COMP SCI/SFWR ENG 4/6E03 - Sample Questions for Test 2

1. A system has three machines operating in parallel and two repairmen. The machines fail independently, with time to fail exponentially distributed with mean 10 hours. A repair takes an exponentially distributed period of time, with mean 8 hours, and requires one repairman. In steady-state:
   (a) What is the expected number of working machines?
   (b) What is the proportion of time that both repairmen are simultaneously busy?

2. Short Answer Questions.
   (a) Requests to a server arrive according to a Poisson process with rate 2 per minute. The last request arrived 20 seconds ago. What is the expected time until the next request arrives?
   (b) Jobs arrive to a system at average rate 15 per hour. On observing the system, the average number occupying it is 10 jobs. What is the mean response time for a job?
   (c) Name the model: Arrivals follow a Poisson process and form a single queue. There are 3 identical processors, each with exponentially distributed processing times. Processing is FCFS and any jobs arriving when there are more than 7 jobs in the system are rejected.

3. Jobs arrive at a system according to a Poisson process with rate 10 per hour. The service requirement of each job has an exponential distribution with mean 5 minutes. For use of the system, each arriving job is charged one dollar, with a 20 cent rebate if the job has to wait before entering service (the jobs wait in one queue). A server costs 1 dollar per hour to operate (this is a fixed cost, and does not depend on the server utilization). To maximize expected revenue for the system, should one or two servers be used?

4. Short Answer Questions

   (i) Very briefly explain why it is not a good idea to operate a queueing system under a heavy load (arrival rate close to service rate).

   (ii) Very briefly explain why an infinite buffer queue is said to be "unstable" when the mean arrival rate is greater than the mean service rate.

   (iii) A professor schedules student appointments every ten minutes. This is on the assumption that she can advise the average student in ten minutes. Will there be a long line outside of her office?

5. A call centre is using the following staffing rule: the average time in queue (before service commences) of a customer must be less than 5 minutes. The minimum staff level should be kept. The time spent with a customer (service time) is exponentially distributed with mean 4 minutes and interarrival times are exponentially distributed.
   (a) For what range of arrival rates can one staff person be used?
   (b) Suppose the staffing rule was changed such that **all** customers should have a wait of less than 5 minutes. Is this achievable? Explain.

6. Jobs arrive to a single server according to a Poisson process with rate 3 per hour. Service times are exponentially distributed with mean $1/\mu$ hours.
(a) Find μ so that the steady-state probability that there is more than 1 customer in the system is .25.
(b) Now assume that $\mu = 5$. The server gets a bonus of \$1 per job that does not have to wait in the queue before entering service. Calculate the expected bonus per hour that the server makes.

7. Arrivals to a single server follow a Poisson process with rate 30 per hour. Service times are exponentially distributed with mean 60/35 minutes.
(a) What fraction of the arrival rate given in the question yields half the expected number in system? Explain why this fraction is more or less than one half.
(b) It is now 9:00. The last job arrived at 8:57. What is the probability that at least one job arrives between now and 9:02?

8. A system operates as follows: requests arrive according to a Poisson process with rate 1 per second. There are two processors, each with exponentially distributed processing times with mean 1.25 seconds per request. The second processor only operates when there are at least T requests waiting for processing. The first processor always operates if possible.
(a) For $T = 3$, calculate the probability that an arriving request has to wait before being processed.
(b) Is the value calculated in (a) an increasing or decreasing function of T? Explain your answer.

9. An existing system consists of a single server. Requests arrive according to a Poisson process with rate three per hour. The processing times are exponentially distributed with mean 18 minutes. You are asked whether it would be beneficial to add a second (identical) processor. The two processors would share a common queue. The costs are as follows: operating costs of 25 dollars per hour per processor and cost of delay of five dollars per hour per request.

10. A system operates as follows. Arrivals occur according to a Poisson process with rate 1 per minute. There is a single server and room for at most three jobs in the system. If there are one or two jobs in the system, the server has service times that are exponentially distributed with mean one minute. However, if there are three jobs in the system, the server speeds up so that the service times are exponentially distributed with mean 30 seconds. Calculate the expected number of jobs in the system (in steady-state).

11. The average waiting time (W) on a database system is three seconds. During a one minute observation interval, the idle time on the system was measured to be 10 seconds. Using an M/M/1 model, determine the mean service time per query.

