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Discrete Geometry and Symmetry

Dedicated to Károly Bezdek and Egon Schulte on the Occasion of Their 60th Birthdays

Springer

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Preface

This volume contains a number of articles on the topics of symmetry and discrete geometry. Most of them were papers presented during the conference 'Geometry and Symmetry', held at the University of Pannonia in Veszprém, Hungary, the week 29 June to 3 July 2015. This conference was arranged in honour of Károly Bezdek and Egon Schulte, on the occasion of the year in which they both turned 60. Many of the papers reflect the remarkable contributions they made to geometry.

The revival of interest in discrete geometry over the past few decades has been influenced by Bezdek and Schulte to a large degree. Although their research interests are somewhat different, one could say that they have complemented each other, and this has resulted in a lively interaction across a wide variety of different fields. Accordingly, the volume includes a range of topics and provides a snapshot of a rapidly evolving area of research. The contributions demonstrate profound interplays between different approaches to discrete geometry.

Kepler was the first to raise the discrete geometry problem of sphere packing. Associated tiling problems were considered at the turn of the century by many researchers, including Minkowski, Voronoi, and Delone. The Hungarian school pioneered by Fejes Tóth in the 1940s initiated the systematic study of packing and covering problems, while numerous other mathematicians contributed to the field, including Coxeter, Rogers, Penrose, and Conway. While the classical problems of discrete geometry have a strong connection to geometric analysis, coding theory, symmetry groups, and number theory, their connection to combinatorics and optimisation has become of particular importance. These areas of research, at the heart of Bezdek's work, play a central role in many of the contributions to this volume.

Kepler, with his discovery of regular non-convex polyhedra, could also be credited with founding of modern polytope theory. The subject went into decline before it was taken up again by Coxeter almost a century ago and later by Grünbaum. Based on their impressive and seminal contributions, the search for deeper understanding of symmetric structures has over the past few decades produced a revival of interest in discrete geometric objects and their symmetries. The rapid development of abstract polytope theory, popularised by McMullen's and

Schulte's research monograph with the same name, has resulted in a rich theory, featuring an attractive interplay of methods and tools from discrete geometry (such as classical polytope theory), combinatorial group theory, and incidence geometry (generators and relations, and Coxeter groups), graph theory, hyperbolic geometry, and topology.

We note with sadness that during the work on this volume, our good friend and colleague Norman W. Johnson (a contributor to this volume) passed away. Since receiving his Ph.D. with Coxeter in 1966, Norman held a position at the Wheaton College in Massachusetts, where he taught until his retirement in 1998.

It is our hope that this volume not only exhibits the recent advances in various areas of discrete geometry, but also fosters new interactions between several different research groups whose contributions are contained within this collection of papers.

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Károly Bezdek—Biosketch

Károly Bezdek was born on 28 May 1955 in Budapest, a son of Károly Bezdek Sr. (who was chief engineer of Hungary's largest steel factory for over 20 years) and Magdolna Cserey (who had a strong interest in the literature and languages). His childhood years were spent in Dunaújváros. This period was challenging for his parents, who had grown up in a totally different Hungary, but despite some of the hardships faced by his family during and after WWII, his parents made all possible efforts to ensure a very educational and enjoyable childhood for Károly and his younger brother András. They encouraged both of their sons to develop interests in learning (across a wide range of subjects) and sports such as fencing and tennis.

Károly and András (who is also a mathematician) scored at the top level in several mathematics and physics competitions for high school and university students in Hungary. The awards won by Károly include first prize in the national KöMal contest (run by the Hungarian mathematics journal for high school students) in 1972/73 and first prize at the National Science Conference for Hungarian Undergraduate Students (TDK) in 1977/78, for his work on optimal circle coverings. As a result of these successes, Károly was admitted to Eötvös Loránd University in Budapest in 1973, without any entrance examination.

His first three years as an undergraduate involved rigorous basic courses, tested in oral exams, but also participation in special seminars on topics representing much of the frontline mathematical research in Hungary. Then in his last two years, he chose to specialise in discrete geometry and completed a Diploma in Mathematics (the equivalent of a master's degree) with a thesis on optimal circle coverings, under the supervision of Professor Károly Böröczky (who held the Chair of Geometry) in 1978.

