

Conjugate Gradient Method on Non-Quadratic Function

Example: Minimize $f(x) = e^{x_1-1} + e^{-x_2+1} + (x_1 - x_2)^2$ starting from $x^1 = (0, 0)^T$.

Solution:

$$\nabla f(x) = \begin{pmatrix} e^{x_1-1} + 2(x_1 - x_2) \\ -e^{-x_2+1} - 2(x_1 - x_2) \end{pmatrix}$$

Step 1:

$$\text{Initial point: } x^1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$g^1 = \nabla f(x^1) = \begin{pmatrix} 0.3679 \\ -2.7183 \end{pmatrix},$$

$$s^1 = -g^1 = \begin{pmatrix} -0.3679 \\ 2.7183 \end{pmatrix},$$

We use Goldstein-Armijo line search to find $x^2 = \arg \min(x^1 + \lambda_1 s^1)$:

$$x^2 = \begin{pmatrix} -0.0862 \\ 0.6371 \end{pmatrix}.$$

Step 2:

$$g^2 = \nabla f(x^2) = \begin{pmatrix} -1.1091 \\ 0.6371 \end{pmatrix},$$

$$\beta_{21} = \frac{\|g^2\|^2}{\|g^1\|^2} = 0.1635,$$

$$s^2 = -g^2 + \beta_{21}s^1 = \begin{pmatrix} 1.0489 \\ -0.1927 \end{pmatrix},$$

Use Goldstein-Armijo line search to find $x^3 = \arg \min(x^2 + \lambda_2 s^2)$.

Step 3: ...