12. A system works as follows. Arrivals occur according to a Poisson process with rate 2 per minute. If there are one or two jobs in the system, a single server processes the jobs,

the processing times being exponentially distributed with mean 40 seconds. If there are three jobs in the system, two servers process the jobs, each server having processing times exponentially distributed with mean 40 seconds. Finally, an arriving job that sees three jobs in the system is rejected. In steady-state, calculate the expected number of jobs in the system.

13. Arrivals occur to a system according to a Poisson process with rate one per minute. The blocking probability for the system is measured to be 0.1. The expected number of jobs in the system is 2.5. What is the expected response time?

14. Students arrive at a single copy machine according to a Poisson process with rate 10 students per hour. The time a student spends at the copying machine is exponentially distributed with a mean of 4 minutes.
(a) What is the probability that a student arriving at the machine will have to wait?
(b) What is the mean number of students waiting for the copier (not including the one making copies)?

15. Consider a single processor implementing a priority discipline. There are two classes of jobs, high priority and low priority. Arriving jobs follow Poisson processes. The processing times are exponentially distributed.

    (i) Low priority jobs are only processed if no high priority jobs are present. That is, any high priority jobs in the queue are served, one at a time, before the processor processes low priority jobs.

    (ii) If the queue only has low priority jobs, an arriving high priority job will instantly displace the low priority job at the processor and take its place.

    (a) Let the arrival rate be 5 jobs per millisecond for each class and the processing rate be 20 jobs (of either class) per millisecond. Calculate the average number of high priority jobs in the system and the average number of jobs of both classes in the system. Use these two answers to deduce the average number of low priority jobs in the system. (Hint: Think carefully how high priority jobs are processed.)
    (b) Try to repeat as much of (a) as possible, if the processing times (in milliseconds) of high priority jobs are uniformly distributed between 0.03 and 0.07. The low priority jobs remain exponentially distributed. If it is not possible to calculate any of the quantities in question, indicate why.

16. You have a single processor implementing FCFS and are given the option of three different processing time distributions: (i) exponential with rate two per minute, (ii) uniformly distributed between 10 and 50 seconds, (iii) equal to exactly 10 or 50 seconds with equal probability. If the performance measure is mean waiting time, which would you choose?

17. A single server systems has arrivals that occur according to a Poisson process with rate λ per hour. The processing times are exponentially distributed with mean $1/\mu$ hours. The cost of waiting jobs is \$ C_1 per job per hour. The cost of operating the system is \$ μC_2

per unit per hour, whether or not it is always in operation. How large should μ be to minimize the total cost?

18. A designer is trying to decide how many processors to have in a system. Each processor works at a rate of 100 jobs per hour and the system is such that a processor upon becoming idle chooses the next waiting job. The system is being designed for an arrival rate of 150 jobs per hour. Assume all underlying distributions (interarrival and service times) are exponential. If the design criterion is that the probability that an arriving job has to wait is no larger than .5, what is the smallest number of processors that can be chosen?

19. Consider two M/M/1 systems with arrival rates $\lambda_1 = 0.5$ per second and processing rates $\mu_1 = 1$ per second and an M/M/2 system with arrival rate $\lambda_2 = 2\lambda_1 = 1$ per second and processing rate $\mu_2 = \mu_1 = 1$ per second for each server. Compare the mean waiting time times for both systems. Discuss your result, giving a physical explanation for why one system is better.
(b) Compare the mean waiting time in an M/D/1 (processing times constant), M/M/1 and an M/G/1 system (with variance of processing times equal to 2 seconds^2), with $\lambda = 0.8$ per second and $\mu = 1$ per second. Give a physical explanation for why the values are different although the arrival rate λ and the processing rates are always the same.

20. An M/M/1 system has a processing rate of 1 per second.
(a) What is the maximum arrival rate that this system can support, if the requirement is that there are, on average, less than 4 jobs in the system, 90 percent of the time?
(b) Suppose that at time t, there are 4 jobs in the system, with the job being processed having begun processing at time $t - 0.5$. What is the expected time of the next departure from the system?

21. A system is described as follows. Arrivals occur according to a Poisson process with rate 1 per second. There is room in the system for 3 jobs (including the one being processed). A single server has exponentially distributed processing times, but the rate depends on the number in system. A good fit is that when there are n jobs in the system, the processing rate is $\sqrt{n}$ jobs per second. Calculate the mean number of jobs in the system, in steady-state.