He was awarded a Ph.D. in 1980 and Candidate of Mathematical Sciences degree in 1985, again with Prof. Károly Böröczky as his advisor in both cases, and later he was awarded a Doctor of Mathematical Sciences degree from the Hungarian Academy of Sciences in 1995 and Habilitation in Mathematics from Eötvös Loránd University in 1997.

Károly became a Faculty Member in the Department of Geometry at Eötvös Loránd University in 1978, served as chair of that department from 1999 to 2006, and earned the position of full professor in 1998. From 1998 to 2001, he served as Széchenyi Professor of Mathematics at Eötvös Loránd University, in a named position awarded to him by the Hungarian government. Although the university never really had a sabbatical system, he was fortunate to be able to travel regularly. During the period 1978 to 2003, he held numerous visiting positions at research institutions in Canada, Germany, the Netherlands, and the USA, including seven years at Cornell University, in Ithaca, NY.

He was invited to take up a Canada Research Chair at the University of Calgary, and he accepted this position in 2003. He is also Director of the Center for Computational and Discrete Geometry in Calgary; for the last few years, he has been an Associate Member of the Alfréd Rényi Institute of Mathematics in Budapest, and he also holds the title of Full Professor at the University of Pannonia in Veszprém.

Károly's research interests are in combinatorial, computational, convex, and discrete geometry, including some aspects of geometric analysis, geometric rigidity, and optimisation. He is the author of more than 110 research papers many of which are highly cited. He also wrote *Classical Topics in Discrete Geometry* (Springer, 2010) and *Lectures on Sphere Arrangements—the Discrete Geometric Side* (Springer, 2013), the monographs that take the reader to the frontiers of the most recent research developments in the relevant parts of discrete geometry.

He has been always interested in teaching, which he finds very rewarding as well. In particular, he has very much enjoyed working with graduate students, who are all very different from each other, but all gifted in many ways, each bringing a new perspective to geometric research. He has supervised five master's students, who he says have become great instructors with the potential to improve mathematics education, and a number of talented undergraduate research students. He has successfully supervised eleven Ph.D. students to date: Tibor Ódor (1991), László Szabó (1995), István Talata (1997), Endre Kiss (2004), Balázs Visy (2002), Márton Naszódi (2007), Zsolt Lángi (2008), Peter Papez (2009), Mate Salat (2009), Ryan Trelford (2014), and Muhammad A. Khan (2017).

Károly says that his work was influenced by a number of great mathematicians, colleagues, and friends, including 1978–1988 by Károly Böröczky, Aladar Heppes, Gábor Fejes Tóth, László Fejes Tóth, Kurt Leichtweiss, Keith Ball, Ted Bisztriczky, Robert Connelly, Oded Schramm, Joerg Wills, Thomas Hales, Alexander Litvak, Oleg Musin, Rolf Schneider, Marjorie Senechal, Egon Schulte, and Elisabeth Werner. He has also enjoyed travelling, often together with his wife Éva and their family, as well as inviting visitors for dinner in their home.

Károly is grateful to Éva for being 'such a fantastic partner and supporter'. Currently, Éva is Director and Teacher at the Gabor Bethlen Hungarian Language School in Calgary, and they have three sons: Dániel, Máté, and Márk. Márk is a third-year undergraduate student majoring in Public Relations at Mount Royal University in Calgary; Dániel has a degree in finance and is now completing a second undergraduate major in Computer Science at the University of Calgary; Máté is a third-year doctoral student in Chemistry at Princeton University.

We are very happy to pay tribute to Károly to his successful career and many contributions to mathematics, especially in geometry.

Egon Schulte—Biosketch

Egon Schulte was born on 7 January 1955 in Heggen (Finnentrop), North Rhine-Westfalia, Germany, to parents Egon and Gisela Schulte. He attended the Volksschule Lenhausen (Finnentrop) and the Katholische Volksschule Herdecke from 1961 to 1965, and the Städtisches Gymnasium Wetter (in the Ruhr region) from 1965 to 1973, completing the Abitur qualification in 1973. It was not until the last year or two in high school that Egon decided to study mathematics. In school, he was always good in mathematics, but was also very much interested in sports. He played very actively in a (European-style) handball team in Herdecke until about 1976 or so. Sports have always been an important part of his life; he has even run marathons.

