# Canada-Japan Workshop on Discrete and Computational Geometry 

July 13-15, 2009<br>Sanjo Conference Hall, University of Tokyo, Japan

Organizers: Hiroshi Imai, Sonoko Moriyama, David Bremner, Antoine Deza



This workshop intends to celebrate and encourage international research collaboration in the area of discrete and computational geometry, particularly between Canada and Japan. The workshop features talks on theoretical and applied aspects of discrete geometry, with an emphasis on computational questions, and computational approaches to mathematics. Topics of particular interest include geometric and combinatorial objects such as matroids, oriented matroids, arrangements of hyperplanes, and convex polyhedra, as well as geometric aspects of optimization.

## Program

## Monday, July 13

| 09h30-10h00 | Welcome reception and opening remarks |  |
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| 10h00-12h00 | Session A1 |  |
|  | A11 Pivoting in linear complementarity: two polynomial-time cases Komei Fukuda (ETH Zurich) | 4 |
|  | A12 Decomposition of multiple coverings into more parts | 5 |
|  | Sébastien Collette (Université Libre de Bruxelles) |  |
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|  | Kokichi Sugihara (Meiji University) |  |
| 12h00-14h00 | Lunch break |  |
| 14h00-15h00 | Session A2 |  |
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|  | A22 More bounds on the diameter of convex polytopes | 8 |
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|  | A23 Realizations of oriented matroids by extended solvability sequence method | 9 |
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| 15h30-17h30 | Session A3 |  |
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|  | Naoki Katoh (Kyoto University) |  |
|  | A32 Connectivity augmentation in planar straight line graphs | 11 |
|  | Csaba Toth (University of Calgary) |  |
|  | A33 Pinning a line by geometric objects | 12 |
|  | Xavier Goaoc (INRIA Nancy) |  |
| Tuesday, July 14 |  |  |
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|  | B12 Minkowski sums of polytopes: bounds on $f$-vectors | 14 |
|  | Christophe Weibel (McGill University) |  |
|  | B13 A cluster pattern approach for improved carpet encoding of surfaces <br> Kamen Kanev (Shizuoka University) | 15 |


| 12h00 | Workshop group photo |  |
| :---: | :---: | :---: |
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| 14h00-15h00 | Session B2 |  |
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|  | B22 On extensions of cyclic arrangements | 17 |
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|  | B23 Round-Tour Voronoi Diagram: A New Generalization of the Voronoi Diagram | 18 |
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|  | B31 Empty pentagons in degenerate point sets | 19 |
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|  | Yoshio Okamoto (Tokyo Institute of Technology) |  |
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|  | Christian Knauer (Free University Berlin) |  |
| 18h00 | Banquet at Innsyoutei |  |
| Wednesday, July 15 |  |  |
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| 10h00-12h00 | Session C1 |  |
|  | C11 Constant working space algorithms for geometric problems Tetsuo Asano (JAIST) | 22 |
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|  | Kazuhisa Makino (University of Tokyo) |  |
| 12h00-14h00 | Lunch break |  |
| 14h00-15h20 | Session C2 |  |
|  | C21 History based pivot rules and unique sink orientations David Avis (McGill University) | 25 |
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# Pivoting in Linear Complementarity: Two Polynomial-Time Cases 

Komei Fukuda<br>Institute for Operations Research and Institute of Theoretical Computer Science<br>ETH Zurich, Switzerland

We study the behavior of simple principal pivoting methods for the P-matrix linear complementarity problem (P-LCP). We solve an open problem of Morris by showing that Murty least-index pivot rule (under any fixed index order) leads to a quadratic number of iterations on Morris highly cyclic P-LCP examples. We then show that on K-matrix LCP instances, all pivot rules require only a linear number of iterations. As the main tool, we employ unique-sink orientations of cubes, a useful combinatorial abstraction of P-LCP.
(Joint work with Jan Foniok, Bernd Gaertner and Hans-Jakob Luethi)

