

Polyhedral Computation

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This volume is dedicated to two pioneers of polyhedral computation:
Leonid Khachiyan (1952–2005) and Victor Klee (1925–2007)

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Preface

A workshop on *Polyhedral Computation* was held in Montréal during the week of October 17–20, 2006. It took place at the Centre de Recherches Mathématiques (CRM) on the campus of the Université de Montréal. It was part of the CRM's theme semester on Combinatorial Optimization held that year.

The last fifteen years have seen significant progress in the development of general purpose algorithms and software for polyhedral computation (e.g., finding lattice points, enumerating vertices, extreme rays, and facets, and triangulating polyhedra). Many polytopes of practical interest have enormous output complexity and are often highly degenerate, posing severe difficulties for known general purpose algorithms. They are, however, highly structured and attention has turned to exploiting this structure, particularly symmetry. Initial applications of this approach have permitted computations previously far out of reach, but much remains to be understood and validated experimentally.

The workshop, one of the first of its kind, brought together researchers with both theoretical and computational expertise in polyhedral computation. Most of the participants have either implemented their ideas in freely available software or have participated in the development of this software by using it to further their own research projects.

The papers in this volume give a good snapshot of the ideas discussed at the workshop and, with one exception, the current state of affairs in this area. The exception is the inclusion of an often cited 1980 technical report of Norman Zadeh, which was never published in a journal and has passed into the folklore of the discipline. This paper illustrates beautifully the work still to be done in the field: it gives a simple pivot rule for the simplex method for which it is still unknown if it yields a polynomial time algorithm.

We would like to thank the CRM for organizing the meeting and for its generous financial support. Additional financial support from GERAD and NSERC is also gratefully acknowledged. We would particularly like to thank Louis Pelletier for tirelessly working out all the logistics and Louise Letendre and André Montpetit for help in putting this volume together and, in particular, for typesetting Zadeh's paper.

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