COMP SCI/SFWR ENG 4/6E03 — Assignment 7 Solutions

1. Plugging into the general formula,

$$p_n = \begin{cases} p_0 (m\rho)^n \binom{K}{n} & 0 \le n < m \\ p_0 \rho^n \binom{K}{n} \frac{n!}{m!} m^m & m \le n \le K \end{cases}$$

   where $\rho = \lambda / m\mu$. Solving for p_0:

$$p_0 = \left(\sum_{n=0}^{m-1} (m\rho)^n \binom{K}{n} + \sum_{n=m}^{K} \rho^n \binom{K}{n} \frac{n!}{m!} m^m \right)^{-1}.$$

   The throughput is

$$Y = \sum_{n=0}^{K} (K-n)\lambda p_n = \lambda (K - L),$$

   where $L = \sum_{n=0}^{K} n p_n$. and the mean waiting time in queue is

$$W = \frac{L_q}{Y},$$

   where

$$L_q = \sum_{n=m+1}^{K} (n-m) p_n.$$

   I have not included a plot of the throughput, however note that it has an asymptote at μ.

2. From the expressions in class, the throughput is

$$\begin{aligned} Y &= \sum_{n=0}^{M} (M-n)\lambda p_n \\ &= \lambda (M - L). \end{aligned}$$

   where $L = \sum_{n=0}^{M} n p_n$. Substituting the expression for p_n, we get

$$L = p_0 \left[\sum_{n=1}^{M} n \binom{M}{n} n! \left(\frac{\lambda}{\mu} \right)^n \right]$$

   and

$$p_0 = \left[1 + \sum_{n=1}^{M} \binom{M}{n} n! \left(\frac{\lambda}{\mu} \right)^n \right]^{-1}.$$

   We need $1 - p_0 = 0.9$, which from the above expression means $M = 13$.

3. For the M/M/1 system, we have $\rho = 0.9$, so $L = 9$. The cost per hour is then

$$25 + (9)(5) = 70.$$

For the M/M/2 system, plugging into the formulas gives $L = 1.13$, so the cost per hour is

$$50 + (1.13)(5) = 55.65.$$

Adding a second server is preferred.