# Decomposition of Multiple Coverings into More Parts 

Greg Aloupis ${ }^{1}$ Jean Cardinal ${ }^{1}$ Sébastien Collette ${ }^{1}$ Stefan Langerman ${ }^{1}$
David Orden ${ }^{2}$ Pedro Ramos ${ }^{2}$
1 Université Libre de Bruxelles (ULB), CP212, Bld. du Triomphe, 1050 Bruxelles, Belgium.
(Supported by the Communauté Francaise de Belgique.)
2 Universidad de Alcalá, Spain. (Supported by grant MTM2008-04699-C03-02.)
We prove that for every centrally symmetric convex polygon $Q$, there exists a constant $\alpha$ such that any $\alpha \mathrm{k}$-fold covering of the plane by translates of $Q$ can be decomposed into $k$ coverings. This improves on a quadratic upper bound proved by Pach and Tòth (SoCG’07). The question is motivated by a sensor network problem, in which a region has to be monitored by sensors with limited battery life.

This work has been accepted to the ACM-SIAM Symposium on Discrete Algorithms (SODA'09).

# Selective Perturbation in Geometric Computation 

Kokichi Sugihara<br>Meiji Institute for Advanced Study of Mathematical Sciences<br>Meiji University, Kawawaki 214-8571, Japan.<br>kokichis@isc.meiji.ac.jp

Degeneracy is one of the most serious issues in practical implementation of geometric algorithms, because it requires exceptional branches of processing and those branches are usually very huge and complicated. Fortunately, however, there is a powerful technique called "symbolic perturbation" to cope with degeneracy. In this technique, we perturb the geometric problem using polynomials of an indeterminate symbol, which intuitively corresponds to an infinitesimally small positive number, and thus remove degeneracy. This technique is usually used with exact computation, and actually makes software engineers free from tedious work to treat degenerate cases.

However, symbolic perturbation is not almighty. Indeed, it removes all kinds of degeneracy automatically, although we often want to treat some of degenerate cases as they are.

For example, consider the problem of constructing the arrangement of given $n$ lines in the plane. Assume that all the lines share a common point of intersection. Then, the total number of intersection is one, which gives a very simple arrangement. On the other hand, if we perturb the input lines, every pair of lines generates a distinct point of intersection, and the total number of increases the complexity of the geometric object.

Another example is found in mesh generation for the finite element method. Suppose that we want to divide a three-dimensional region into tetrahedra with given vertices. The Delaunay tetrahedrization is one of standard methods for this purpose, because it can avoid thin tetrahedra in many cases. Assume that four vertices are on a common plane and at the same time on a common circle, which means a degenerate case for the Delaunay tetrahedrization. If we perturb the vertices, those four points come to general position and hence can form a Delaunay tetrahedron. However, this kind of a thin tetrahedron should not appear in the finite element method, because it causes large numerical errors in solving partial differential equations. Thus we do not want to perturb those four vertices, although we want to perturb other kinds of degenerate configurations. We can find many other examples of similar unwanted side effects causes by perturbation.

So, it is important to establish some method for selective perturbation, by which we could remove only those degenerates cases that we want to remove and leave other degenerate cases as they are. In this presentation we consider a general framework by which we can perturb degeneracy in such a selective manner.

We already succeeded in selective perturbation for some individual geometric problems such as Delaunay tetrahedrization of a three-dimensional region and collision detection in electric-wire layout. Our solutions are rather heuristic, but we can observe a common property, that is, selective perturbation can be achieved by perturbing the problem type rather than perturbing numerical data in the problem. We would like to name this kind of perturbation "hyper-perturbation", and consider whether it can be extended to a general framework for selective perturbation.

This work is partly supported by the Grant-in-Aid for Exploratory Research No. 19650003 of the Japanese Society for Promotion of Science.

## Highway Insertion under the Generalized City Metric

Matias Korman

Imagine that we travel in such a city starting from $p$ towards a destination $q$ : we normally should not go along the geometric shortest path, which is the straight line segment $p q$. In a city that has some kind of transportation facility that speeds up the travel time of its passengers we normally take detours (to ride a train, bus, etc . . . ). Another factor that influences our paths is the presence of obstacles such as large buildings: any reasonable path must avoid obstacles, and thus we are interested in taking the smallest detour possible to reach our destination. We consider the following problem derivated from urban planning: given two sets of points $\mathcal{S}$ and $\mathcal{T}$ called sources and sinks respectively, a set of isothetic (i.e: horizontal or vertical) segments $\mathcal{H}$ called highways and a set of polygons $\mathcal{O}$ called obstacles, we locate another highway $h$ that minimizes an objective function related to the generalized city distance between sources and sinks, considering that we can travel $v$ times faster on a highway and that paths must avoid obstacles. In Figure 1 we can see a sample problem with two sources (white points), connected to the sink (dark points). In the example, there are three highways and one obstacle (grey region). In Figure 2, a new highway has been located and the shortest paths have been updated accordingly.