22. (a) Consider an M/M/1/4 system. The observed mean number in system is 2, the mean waiting time (from arrival to departure) is 4 seconds. The arrival rate, λ (without taking the losses into account) is $\lambda = 0.75$ jobs per second. What is the loss (blocking) probability?
(b) Consider an M/M/c system with arrival rate 5 per minute. Each processor has a mean processing time of 20 seconds. How many processors are required so that the system is stable (i.e. the steady-state probabilities p_n exist)?
(c) If you could convert the system in (b) to c M/M/1's, where an arrival joins each queue with probability $1/c$, would you make the change? Briefly explain why or why not.

23. A system operates with a central server that processes requests from a large number of users. The operating cost of the server is 20 dollars per hour, whether it is working or not. The cost of a waiting user is 15 dollars per hour per user.
(a) If the processing time of a job is exponentially distributed with mean three minutes and the arriving requests follow a Poisson process with rate 15 per hour, should another server be purchased?
(b) If the processing times and interarrival times are both deterministic (the values are the same as the means in (a)), would your recommendation change?

24. Consider a system with four nodes. External arrivals occur to the first node according to a Poisson process with rate 9 per hour. From node one, with probability .17, jobs go to node four, with probability .47 they go to node two, otherwise they go to node three. From node three, all jobs go to node four. From node two, with probability .95 a job goes to node four, otherwise it returns to node one. There is a single processor at each node. The processing times at each node are exponentially distributed, with the following means: node one - 6 minutes, node two - 12 minutes, node three - 16 minutes, and node four - 6 minutes.

 (i) Calculate the expected number of jobs at node three.

 (ii) You wish to improve the expected time a job spends in the network. If you are allowed to increase only one processing rate, which would it be?

25. Consider a system with three nodes. Arrivals occur to node 1 according to a Poisson process of rate 2 per second. Arrivals occur to node 2 according to a Poisson process of rate 1 per second. After processing at node 1, jobs return to node 1 with probability 0.25, otherwise they go to node 3. After processing at node 2, jobs return to node 2 with probability 0.2, otherwise they go to node 3. After processing at node 3, jobs leave the system. The service policy at each node is FCFS. The processing times at each node are exponentially distributed with rates 3 per second, 2 per second and 5 per second, respectively.
(a) Calculate the expected number of jobs at node 3.
(b) You wish to improve the expected time in system for jobs that originally arrive to node 2. To do this, you are allowed to increase the processing rate of one node by ten percent. Which processor would you improve?

26. We consider a system that allows a constant level of N programs (jobs) sharing main memory. It has 2 servers, server 1 being the CPU and server 2 being the I/O. The mean service time for a job at the CPU is 0.02 seconds, the mean service time for a job at the I/O subsystem is 0.04 seconds (assume all times are exponentially distributed). After a job completes at the I/O subsystem, it always goes to the CPU. After completion of a job at the CPU, a job exits the system with probability 0.05, otherwise it goes to I/O. When a job exits a system, another immediately enters at the CPU, thus there are always exactly N jobs in the system. For $N = 2$, calculate the utilization of the CPU (proportion of time CPU is busy).

27. Consider the following situation. There are three nodes, the first having two servers and the second and third one server each. Both servers at the first node have exponentially distributed service times with mean 10 seconds. At the second and third centers, the service times follow an exponential distribution with mean 8 seconds. Service is FCFS (if you are not explicitly given a policy, assume FCFS). Two jobs circulate through the network: after exiting the first node, a job goes to the second node with probability .25 and to the third node otherwise. From the second and third nodes, jobs always return to the first node.
(a) In steady-state, calculate the probability that both jobs are at the first center.
(b) Calculate the throughput at the first node.

28. A network consists of three single server nodes. External arrivals to the first node follow a Poisson process with rate 10 per minute. After service at node 1, jobs go to node 2 with probability .5 and go to node 3 otherwise. After service at node 2, jobs return to node 2 with probability .7 and go to node 3 otherwise. After service at node 3, jobs always exit the system. The service times at each node are exponentially distributed, with the following means: Node 1 - 5 seconds, Node 2 - 2 seconds, Node 3 - 4 seconds.
(a) Find the expected number of customers at node 2.
(b) If you could increase the service rate at one node, which node would be your **last** choice to do so?

