**Multivariate polynomial interpolation**

**Example:** Give the interpolation polynomial $Q(x)$ of the function $f(x) = -x_1 + x_2 + \exp(x_1 + x_2)$ through the points $x^1 = (0.5, -0.5)^T$, $x^2 = (0, 0)^T$, $x^3 = (0.5, 0)^T$ by using the basis functions $\phi_1(x) = 1$, $\phi_2(x) = x_1 x_2$ and $\phi_3(x) = x_1 + x_2$.

Write the interpolation polynomial as

$$Q(x) = a_1 + a_2 x_1 x_2 + a_3 (x_1 + x_2)$$

Evaluate $f(x)$ at the interpolation points:

$$f(x^1) = -1 + e^0 = 0$$
$$f(x^2) = e^0 = 1$$
$$f(x^3) = -0.5 + e^{0.5} = 1.1487$$

$Q(x)$ should have the same value as $f(x)$ at the interpolation points: So,

$$Q(x^1) = a_1 - 0.25 a_2 = 0 = f(x^1)$$
$$Q(x^2) = a_1 = 1 = f(x^2)$$
$$Q(x^3) = a_1 + 0.5 a_3 = 1.1487 = f(x^3)$$

$\implies a_1 = 1$, $a_2 = 4$, $a_3 = 0.2974$. The interpolation polynomial is:

$$Q(x) = 1 + 4x_1 x_2 + 0.2974(x_1 + x_2)$$