In this talk we will discuss and summarize the latest highway location algorithms.


Figure 1: Example of generalized city metric


Figure 2: Updated diameter once $h$ is inserted

# More bounds on the diameter of convex polytopes 

William Hua,<br>McMaster University Hamilton, Ontario, Canada.

Let $\Delta(d, n)$ be the maximum possible edge diameter over all polytopes defined by $n$ inequalities in dimension $d$. The conjecture of Hirsch, formulated in 1957, states that $\Delta(d, n)$ is not greater than $n-d$. No polynomial bound is known for $\Delta(d, n)$, the best one being quasipolynomial and due to Kalai and Kleitman in 1992. Goodey showed in 1972 that $\Delta(4,10)=5$ and $\Delta(5,11)=6$. Recently, Bremner and Schewe proved that $\Delta(4,11)=\Delta(6,12)=6$. In this follow-up talk, we show that $\Delta(4,12)=7$ and present evidence that $\Delta(5,12)=\Delta(6,13)=7$.
based on joint-work with David Bremner, University of New Brunswick, Antoine Deza, McMaster University, and Lars Schewe, TU Darmstadt.

# Realizations of oriented matroids by extended solvability sequence method 

Hiroyuki Miyata<br>University of Tokyo

We consider the realizability problem of oriented matroids, that is to decide whether a given oriented matroid is representable by a real hyperplane arrangement. This problem is known to be equivalent to the existential theory of the reals and thus NP-hard. On the other hand, there are some sufficient conditions that are easy to check for realizability or nonrealizability of oriented matroids. In this talk, we propose a new realizability certificate of oriented matroids, which can be viewed as an extension of the existing realizability certificate called solvability sequence method.

We show the effectiveness of the method by applying it to the classification of rank-4 oriented matroids of eight elements. Among 5321 such oriented matroids which could not be classified before out of the grand total of 181472 cases, we managed to recognize 5020 realizable oriented matroids. The number of unclassified oriented matroids in this class is now 301.

This is joint work with Sonoko Moriyama and Komei Fukuda.

# A Proof of the Molecular Conjecture 

Naoki Katoh<br>Kyoto University

A $d$-dimensional body-and-hinge framework is a structure consisting of rigid bodies connected by hinges in $d$-dimensional space. The generic infinitesimal rigidity of a body-and-hinge framework has been characterized in terms of the underlying multigraph independently by Tay and Whiteley as follows: A multigraph $G$ can be realized as an infinitesimally rigid body-and-hinge framework by mapping each vertex to a body and each edge to a hinge if and only if $\left(\binom{d+1}{2}-1\right) G$ contains $\binom{d+1}{2}$ edge-disjoint spanning trees, where $\left(\binom{d+1}{2}-1\right) G$ is the graph obtained from $G$ by replacing each edge by $\left.\binom{d+1}{2}-1\right)$ parallel edges. In 1984 they jointly posed a question about whether their combinatorial characterization can be further applied to a nongeneric case. Specifically, they conjectured that $G$ can be realized as an infinitesimally rigid body-and-hinge framework if and only if $G$ can be realized as that with the additional "hinge-coplanar" property, i.e., all the hinges incident to each body are contained in a common hyperplane. This conjecture is called the Molecular Conjecture due to the equivalence between the infinitesimal rigidity of "hinge-coplanar" body-and-hinge frameworks and that of bar-and-joint frameworks derived from molecules in 3-dimension. In 2-dimensional case this conjecture has been proved by Jackson and Jordán in 2006. In this paper we prove this long standing conjecture affirmatively for general dimension. Also, as a corollary, we obtain a combinatorial characterization of the 3-dimensional bar-and-joint rigidity matroid of the square of a graph.