29. A closed network operates as follows. It has 4 nodes, the processing rates being 5, 10, 10, 1 at the nodes, respectively. After finishing processing at node 1, jobs proceed to either node 2 or 3 with equal probability. After processing at either nodes 2 or 3, with probability 1/10 a job visits node 4, otherwise it returns to node 1. After processing at node 4, jobs return to node 1.

    Calculate the throughput of this system with 2 jobs circulating. If you were to improve one processing rate, which would it be? Would you expect your recommendation to change as we increase the number of jobs in the system?

30. Jobs arrive to node two of a network according to a Poisson process with rate one per minute. With probability 0.5, jobs exiting node two move to node one, otherwise they exit the system. Jobs exiting node one always go to node three. Jobs exiting node three either go to node one or exit the system, with equal probability. Processing times at nodes one, two and three are exponentially distributed with means 30 seconds, 40 seconds and 50 seconds, respectively. Calculate
(a) the utilization of node two.
(b) the mean number of jobs in the network.

31. Jobs arrive to node one of a network according to a Poisson process with rate two per minute. With probability 0.4, jobs exiting node one go to node two, with probability 0.5, jobs exiting node one go to node three, otherwise they exit the system. Jobs exiting node two go to node three or node one with equal probability. Jobs exiting node three go to node two or node one with equal probability. Processing times at nodes one, two

and three are exponentially distributed with means 3 seconds, 5 seconds and 4 seconds, respectively. Calculate
(a) the mean time a job spends in the network from arrival to exit
(b) the system bottleneck.

32. You have a single processor implementing FCFS and are given the option of three different processing time distributions: (i) exponential with rate two per minute, (ii) uniformly distributed between 10 and 50 seconds, (iii) equal to exactly 10 or 50 seconds with equal probability. If the performance measure is mean waiting time, which would you choose?

33. Consider a closed queueing network with $N = 3$ nodes and $M = 2$ jobs. The processing times are exponentially distributed with rates $\mu_1 = 0.72$, $\mu_2 = 0.64$, $\mu_3 = 1.00$. The routing probabilities are $r_{31} = 0.4$, $r_{32} = 0.6$, $r_{13} = 1.0$, $r_{23} = 1.0$.
(a) Calculate the throughput of the system.
(b) Which node is the bottleneck?

34. Consider two M/M/1 systems with arrival rates $\lambda_1 = 0.5$ per second and processing rates $\mu_1 = 1$ per second and an M/M/2 system with arrival rate $\lambda_2 = 2\lambda_1 = 1$ per second and processing rate $\mu_2 = \mu_1 = 1$ per second for each server. Compare the mean waiting time times for both systems. Discuss your result, giving a physical explanation for why one system is better.
(b) Compare the mean waiting time in an M/D/1 (processing times constant), M/M/1 and an M/G/1 system (with variance of processing times equal to 2 seconds^2), with $\lambda = 0.8$ per second and $\mu = 1$ per second. Give a physical explanation for why the values are different although the arrival rate λ and the processing rates are always the same.

35. Construct a closed queueing network with two jobs and two servers such that the throughput is one per second. (You are free to choose all network parameters other than the number of jobs and servers.)

36. A queueing network is described as follows. Arrivals occur to node one according to a Poisson process with rate one per minute. Processing times at node one are exponentially distributed with mean 20 seconds. After exiting node one, jobs go to nodes two or three with equal probability. The processing times at nodes two and three are exponentially distributed with means of 60 and 70 seconds, respectively. After processing at node two or three, a job leaves the system with probability 0.75, otherwise it returns to node one. Calculate the network bottleneck.

37. Consider a network where external arrivals occur to node 1 at a rate of γ jobs per second. After processing at node 1, a job is routed to node 2 with probability 0.4 or to node 3 with probability 0.3. Otherwise, it leaves the network. After processing at node 2 or node 3, jobs always return to node 1. Processing times at nodes 1, 2, and 3 are exponentially distributed with rates 10 per second, 2 per second, and 2 per second, respectively.
(a) If the utilization of node 3 is measured to be 0.7, what is γ?

(b) Using the value of γ from (a), if the bottleneck processor has its processing rate doubled, calculate the mean number of jobs in the network.