From 1973 to 1978, he studied at the University of Dortmund, graduating with a 'Diploma' in Mathematics in 1978. Egon's Diplom thesis was on *Konstruktion regulärer Hüllen konstanter Breite* (regular hulls of constant width), a topic in convex geometry, and was published as his first paper in *Monatshefte der Mathematik.* His advisor was Ludwig Danzer, who also was advisor for his doctoral dissertation on *Regular Inzidenzkomplexe* (regular incidence complexes), which began Egon's lifelong interest in regular abstract polytopes. Egon graduated as a Doctor of Natural Sciences (in Mathematics) at the University of Dortmund in 1980. All three of Egon's main qualifications (Abitur, Diplom, and Doctorate) were awarded 'Auszeichnung' (distinction).

Prospects for academic positions in Germany were not good in the late 1970s and 1980s, especially in pure mathematics. Egon took a position as Wissenschaftlicher Assistent at the University of Dortmund from 1978 to 1983, and again from 1984 to 1987, but the period in between was very important for him, in that he found a very clear direction for himself, thanks largely to a visit by Branko Grünbaum to Dortmund in 1982. This had a profound influence on Egon, both mathematically and career-wise. He spent the 1983/84 academic year at the University of Washington, Seattle, and he describes the year as 'fantastic'. It introduced him to life in the USA and ultimately set him on a path towards a career there.

After Seattle, he returned to Germany for three years, gained Habilitation in Mathematics at the University of Dortmund in 1985, with a thesis on *Monotypische Pflasterungen und Komplexe* (monotypic tilings and complexes), and gained the title of 'Privatdozent'. Then, 1987 marked a new beginning for Egon, by moving to Boston, where he has been ever since. He worked as Visiting Assistant Professor at the Massachusetts Institute of Technology from 1987 to 1989 and then as an Associate Professor at Northeastern University from 1989 to 1992. Since 1992, he has been a Professor of Mathematics at Northeastern University, with tenure since 1993.

A few years after moving to Boston, Egon married Ursula Waser. They had two children: Sarah Marlen Schulte (born in 1992) and Isabelle Sophie Schulte (born in 1994), and both have studied at Northeastern. Sarah studied International Affairs and is now in her third year of Law School, and Isabelle graduated in 2017 with a major in Chemistry. Egon and Ursula separated in 2013 but remain good friends.

Mathematics has been Egon's passion ever since he began university. Looking back, he would say that over the years there were four people who strongly influenced his mathematical work and development: Ludwig Danzer, Branko Grünbaum, Harold Scott MacDonald (Donald) Coxeter, and Peter McMullen. Of course, he was positively influenced by many others as well. He is co-author with Peter McMullen of the outstanding book *Abstract Regular Polytopes*, has published well over 100 research articles (on a range of topics spanning discrete geometry, combinatorics and group theory), and edited six special issues of journals.

He is a popular invited lecture at conferences, has also organised or co-organised several conferences and workshops (or special sessions), and served on the editorial boards of many journals. He has won several grants, including many from the NSA and NSF in the USA, and a recent one from the Simons Foundation. And to date, he has successfully supervised 12 Ph.D. students: Barbara Nostrand (1993), Sergey Bratus (1999), Daniel Pellicer (2007), Anthony Cutler (2009), Mark Mixer (2010), Gabriel Cunningham (2012), Ilanit Helfand (2013), Andrew Duke (2014), Undine Leopold (2014), Ilya Scheidwasser (2015), Abigail Dalton-Williams (2015), and Nicholas Matteo (2015).

On top of all this, Egon is well-liked and highly respected by his friends and colleagues around the world for his positive attitude, his enthusiasm for mathematics, his engaging personality, and his encouragement of the next generation.

As far as choice of research topics is concerned, he says he usually followed his own interests and instincts and did not pay too much attention to trends and fashions. This had its rewards, but he says at times it came at a high price: ‘It might have been smarter to follow more trendy mathematics’, but we have the impression he does not regret his choices.