# Connectivity augmentation in planar straight line graphs 

Csaba Toth<br>University of Calgary

The connectivity augmentation problems for abstract graphs have a long history and many application. They have polynomial time solutions. A planar straight line graph (PSLG) is a planar graph $G(V, E)$ together with a straight line embedding in the plane. In the embedding preserving connectivity augmentation problem, we are given a PSLG and an integer k , and want to find a smallest set $F$ of edges such that $G^{\prime}(V, E \cup F)$ is $k$-connected (or $k$-edge connected) PSLG, and the embedding of $E$ is preserved in $G^{\prime}$. This problem is NP-hard already for $k=2$. It is more restrictive than the planarity preserving connectivity augmentation, in which the input and output graphs have to be planar but the input does not come with an actual plane embedding.

This talk presents combinatorial bounds on the number of new edges necessary for the embedding preserving connectivity augmentation problem for $k=2$ and $k=3$. We characterize the PLSGs that have embedding preserving augmentations to a 3 -connected and, respectively, 3-edge connected PLSG. Tight bounds are given on the minimum number of new edges sufficient for augmenting any PLSG with $n$ vertices to a 2 - or 3-edge connected PSLG, if such an augmentation is possible.

## Pinning a line by geometric objects

Xavier Goaoc<br>INRIA Nancy

A line is pinned by a family of geometric objects if (i) it intersects every member of the family and (ii) any perturbation of the line misses at least one member of the family. Pinnings of lines naturally arise in various areas of discrete and computational geometry: geometric optimization, grasping and fixturing, geometric transversal theory... I will discuss various Helly-type theorems related to pinning.

# Distance $k$-sectors and zone diagrams 

Takeshi Tokuyama
Tohoku University
We prove the existence and uniqueness of the zone diagram of a given set of sites in Euclidean space. This was known for point sites in the plane, but our proof is simpler even for this specific case. We also show the existence of a distance $k$-sector between two sites. Both proofs rely on the Knaster-Tarski theorem on fixed points of monotone functions. We also discuss the property of metric spaces so that the theory can be extended.

This is a joint work with J. Matousek, A. Kawamura, K. Imai, and Y. Muramatsu.

# Minkowski sums of polytopes: bounds on $f$-vectors 

Christophe Weibel<br>McGill University

Minkowski sums of polytopes is a simple operation with various interesting properties. The study of the $f$-vector of sums with relation to that of their summands is of particular interest. We present in this talk some recent results on the number of faces of Minkowski sums of polytopes, as well as some conjectures and open problems. In particular, we prove the exact maximal number of faces for sums of any number of 3-dimensional polytopes, and we present a conjecture extending this result to higher dimensions.

# A Cluster Pattern Approach for Improved Carpet Encoding of Surfaces 

Valentin E. Brimkov ${ }^{1}$ Kamen D. Kanev ${ }^{2}$ Reneta P. Barneva ${ }^{3}$<br>1 SUNY Buffalo State College, USA.<br>2 University of Shizuoka, Japan.<br>3 SUNY Fredonia, USA.

Since the invention of computer mouse point-and-click functionality has been widely adopted, in particular, for work with images and GUIs. In this interaction model the user employs a relative-displacement pointing device with a cursor feedback and clickable buttons invoking actions. It would be useful to provide similar functionality for printed materials, such as books, handouts, and labels, as well. The idea is to have direct point-and-click functionality that does not require feedback on the computer screen or tablet-like devices, but rather uses an embedded marking which defines a coordinate system within the printed document. In this presentation we will introduce an original approach and related technology for building direct point-and-click interfaces, known as CLUSPI (Cluster Pattern Interface). It is based on an unobtrusive image layer defining a coordinate system, a camera-based pointing device, and decoding software. CLUSPI technology was co-invented and patented by one of the current authors and here we present the theoretical (mathematical) foundations of the developed methods. We also describe some theoretical challenges and propose solutions to them.

# On the infinitesimal rigidity of bar-and-slider frameworks 

Shin-ichi Tanigawa<br>Kyoto University

A bar-and-slider framework is a bar-and-joint framework whose some joints are connected by line-sliders and each joint is allowed to move only along the lines. Streinu and Theran (SoCG 2008) proposed a combinatorial characterization of the infinitesimal rigidity of generic bar-and-slider frameworks in two dimension. In this talk we propose a generalization of their result such that the direction of each slider is predetermined. In particular, we prove that, even thought the directions of the sliders are degenerate, i.e., some sliders may have a same direction, it can be combinatorially decidable whether the framework is infinitesimally rigid or not.

