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Marston D. E. Conder · Antoine Deza
Asia Ivić Weiss
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Discrete Geometry and Symmetry

Dedicated to Károly Bezdek and Egon Schulte on the
Occasion of Their 60th Birthdays

 Springer

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Preface

This volume contains a number of articles on the topics of symmetry and discrete geometry. Most of them were papers presented during the conference ‘Geometry and Symmetry’, held at the University of Pannonia in Veszprém, Hungary, the week 29 June to 3 July 2015. This conference was arranged in honour of Károly Bezdek and Egon Schulte, on the occasion of the year in which they both turned 60. Many of the papers reflect the remarkable contributions they made to geometry.

The revival of interest in discrete geometry over the past few decades has been influenced by Bezdek and Schulte to a large degree. Although their research interests are somewhat different, one could say that they have complemented each other, and this has resulted in a lively interaction across a wide variety of different fields. Accordingly, the volume includes a range of topics and provides a snapshot of a rapidly evolving area of research. The contributions demonstrate profound interplays between different approaches to discrete geometry.

Kepler was the first to raise the discrete geometry problem of sphere packing. Associated tiling problems were considered at the turn of the century by many researchers, including Minkowski, Voronoi, and Delone. The Hungarian school pioneered by Fejes Tóth in the 1940s initiated the systematic study of packing and covering problems, while numerous other mathematicians contributed to the field, including Coxeter, Rogers, Penrose, and Conway. While the classical problems of discrete geometry have a strong connection to geometric analysis, coding theory, symmetry groups, and number theory, their connection to combinatorics and optimisation has become of particular importance. These areas of research, at the heart of Bezdek’s work, play a central role in many of the contributions to this volume.

Kepler, with his discovery of regular non-convex polyhedra, could also be credited with founding of modern polytope theory. The subject went into decline before it was taken up again by Coxeter almost a century ago and later by Grünbaum. Based on their impressive and seminal contributions, the search for deeper understanding of symmetric structures has over the past few decades produced a revival of interest in discrete geometric objects and their symmetries. The rapid development of abstract polytope theory, popularised by McMullen’s and

Schulte's research monograph with the same name, has resulted in a rich theory, featuring an attractive interplay of methods and tools from discrete geometry (such as classical polytope theory), combinatorial group theory, and incidence geometry (generators and relations, and Coxeter groups), graph theory, hyperbolic geometry, and topology.

We note with sadness that during the work on this volume, our good friend and colleague Norman W. Johnson (a contributor to this volume) passed away. Since receiving his Ph.D. with Coxeter in 1966, Norman held a position at the Wheaton College in Massachusetts, where he taught until his retirement in 1998.

It is our hope that this volume not only exhibits the recent advances in various areas of discrete geometry, but also fosters new interactions between several different research groups whose contributions are contained within this collection of papers.

Auckland, New Zealand
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Contents

The Geometry of Homothetic Covering and Illumination	1
Károly Bezdek and Muhammad A. Khan	
1 Shedding Some ‘Light’	1
2 Progress on the Illumination Conjecture	5
2.1 Results in \mathbb{E}^3 and \mathbb{E}^4	5
2.2 General Results	7
3 On Some Relatives of the Illumination Number	11
3.1 Illumination by Affine Subspaces	11
3.2 ‘X-raying’ the Problem	12
3.3 Other Relatives	14
4 Quantifying Illumination and Covering	16
4.1 The Illumination and Covering Parameters	16
4.2 The Covering Index	19
4.3 Cylindrical Covering Parameters	22
5 A Computer-Based Approach	24
References	27
Stability of the Simplex Bound for Packings by Equal Spherical Caps Determined by Simplicial Regular Polytopes	31
Károly Böröczky, Károly J. Böröczky, Alexey Glazyrin and Ágnes Kovács	
1 Introduction	32
2 Some Simple Preparatory Statements	33
3 The Proof of Theorem 1.1 in the Case of Simplices	35
4 The Linear Programming Bound	37
5 The Proof of Theorem 1.1 in the Case of Crosspolytopes	39
6 Spherical Dirichlet-Voronoi and Delone Cell Decomposition	41
7 Volume Estimates Related to the Simplex Bound	47
8 The Case of the Icosahedron	50
9 The Case of the 600-Cell	54
References	59

