COMP SCI/SFWR ENG 4/6E03 — Solutions to Assignment 4

1. (a) This is the $(0,0)$ entry of P, or 0.3.

   (b) This is the $(0,1)$ entry of P^5, or 0.5835.

   (c)

$$\begin{aligned} P(X_1 \neq X_0) &= P(X_1 = 1, X_0 = 0) + P(X_1 = 0, X_0 = 1) \\ &= P(X_1 = 1 | X_0 = 0) P(X_0 = 0) + P(X_1 = 0 | X_0 = 1) P(X_0 = 1) \\ &= (0.7)(0.3) + (0.5)(0.7) \\ &= 0.56 \end{aligned}$$

2. (a)

$$P = \begin{bmatrix} 0 & 1-p & p \\ 1 & 0 & 0 \\ 0 & 1-\alpha & \alpha \end{bmatrix}$$

   (b) Solving for the steady-state distribution, we must solve the equations:

$$\begin{aligned} \pi_2 &= \pi_1 \\ p\pi_1 + (1-\alpha)\pi_3 &= \pi_3 \\ \pi_1 + \pi_2 + \pi_3 &= 1 \end{aligned}$$

   which yields

$$\begin{aligned} \pi_1 &= \frac{\alpha}{2\alpha + p} \\ \pi_2 &= \frac{\alpha}{2\alpha + p} \\ \pi_3 &= \frac{p}{2\alpha + p} \end{aligned}$$

   (c)

$$\frac{1}{\pi_2} = \frac{2\alpha + p}{\alpha}$$

   (d) By inspection, this is true if $\alpha = p > 0$.

3. We have that the CPU is in one of two states, u or h (in that order), so P is

$$P = \begin{bmatrix} p_u & 1-p_u \\ 1-p_h & p_h \end{bmatrix}$$

Requirement (b) tells us that $1/(1-p_h) = 100$, or $p_h = 0.99$. This is by comparing the requirement with a geometric distribution. So, we have

$$\begin{aligned} \pi_u p_u + 0.01\pi_h &= \pi_u \\ \pi_h + \pi_u &= 1 \end{aligned}$$

So, (a) yields $\pi_u = 0.9$, $\pi_h = 0.1$. So,

$$0.9p_u + (0.01)(0.1) = 0.9$$

or $p_u = .9989$.