# On extensions of cyclic arrangements 

Feng Xie<br>McMaster University Hamilton, Ontario, Canada.

The average diameter of a bounded cell of a simple arrangement is conjectured to be not greater than the dimension. This conjecture relates to the average case performance of the simplex method of linear programming. Preliminary results indicate that the arrangements maximizing the average diameter are obtained by adding to the cyclic arrangements one hyperplane with the vertices of the cyclic arrangement on the same side of the added hyperplane. The combinatorics of the addition of a pseudo-hyperplane to the cyclic arrangement are well-studied and correspond to higher Bruhat orders. In this talk, we present a computational framework to investigate these special extensions of the cyclic arrangement, and some preliminary results.

Joint work with Antoine Deza, McMaster University

# Round-Tour Voronoi Diagram: A New Generalization of the Voronoi Diagram 

Hidenori Fujii<br>Department of Mathematical Informatics, University of Tokyo

The Voronoi diagram is one of the most fundamental concepts in computational geometry because of its useful generalizations and applications. Generalization of the Voronoi diagram can be classified in three groups.

The first group comprises generalizations of the distance. The Euclidean distance, which is used for the ordinary Voronoi diagram, can be replaced by the $L_{p}$ distance, the collisionavoidance distance, the power distance, the weighted distance, or the boat-sail distance, to mention a few.

The second group comprises generalizations of the underlying space. The ordinary Voronoi diagram is in Euclidean space; however, the space can be replaced by a spherical surface, polygonal surface, or network.

The third group comprises generalizations of the generators. The ordinary Voronoi diagram is defined for points; however, they can be replaced by general figures such as circles, line segments, and polygons, or replaced by subsets of points in higher-order Voronoi diagrams.

In this paper we proposes a new generalization of the Voronoi diagram. Suppose that restaurants and bookstores are located in a city, and we want to visit both a restaurant and bookstore and return to our house. To each pair of a restaurant and a bookstore, we can assign a region such that a resident in this region can visit the restaurant and bookstore in a round tour that is shorter than that for a visit to any other pair. The city is partitioned into these regions according to which pair of a restaurant and bookstore permits the shortest round tour. We call this partitioning a "round-tour Voronoi Diagram" for the restaurants and bookstores. We study the basic properties of this Voronoi diagram and consider an efficient algorithm for its approximate construction.

## Empty pentagons in degenerate point sets

Stefan Langerman<br>Université Libre de Bruxelles

We prove the following generalized empty pentagon theorem: for every integer $\ell \geq 2$, every sufficiently large set of points in the plane contains $\ell$ collinear points or an empty pentagon.

Joint work with Zachary Abel, Brad Ballinger, Prosenjit Bose, Sébastien Collette, Vida Dujmovic, Ferran Hurtado, Scott D. Kominers, and Attila Por.

# Discrete Geometry of multi-criteria optimization problems 

Yoshio Okamoto<br>Tokyo Institute of Technology

This talk will discuss the benefit of a viewpoint from discrete and computational geometry for multi-criteria optimization. In particular, the connections between some multi-criteria optimization problems and techniques from geometry are presented. The first one is an application of the reverse search technique proposed by Avis and Fukuda to enumerating all extremal efficient solutions to multi-criteria linear programming. The second one is an application of a lattice-point counting algorithm by Barvinok and Woods to enumerating all pareto-optimal solutions to multi-criteria integer programming.

# Parameterized complexity of geometric problems 

Christian Knauer<br>Free University Berlin

Parameterized complexity aims to design exact algorithms whose running times depend on certain parameters of the input data that are naturally related to the problem at hand and in a way capture its complexity. A problem is called fixed-parameter tractable (FPT) with respect to a parameter $k$ if there is an efficient algorithm to solve the problem for the cases where the parameter $k$ is small. Another objective of this theory is to show that such algorithms are unlikely to exist for certain problems (and parameters).