Vertex-Transitive Haar Graphs That Are Not Cayley Graphs	61
Marston D. E. Conder, István Estélyi and Tomaz Pisanski	
1 Introduction	62
2 The Graphs $D(n, r)$ and Their Properties	63
3 The Graphs $D(n, r)$ as Haar Graphs	65
4 Vertex-Transitive Haar Graphs That Are Not Cayley Graphs	67
References	69
On the Volume of Boolean Expressions of Large Congruent Balls	71
Balázs Csikós	
1 Introduction	71
2 Comparison of the Volume of a Union of Balls and the Volume of Its Convex Hull	74
3 Combinatorics of Boolean Expressions	76
4 Asymptotics for the Volume of Boolean Expressions of Large Balls	80
5 Properties of Boolean Intrinsic Volumes	81
6 Monotonocity of the Boolean Intrinsic Volume $V_{f,1}$	85
References	86
Small Primitive Zonotopes	87
Antoine Deza, George Manoussakis and Shmuel Onn	
1 Introduction	87
2 Primitive Zonotopes	88
2.1 Zonotopes Generated by Short Primitive Vectors	88
2.2 Combinatorial Properties of the Primitive Zonotopes	91
2.3 Primitive Zonotopes as Lattice Polytopes with Large Diameter	92
2.4 Primitive Zonotopes and Convex Matroid Optimization	93
3 Small Primitive Zonotopes $H_q(d, p)$ and $H_q^+(d, p)$	95
3.1 Small Primitive Zonotopes $H_q(d, p)$	95
3.2 Small Positive Primitive Zonotopes $H_q^+(d, p)$	97
4 Complexity Issues	99
4.1 Complexity Properties	99
4.2 Open Problems	100
5 Proofs for Sections 2.2 and 3	101
5.1 Proof for Section 2.2	101
5.2 Proof for Section 3	104
References	106
Delone Sets: Local Identity and Global Symmetry	109
Nikolay Dolbilin	
1 Introduction	109
2 Basic Definitions and Results	111

3	Proof of the Local Criterion for Crystal	116
4	Proof of Theorem 4	121
5	Proofs of Theorems 1, 2 and 3	123
	References	124
	The Twist Operator on Maniplexes	127
	Ian Douglas, Isabel Hubbard, Daniel Pellicer and Steve Wilson	
1	Introduction	128
2	Polyhedra, Maps, Maniplexes and Polytopes	128
	2.1 Maniplexes	130
	2.2 Polytopes	132
3	Symmetry	132
4	The Twist	134
	4.1 The Krughoff Cubes	134
	4.2 The Twist in 4 dimensions	135
	4.3 The General Twist	136
5	Chirality	137
6	The Maniplex $\hat{2}^{\mathcal{M}}$	139
	6.1 Color-Coded Extensions	141
7	Example of Twist on Rank 5	142
	7.1 The Map $n\mathcal{M}$	142
	7.2 A Series of 5-Maniplex Examples	143
8	Open Questions	144
	References	145
	Hexagonal Extensions of Toroidal Maps and Hypermaps	147
	Maria Elisa Fernandes, Dimitri Leemans and Asia Ivić Weiss	
1	Introduction	148
2	Preliminaries	150
	2.1 Hypertopes	150
	2.2 Regular and Chiral Hypertopes as C^+ -Groups	151
	2.3 B-Diagrams	153
3	Rank 4 Universal Locally Toroidal Hypertopes	155
4	Locally Toroidal Regular and Chiral Polytopes of Type $\{6, 3, 6\}$	158
5	Polytopes of Type $\{3, 6, 3\}$	160
6	Hexagonal Extensions of Toroidal Hypermap $(3, 3, 3)$	160
7	Nonlinear Hexagonal Extensions of the Tetrahedron	163
8	4-Circuits with Hexagonal Residues	166
9	Future Work and Open Problems	168
	References	169

Noncongruent Equidissections of the Plane	171
D. Frettlöh	
1 Introduction	171
1.1 Notation	172
2 Basic Observations	172
3 Variants of the Problem	175
4 Main Results	176
References	180
Pascal’s Triangle of Configurations	181
Gábor Gévay	
1 Introduction	181
2 The Configurations $DCD(n, d)$ and Their Geometric Realization	183
3 Pascal’s Triangle of Configurations $DCD[n, d]$	185
4 Generating the Entries as Incidence Sums	187
5 Incidence Theorems	192
6 Point-Circle Realizations	195
References	198
Volume of Convex Hull of Two Bodies and Related Problems	201
Ákos G. Horváth	
1 Introduction	201
2 Maximal Volume Polytopes Inscribed in the Unit Sphere	202
2.1 3-Dimensional Results	203
2.2 The Cases of Higher Dimensions	209
3 Volume of the Convex Hull of Two Connecting Bodies	214
3.1 On the Volume Function of the Convex Hull of Two Convex Body	214
3.2 Simplices in the 3-Space	220
References	223
Integers, Modular Groups, and Hyperbolic Space	225
Norman W. Johnson	
1 Linear Fractional Transformations	225
2 Complex Modular Groups	227
3 Quaternionic Modular Groups	229
4 Integral Octonions	231
5 Summary	233
References	233
Monge Points, Euler Lines, and Feuerbach Spheres in Minkowski Spaces	235
Undine Leopold and Horst Martini	
1 Introduction	236
2 Orthocentric Simplices and the Monge Point in Euclidean Space	238

