COMP SCI/SFWR ENG 4/6E03 — Assignment 8 Solutions

1. The assumption that we need to make is probabilistic routing. So, assume that the dispatcher assigns requests to the processors with equal probability. A similar assumption is made for the routing after the disk. Note that this model does not exactly match the system, in fact it adds some variability. Using the insight developed earlier that variability degrades performance, this means the results developed should be conservative. (If you are ambitious, you can check this by simulation.) In any case, if we let node 1 be the users, node 2 the dispatcher, nodes 3 and 4 the two processors and node 5 the disk, the traffic equations are (where p is an unknown routing probability):

$$\begin{aligned}\lambda_1 &= (1-p)(\lambda_3+\lambda_4)\\ \lambda_2 &= \lambda_1\\ \lambda_3 &= 0.5\lambda_2+0.5\lambda_5\\ \lambda_4 &= 0.5\lambda_2+0.5\lambda_5\\ \lambda_5 &= p(\lambda_3+\lambda_4)\end{aligned}$$

Here, setting $\lambda_1 = 1$ gives $\lambda_2 = 1$, $\lambda_5 = p/(1-p)$, $\lambda_3 = \lambda_4 = 1/2 + p/(2(1-p))$. The data in the question gives $\lambda_5/\lambda_1 = 3$, so $p = 3/4$ and thus $\lambda_5 = 3$ and $\lambda_3 = \lambda_4 = 2$.

Using the MVA algorithm, the actual system throughput is 241.3 requests per second, the utilization of the four resources are .19, .58, .58 and .87, respectively. Finally, the total response time is the sum of the waiting time at the dispatcher plus twice that at each of the processors plus three times that at the disk, or 36.2 milliseconds.

2. (a) This is a single class network (all jobs use the same routing probabilities), each node is load independent. Solving for the visit ratios, we get $V_0 = 1$, $V_1 = 0.3$, and $V_2 = 0.6$. So, we get

$$p(n_1,n_2,n_3) = \frac{1}{G}(1/2)^{n_1}(0.3/1.2)^{n_2}(0.6/1.2)^{n_3}.$$

Solving for G yields $G = .7656$.
(b) .6939

3. First solve the traffic equations (using the time unit of minutes):

$$\begin{aligned}\lambda_1 &= 10\\ \lambda_2 &= 0.5\lambda_1+0.7\lambda_2\\ \lambda_3 &= 0.5\lambda_1+0.3\lambda_2\end{aligned}$$

Solving these gives $\lambda_1 = 10$, $\lambda_2 = 50/3$ and $\lambda_3 = 10$. First, check that $\lambda_1/\mu_1 = 5/6$, $\lambda_2/\mu_2 = 5/9$ and $\lambda_3 = 2/3$. So, all $\rho_i = \lambda_1/\mu_1$ are less than one and we can proceed.
(a) $L_2 = \rho_2/(1-\rho_2) = 5/4$.
(b) The largest utilization is for server 1, so increase that one.

4. The mean processing times are the same for each, so using the P-K formula, choose the option with the lowest processing time variance. Note that we have to be careful to use consistent units, so here I will use seconds. For (i) we get 900, for (ii) 133.33, and for (iii) 400, so choose (ii).