Not many geometric problems have been studied from the parameterized complexity point of view. Most research has focused on special (combinatorial) parameters for geometric problems, like, e.g., the number of inner points (i.e., points in the interior of the convex hull) for the TSP problem or for the problem of computing minimum convex decompositions. Also, on the negative side, only few connections between geometric problems and known hard parameterized problems are known to date.

# Constant working space algorithms for geometric problems 

Tetsuo Asano
JAIST, Japan
A main interest in this talk is how to implement algorithms using only constant working space. More formally, it uses only constant amount of working space in addition to a read-only array storing input data and outputs are put on a write-only array. Such algorithms have been studied in the community of complexity theory under the name of log-space computation. In our case we are more interested in efficient implementation of those algorithms. In this talk I will describe some constant working space algorithms for several geometric problems, such as those of computing convex hull, triangulation, Voronoi diagrams, and minimum spanning tree for a set of points in the plane.

# The duality between maximum margin separation and norm minimization 

David Bremner<br>University of New Brunswick

The duality between the problem finding the closest pair of points, one in each set, and the problem of finding the widest slab separating the two sets is well known in theory and widely applied in practice. In this talk I discuss generalizing this duality to the inseparable case, and to general Minkowski metrics. The work described is joint with Peter Gritzmann, TU Muenchen.

# Computational Aspects of Monotone Dualization 

Kazuhisa Makino<br>University of Tokyo

Dualization of a monotone Boolean function represented by a conjunctive normal form (CNF) is a problem which, in different disguise, is ubiquitous in many areas including Computer Science, Artificial Intelligence, and Game Theory to mention some of them. It is also one of the few problems whose precise tractability status (in terms of polynomial-time solvability) is still unknown, and now open for more than 25 years. In this talk, we briefly survey computational results for the monotone dualization problem, and show that several generation problems such as 1) generating all minimal integer solutions to a monotone system of linear inequalities, 2) generating all maximal hyper-rectangles which contain a specified fraction of points of a given set in $R^{n}$ and 3) generating all inefficient points of a given discrete probability distribution, are polynomially equivalent to the monotone dualization problem, and can be solved in incremental quasi-polynomial time.

# History based pivot rules and unique sink orientations 

David Avis<br>McGill University

In 1980, Zadeh proposed the least entered rule for pivot selection in the simplex method : enter the improving variable that has been entered least often. He also offered $\$ 1000$ "to the first person who can find a counterexample to the least entered rule, or prove it to be polynomial" - an offer that still stands. Zadeh's history based rule is designed to defeat Klee-Minty type examples, based on deformed products, that cause other pivot rules to use an exponential number of pivots.

We study Zadeh's rule, and other similar history based rules, on acyclic unique source and sink orientations (USOs). USOs are acyclic orientations of the hypercube with the property that every face has exactly one source and sink. For dimensions up to seven, we construct all acyclic USOs where Zadeh's rule can be made to follow a Hamiltonian path. The number of such examples grows very rapidly in the dimension. We also give results on two other history based rules: the least used direction rule and the least time in the basis rule.

Joint work with Sonoko Moriyama and Yoshitake Matsumoto.

# A continuous $d$-step conjecture for polytopes 

Antoine Deza
McMaster University Hamilton, Ontario, Canada.
The curvature of a polytope, defined as the largest possible total curvature of the associated central path, can be regarded as the continuous analogue of its diameter. We prove the analogue of the result of Klee and Walkup. Namely, we show that if the order of the curvature is less than the dimension $d$ for all polytope defined by $2 d$ inequalities and for all $d$, then the order of the curvature is less that the number of inequalities for all polytopes.

Joint work with Yuriy Zinchenko (University of Calgary) and Tamás Terlaky (Lehigh University).