3 The Monge Point of Simplices in Minkowski Spaces 239

4 Euler Lines and Generalized Feuerbach Spheres of Minkowskian
Simplices 242

5 Generalizations for Polygons in the Plane 247

6 Concluding Remarks and Open Problems 252

References 254

**An Algorithm for Classification of Fundamental Polygons for a Plane
Discontinuous Group 257**

Zoran Lučić, Emil Molnár and Nebojša Vasiljević

1 Introduction 258

2 Paired Polygon 265

3 Discrete Structures and the Algorithm 266

 3.1 Descriptor of Paired Polygon 266

 3.2 Starting Descriptor 268

 3.3 Tree Decomposition 268

 3.4 Blank Derivation and Qualifier 271

 3.5 Algorithm 272

 3.6 Boundaries and Genus 0 273

4 Program COMCLASS 273

5 Closing Remarks 277

References 277

Self-inscribed Regular Hyperbolic Honeycombs 279

Peter McMullen

1 Introduction 279

2 Regular Polytopes and Automorphism Groups 280

3 The Coxeter Group $[3^{n-2}, 2r]$ 281

4 The Tessellation $\{3, \infty\}$ 283

5 Honeycombs Inscribed in $\{3, 3, 6\}$ 283

6 The Coxeter Group $[3^{n-3}, 4, q]$ 285

7 Honeycombs Inscribed in $\{3, 4, 4\}$ 287

8 Honeycombs Inscribed in $\{3, 3, 3, 4, 3\}$ 289

9 Quotients 291

References 292

Sphere-of-Influence Graphs in Normed Spaces 293

Márton Naszódi, János Pach and Konrad Swanepoel

References 296

On Symmetries of Projections and Sections of Convex Bodies 297
 Dmitry Ryabogin

- 1 Introduction: Questions on Bodies with Congruent Projections and Sections 297
 - 1.1 Notation 298
- 2 Translations Only 299
 - 2.1 Projections 299
 - 2.2 Sections 299
- 3 Directly Congruent Projections 300
 - 3.1 Symmetric Bodies 300
 - 3.2 Golubyatnikov’s Approach 301
 - 3.3 One Body, A Rotational Symmetry 302
 - 3.4 One Body, A Direct Rigid Motion Symmetry 304
 - 3.5 Main Results 305
- 4 Other Groups of Symmetries 307
 - 4.1 Adding Reflections, Symmetries of $O(n)$ 307
 - 4.2 Groups of Symmetries Containing $O(n)$ 308
- 5 Concluding Remarks 308
- References 309

Regular Incidence Complexes, Polytopes, and C-Groups 311
 Egon Schulte

- 1 Introduction 311
- 2 Incidence Complexes 312
- 3 Flag-Transitive Subgroups of the Automorphism Group 314
- 4 Regular Complexes from Groups 318
- 5 Regular Polytopes and C-groups 323
- 6 Extensions of Regular Complexes 325
- 7 Abstract Polytope Complexes 327
- 8 Notes 330
- References 331

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Károly Bezdek—Biosketch



Károly Bezdek was born on 28 May 1955 in Budapest, a son of Károly Bezdek Sr. (who was chief engineer of Hungary's largest steel factory for over 20 years) and Magdolna Cserey (who had a strong interest in the literature and languages). His childhood years were spent in Dunaújváros. This period was challenging for his parents, who had grown up in a totally different Hungary, but despite some of the hardships faced by his family during and after WWII, his parents made all possible efforts to ensure a very educational and enjoyable childhood for Károly and his younger brother András. They encouraged both of their sons to develop interests in learning (across a wide range of subjects) and sports such as fencing and tennis.