38. Consider the following situation. There are three nodes, the first having two servers and the second and third one server each. Both servers at the first node have exponentially distributed service times with mean 10 seconds. At the second and third centers, the service times follow an exponential distribution with mean 8 seconds. Service is FCFS (if you are not explicitly given a policy, assume FCFS). Two jobs circulate through the network: after exiting the first node, a job goes to the second node with probability .25 and to the third node otherwise. From the second and third nodes, jobs always return to the first node.
(a) In steady-state, calculate the probability that both jobs are at the first center.
(b) Calculate the throughput at the first node.

39. A closed system has 2 jobs circulating. Node 1 is an infinite server node, with processing rate 3. Nodes 2 and 3 are processor sharing nodes, with processing rate 1. All processing time distributions are exponential. After processing at node 1, jobs are equally likely to go to either node 2 or 3. After processing at nodes 2 or 3, jobs return to node 1.
(a) Calculate the probability that node 1 is idle.
(b) Could any of the system assumptions be relaxed while maintaining the same technique in (a)?

40. Short answer questions. Explain your answers for each.
(a) True or False. MVA may be used to calculate the probability that there are zero jobs at a node.
(b) True or False. All things being equal, increasing variability improves system performance.
(c) True or False. In an arbitrary network, the node with the smallest processing rate is the bottleneck.
(d) You calculate the throughput of a system to be five jobs per minute. You also observe that the average number of jobs in the system is two. What is the average time that a job spends in the system?
(e) Give an example of a network that is **not** product form.

41. Parts (a) and (b) are unrelated.
(a) A single server system has arrivals occurring according to a Poisson process with rate one per minute. You have two design choices. The first has processing times that are exponentially distributed with mean two per minute, the second has processing times that are uniformly distributed between 20 seconds and 40 seconds. If the mean waiting time is the performance measure of interest, which would you choose?
(b) Give an example of a performance measure that is difficult to calculate using the basic MVA algorithm.

42. Answer True or False to each part. There are no part marks, so you do not need to justify your answer.

(a) Increasing variability in processing times tends to improve performance.
(b) BCMP networks include those with state dependent routing.
(c) Resource utilization can be calculated using the MVA algorithm.

43. Users are connected to a database server through a network. There are two users, each having a think time that is exponentially distributed with mean 20 seconds. A user request goes to the database server through a network, where the network processing time is exponentially distributed with mean 0.1 seconds. The database server takes a period of time that is exponentially distributed with mean five seconds to process a request. Requests returning from the database server to the users must go via the network, where the processing time is exponentially distributed with mean one second. The network is modelled as a PS node, while the database server is FCFS. Calculate the utilization of the database server.

44. Consider the following cyclic closed network with 2 jobs circulating. Node 1 is an IS node with the identical processors each having rate 1 per minute. Nodes 2 and 3 are FCFS nodes whose processing times are exponentially distributed with means 30 seconds and 1 minute, respectively. Calculate the throughput of the network.

45. A system has four single server FCFS nodes, the CPU (node one), Printer (node two), Disk (node three), and an Input Device (node four). Processing times at each of the nodes are exponentially distributed with means $1/\mu_1 = 0.04$ seconds, $1/\mu_2 = 0.03$ seconds, $1/\mu_3 = 0.06$ seconds, and $1/\mu_4 = 0.05$ seconds. Arrivals to the system occur to node 4 at a rate of 4 jobs per second. The non-zero routing probabilities are $p_{12} = p_{13} = 0.5$, $p_{41} = p_{21} = 1$, and $p_{31} = 0.6$.
(a) Compute the probability that there are exactly three jobs at the CPU, two jobs at the printer, four jobs at the disk, and one job at the input device.
(b) Calculate the mean time that a job spends in the system (from arrival to departure).
(c) If the bottleneck device has its processing rate doubled, is there a new bottleneck device? If so, which device is it?

46. You are asked to design a system for which you receive 1 dollar in revenue for every arriving job that is served without waiting before processing and 50 cents for every job that is processed but has to wait before processing. Arrivals occur according to a Poisson process with rate 54 per hour and the processing times are exponentially distributed with mean 1 minute. You are considering the following two choices:

    (a) A single processor, admit all arrivals, processing in FIFO order.