## Participants

| Tetsuo Asano | JAIST | t-asano@jaist.ac.jp |
| :---: | :---: | :---: |
| David Avis | McGill University | avis@cs.mcgill.ca |
| David Bremner | University of New Brunswick | bremner@unb.ca |
| Chun-Pi Chang | Polytechnic Insitute of NYU | cchang05@students.poly.edu |
| Jinhee Chun | Tohoku University | jinhee@dais.is.tohoku.ac.jp |
| Sébastien Collette | Université Libre de Bruxelles | sebastien.collette@ulb.ac.be |
| Antoine Deza | McMaster University | deza@mcmaster.ca |
| Norie Fu | University of Tokyo | f_norie@is.s.u-tokyo.ac.jp |
| Hidenori Fujii | University of Tokyo | fujii@ipl.t.u-tokyo.ac.jp |
| Yoshinori Fujii | University of Tokyo | yoshinori_fujii@mist.i.u-tokyo.ac.jp |
| Komei Fukuda | ETH Zurich | fukuda@ifor.math.ethz.ch |
| Xavier Goaoc | INRIA | goaoc@loria.fr |
| Xin Han | University of Tokyo | xhan@mist.i.u-tokyo.ac.jp |
| Takuya Hara | University of Tokyo | takuya_hara@mist.i.u-tokyo.ac.jp |
| William Hua | McMaster University | huaw@mcmaster.ca |
| Shinji Imahori | The University of Tokyo | imahori@mist.i.u-tokyo.ac.jp |
| Hiroshi Imai | University of Tokyo | imai@is.s.u-tokyo.ac.jp |
| Naonori Kakimura | University of Tokyo | kakimura@mist.i.u-tokyo.ac.jp |
| Kamen Kanev | Shizuoka University | kanev@rie.shizuoka.ac.jp |
| Ryosei Kasai | Tohoku University | ryosei@dais.is.tohoku.ac.jp |
| Naoki Katoh | Kyoto University | naoki.katoh@gmail.com |
| Yasushi Kawase | University of Tokyo | kawase@stat.t.u-tokyo.ac.jp |
| Kei Kimura | University of Tokyo | kimukei2@hotmail.com |
| Christian Knauer | Freie Universitat Berlin | christian.knauer@inf.fu-berlin.de |
| Yusuke Kobayashi | University of Tokyo | Yusuke_Kobayashi@mist.i.u-tokyo.ac.jp |
| Matias Korman | Tohoku University | mati@dais.is.tohoku.ac.jp |
| Stefan Langerman | Université Libre de Bruxelles | stefan.langerman@ulb.ac.be |
| Ming-Hua Lin | Shih-Chien University | mhlin@mail.usc.edu.tw |
| Kazuhisa Makino | University of Tokyo | makino@mist.i.u-tokyo.ac.jp |
| Hiroyuki Miyata | University of Tokyo | hmiyata@is.s.u-tokyo.ac.jp |
| Sonoko Moriyama | University of Tokyo | moriso@is.s.u-tokyo.ac.jp |
| Kazuo Murota | University of Tokyo | murota@mist.i.u-tokyo.ac.jp |
| Ryota Narita | Tohoku University | narita@dais.is.tohoku.ac.jp |
| Yoshio Okamoto | Tokyo Institute of Technology | okamoto@is.titech.ac.jp |
| Vera Rosta | Renyi Institute | rosta@renyi.hu |
| Nirin Suarod | University of Tokyo | tos_nt@hotmail.com |
| Kokichi Sugihara | Meiji University | kokichis@isc.meiji.ac.jp |
| Vorapong Suppakitpaisarn | University of Tokyo | mr_t_dtone@is.s.u-tokyo.ac.jp |
| Mizuyo Takamatsu | University of Tokyo | mizuyo_takamatsu@mist.i.u-tokyo.ac.jp |
| Shin-ichi Tanigawa | Kyoto University | is.tanigawa@archi.kyoto-u.ac.jp |
| Toshihiro Tanuma | University of Tokyo | to-tanuma@is.s.u-tokyo.ac.jp |
| Takeshi Tokuyama | Tohoku University | tokuyama@dais.is.tohoku.ac.jp |
| Csaba D. Toth | University of Calgary | cdtoth@ucalgary.ca |
| Jung-Fa Tsai | National Taipei Univ. of Tech. | jftsai@ntut.edu.tw |
| Taiju Tsuiki | University of Tokyo | Taiju_Tsuiki@mist.i.u-tokyo.ac.jp |
| Christophe Weibel | McGill University | christophe.weibel@gmail.com |
| Feng Xie | McMaster University | xief@mcmaster.ca |

Kazuo Yonekura University of Tokyo kazuo_yonekura@mist.i.u-tokyo.ac.jp