Károly and András (who is also a mathematician) scored at the top level in several mathematics and physics competitions for high school and university students in Hungary. The awards won by Károly include first prize in the national KöMal contest (run by the Hungarian mathematics journal for high school students) in 1972/73 and first prize at the National Science Conference for Hungarian Undergraduate Students (TDK) in 1977/78, for his work on optimal circle coverings. As a result of these successes, Károly was admitted to Eötvös Loránd University in Budapest in 1973, without any entrance examination.

His first three years as an undergraduate involved rigorous basic courses, tested in oral exams, but also participation in special seminars on topics representing much of the frontline mathematical research in Hungary. Then in his last two years, he chose

to specialise in discrete geometry and completed a Diploma in Mathematics (the equivalent of a master's degree) with a thesis on optimal circle coverings, under the supervision of Professor Károly Böröczky (who held the Chair of Geometry) in 1978.

He was awarded a Ph.D. in 1980 and Candidate of Mathematical Sciences degree in 1985, again with Prof. Károly Böröczky as his advisor in both cases, and later he was awarded a Doctor of Mathematical Sciences degree from the Hungarian Academy of Sciences in 1995 and Habilitation in Mathematics from Eötvös Loránd University in 1997.

Károly became a Faculty Member in the Department of Geometry at Eötvös Loránd University in 1978, served as chair of that department from 1999 to 2006, and earned the position of full professor in 1998. From 1998 to 2001, he served as Széchenyi Professor of Mathematics at Eötvös Loránd University, in a named position awarded to him by the Hungarian government. Although the university never really had a sabbatical system, he was fortunate to be able to travel regularly. During the period 1978 to 2003, he held numerous visiting positions at research institutions in Canada, Germany, the Netherlands, and the USA, including seven years at Cornell University, in Ithaca, NY.

He was invited to take up a Canada Research Chair at the University of Calgary, and he accepted this position in 2003. He is also Director of the Center for Computational and Discrete Geometry in Calgary; for the last few years, he has been an Associate Member of the Alfréd Rényi Institute of Mathematics in Budapest, and he also holds the title of Full Professor at the University of Pannonia in Veszprém.

Károly's research interests are in combinatorial, computational, convex, and discrete geometry, including some aspects of geometric analysis, geometric rigidity, and optimisation. He is the author of more than 110 research papers many of which are highly cited. He also wrote *Classical Topics in Discrete Geometry* (Springer, 2010) and *Lectures on Sphere Arrangements—the Discrete Geometric Side* (Springer, 2013), the monographs that take the reader to the frontiers of the most recent research developments in the relevant parts of discrete geometry.

He has been always interested in teaching, which he finds very rewarding as well. In particular, he has very much enjoyed working with graduate students, who are all very different from each other, but all gifted in many ways, each bringing a new perspective to geometric research. He has supervised five master's students, who he says have become great instructors with the potential to improve mathematics education, and a number of talented undergraduate research students. He has successfully supervised eleven Ph.D. students to date: Tibor Ódor (1991), László Szabó (1995), István Talata (1997), Endre Kiss (2004), Balázs Visy (2002), Márton Naszódi (2007), Zsolt Lángi (2008), Peter Papez (2009), Mate Salat (2009), Ryan Trelford (2014), and Muhammad A. Khan (2017).

Károly says that his work was influenced by a number of great mathematicians, colleagues, and friends, including 1978–1988 by Károly Böröczky, Aladar Heppes, Gábor Fejes Tóth, László Fejes Tóth, Kurt Leichtweiss, Keith Ball, Ted Bisztriczky, Robert Connelly, Oded Schramm, Joerg Wills, Thomas Hales, Alexander Litvak, Oleg Musin, Rolf Schneider, Marjorie Senechal, Egon Schulte,

and Elisabeth Werner. He has also enjoyed travelling, often together with his wife Éva and their family, as well as inviting visitors for dinner in their home.

Károly is grateful to Éva for being ‘such a fantastic partner and supporter’. Currently, Éva is Director and Teacher at the Gabor Bethlen Hungarian Language School in Calgary, and they have three sons: Dániel, Máté, and Márk. Márk is a third-year undergraduate student majoring in Public Relations at Mount Royal University in Calgary; Dániel has a degree in finance and is now completing a second undergraduate major in Computer Science at the University of Calgary; Máté is a third-year doctoral student in Chemistry at Princeton University.

We are very happy to pay tribute to Károly to his successful career and many contributions to mathematics, especially in geometry.

