

- *Multivariate polynomial interpolation*

**Example:** Give the interpolation polynomial  $Q(x)$  of the function  $f(x) = -x_1 + x_2 + \exp(x_1 + x_2)$  through the points  $x^1 = (0.5, -0.5)^T$ ,  $x^2 = (0, 0)^T$ ,  $x^3 = (0.5, 0)^T$  by using the basis functions  $\phi_1(x) = 1$ ,  $\phi_2(x) = x_1x_2$  and  $\phi_3(x) = x_1 + x_2$ .

Write the interpolation polynomial as

$$Q(x) = a_1 + a_2x_1x_2 + a_3(x_1 + x_2)$$

Evaluate  $f(x)$  at the interpolation points:

$$\begin{aligned}f(x^1) &= -1 + e^0 = 0 \\f(x^2) &= e^0 = 1 \\f(x^3) &= -0.5 + e^{0.5} = 1.1487\end{aligned}$$

$Q(x)$  should have the same value as  $f(x)$  at the interpolation points: So,

$$\begin{aligned}Q(x^1) &= a_1 - 0.25a_2 = 0 = f(x^1) \\Q(x^2) &= a_1 = 1 = f(x^2) \\Q(x^3) &= a_1 + 0.5a_3 = 1.1487 = f(x^3)\end{aligned}$$

$\implies a_1 = 1$ ,  $a_2 = 4$ ,  $a_3 = 0.2974$ . The interpolation polynomial is:

$$Q(x) = 1 + 4x_1x_2 + 0.2974(x_1 + x_2)$$