) Examine if the point $x = (-2, 1)^T$ is a local/global minimum of $f(x)$ by using its gradient/Hessian information.

   (e) Give the directional derivative of the function $f(x)$ at the point $(2, -1)^T$ in the direction $(2, 1)^T$. Examine if this direction is a descent or an ascent direction.

10p

2. Give a sequence that converges to 2 with order $\sqrt{2}$ and prove its order of convergence.

3p

3. Consider the function $f(x_1, x_2) = \left(\frac{2x_1 - x_2}{x_1^2 + 1}\right)^2$.

   (a) Let $x^0 = (2, 0)^T$. Apply one cycle of the pattern search algorithm to minimize the function when the search directions are given as $(-1, 1)^T$ and $(1, 1)^T$. Further, let the damping factor be $\alpha = 0.5$.

   (b) Make one step of the Goldstein-Armijo line-search (one function evaluation) from the point $(2, 0)^T$ in the direction $(1, -1)^T$, when $\mu_1 = 0.1$, $\mu_2 = 0.9$ and $\alpha_0 = 0.8$. Let the multiplication factor be 2, if the step-length has to be increased; 0.5 if it has to be decreased. Describe what action is taken at the end of the step based on the result of your computations.

8p

SEE THE OTHER SIDE FOR MORE QUESTIONS!

4. Which of the following statements is true/false.
   Give a one-sentence justification of your answer.
   (a) Every monotone non-decreasing function is convex.
   (b) The sum of two convex functions is convex.
   (c) The cost of one iteration of the Trust Region algorithm is $O(n^3)$ arithmetic operations.
   (d) The Trust Region algorithms alternately uses gradient and Newton steps.
   (e) The Hessian of a convex functions has only nonpositive eigenvalues.
   (f) The union of convex sets is convex.
   (g) Newton's method (for zero finding in 1-dimension) does not converge quadratically to find the root of the function x^2.
   (h) The function $\log(|x|)$ is a convex function.
   (i) The relative interior of the set consisting of the single point $(0,0,0)^T$ in $\mathbb{R}^3$ is nonempty.
   (j) A Superlinear convergence is faster than quadratic convergence.

10p

5. Prove that if the functions $f(x), g(x) : \mathbb{R}^n \to \mathbb{R}$ are convex, then $h(x) = \max\{f(x), g(x)\}$ is a convex function too.

4p

THE END