Egon Schulte—Biosketch



Egon Schulte was born on 7 January 1955 in Heggen (Finnentrop), North Rhine-Westfalia, Germany, to parents Egon and Gisela Schulte. He attended the Volksschule Lenhausen (Finnentrop) and the Katholische Volksschule Herdecke from 1961 to 1965, and the Städtisches Gymnasium Wetter (in the Ruhr region) from 1965 to 1973, completing the Abitur qualification in 1973. It was not until the last year or two in high school that Egon decided to study mathematics. In school, he was always good in mathematics, but was also very much interested in sports. He played very actively in a (European-style) handball team in Herdecke until about 1976 or so. Sports have always been an important part of his life; he has even run marathons.

From 1973 to 1978, he studied at the University of Dortmund, graduating with a ‘Diploma’ in Mathematics in 1978. Egon’s Diplom thesis was on *Konstruktion regulärer Hüllen konstanter Breite* (regular hulls of constant width), a topic in convex geometry, and was published as his first paper in *Monatshefte der Mathematik*. His advisor was Ludwig Danzer, who also was advisor for his doctoral dissertation on *Regular Inzidenzkomplexe* (regular incidence complexes), which began Egon’s lifelong interest in regular abstract polytopes. Egon graduated as a Doctor of Natural Sciences (in Mathematics) at the University of Dortmund in 1980. All three of Egon’s main qualifications (Abitur, Diplom, and Doctorate) were awarded ‘Auszeichnung’ (distinction).

Prospects for academic positions in Germany were not good in the late 1970s and 1980s, especially in pure mathematics. Egon took a position as Wissenschaftlicher Assistent at the University of Dortmund from 1978 to 1983, and again from 1984 to 1987, but the period in between was very important for him, in that he found a very clear direction for himself, thanks largely to a visit by Branko Grünbaum to Dortmund in 1982. This had a profound influence on Egon, both mathematically and career-wise. He spent the 1983/84 academic year at the University of Washington, Seattle, and he describes the year as ‘fantastic’. It introduced him to life in the USA and ultimately set him on a path towards a career there.

After Seattle, he returned to Germany for three years, gained Habilitation in Mathematics at the University of Dortmund in 1985, with a thesis on *Monotypische Pflasterungen und Komplexe* (monotypic tilings and complexes), and gained the title of ‘Privatdozent’. Then, 1987 marked a new beginning for Egon, by moving to Boston, where he has been ever since. He worked as Visiting Assistant Professor at the Massachusetts Institute of Technology from 1987 to 1989 and then as an Associate Professor at Northeastern University from 1989 to 1992. Since 1992, he has been a Professor of Mathematics at Northeastern University, with tenure since 1993.

A few years after moving to Boston, Egon married Ursula Waser. They had two children: Sarah Marlen Schulte (born in 1992) and Isabelle Sophie Schulte (born in 1994), and both have studied at Northeastern. Sarah studied International Affairs and is now in her third year of Law School, and Isabelle graduated in 2017 with a major in Chemistry. Egon and Ursula separated in 2013 but remain good friends.

Mathematics has been Egon’s passion ever since he began university. Looking back, he would say that over the years there were four people who strongly influenced his mathematical work and development: Ludwig Danzer, Branko Grünbaum, Harold Scott MacDonal(d) Coxeter, and Peter McMullen. Of course, he was positively influenced by many others as well. He is co-author with Peter McMullen of the outstanding book *Abstract Regular Polytopes*, has published well over 100 research articles (on a range of topics spanning discrete geometry, combinatorics and group theory), and edited six special issues of journals.

He is a popular invited lecture at conferences, has also organised or co-organised several conferences and workshops (or special sessions), and served on the editorial boards of many journals. He has won several grants, including many from the NSA and NSF in the USA, and a recent one from the Simons Foundation. And to date, he has successfully supervised 12 Ph.D. students: Barbara Nostrand (1993), Sergey Bratus (1999), Daniel Pellicer (2007), Anthony Cutler (2009), Mark Mixer (2010), Gabriel Cunningham (2012), Ilanit Helfand (2013), Andrew Duke (2014), Undine Leopold (2014), Ilya Scheidwasser (2015), Abigail Dalton-Williams (2015), and Nicholas Matteo (2015).

On top of all this, Egon is well-liked and highly respected by his friends and colleagues around the world for his positive attitude, his enthusiasm for mathematics, his engaging personality, and his encouragement of the next generation.

As far as choice of research topics is concerned, he says he usually followed his own interests and instincts and did not pay too much attention to trends and fashions. This had its rewards, but he says at times it came at a high price: ‘It might have been smarter to follow more trendy mathematics’, but we have the impression he does not regret his choices.