    (b) A single processor, limit the number in system to 5, serve arrivals in FIFO order.

    Which would you choose if your goal is to maximize the rate of revenue?

47. A single processor has room for at most three in the system. Processing times are exponentially distributed with mean 1, while the interarrival times are modelled as exponentially

distributed with the following rates:

number in system	arrival rate
0	1.5
1	1.0
2	0.5

Calculate the utilization of the processor.

48. There are two independent arrival streams (following independent Poisson processes) with respective arrival rates $\lambda_1 = 20$ and $\lambda_2 = 15$ per hour. Processing times for both types of jobs are exponentially distributed with mean 2 minutes. You have two processors and have the choice of dedicating one processor to each arrival stream, or combining the two arrival streams into a single queue and serving the arrivals in a FIFO manner at the first available processor of the two. Which configuration would you choose? Quantify the resulting performance difference between the two configurations.

49. Consider the following closed queueing network. There are three nodes, each having a single processor and exponentially distributed processing times. Departures from node 1 are equally likely to go to the other two nodes. Departures from node 2 go to node 1 with probability 0.8, otherwise they go to node 3. Departures from node 3 always go to node 1. The mean processing times at the three nodes are 0.5, 1 and 1.2, respectively. Two jobs circulate in the network.
(a) What is the system bottleneck?
(b) Calculate the throughput at node 1.
(c) Calculate the mean waiting time at node 2.

50. An M/M/1/3 system has an arrival rate of 8 per hour and mean processing time of 6 minutes. The profit received from each arrival that is admitted into the system is 5 dollars. For an extra 60 dollars per day, you can convert it to an M/M/1/4. Would you make the change?

51. Consider the following network. Arrivals occur from outside to node 1 according to a Poisson process with rate 2 per minute. After processing at node 1, jobs go to node 2 with probability 1/3 and to node 3 with probability 2/3. After processing at node 2, jobs return to node 1 with probability 1/2, otherwise they leave the network. After processing at node 3, jobs return to node 1 with probability 1/2, otherwise they leave the network. All processing times are exponentially distributed, each node has a single processor that uses FCFS. The mean processing times are node 1 - 7.5 seconds, node 2 - 40 seconds and node 3 - 15 seconds.
(a) Find the probability that there is exactly one job in the network.
(b) If you were to improve the processing rate at exactly one node, which would it be?

52. As a first cut at dimensioning a database server, we assume that the server has arrivals occurring as a Poisson process with rate 2 per minute and that the processing times are

exponentially distributed with rate μ.
(a) If one pays a fixed cost of μ per minute for the capital investment and 10 for each arrival that has to wait to begin processing, what would you choose for μ?
(b) Suppose that the processing times are constant. How would your choice of μ change? You only need to indicate if it would be larger, smaller or the same. Justify your response by using insights from class.

53. A system has room for at most two in system (i.e. arrivals finding two in the system are lost). Arrivals follow a Poisson process with rate 1 per minute. The processing times are exponentially distributed with mean 40 seconds if there is one job in the system and 20 seconds if there are two jobs in the system. Find the expected number of lost arrivals in a one hour interval.

54. True or False.
(a) BCMP networks allow for routing choices which can be made based on queue length information.
(b) Arrivals occur according to a Poisson process with rate one every 8 minutes. It is now 6:00 and the last arrival occurred at 5:57. The expected time of the next arrival is 6:05.
(c) One M/M/4 queue has lower mean waiting time in system than 4 M/M/1 queues. (The arrival rate is 4λ to the M/M/4, while the arrival rate is λ to each M/M/1. All servers have rate μ.)
(d) One M/M/4 queue has lower average server utilization than 4 M/M/1 queues. (The arrival rate is 4λ to the M/M/4, while the arrival rate is λ to each M/M/1. All servers have rate μ.)
(e) A system has an arrival rate of 2 per minute and an average waiting time in system of 10 minutes. The average number in system is 20.

55. The following closed network has 3 nodes and 2 jobs circulating. From node 1, with probability 1/3 a job visits node 2, otherwise it visits node 3. After processing at nodes 2 or 3, a job returns to node 1. The processing times are all exponentially distributed, with mean 10 seconds at node 1, and 20 seconds at both nodes 2 and 3.
(a) Find the throughput at the bottleneck node.
(b) Removing the constraint that there are 2 jobs circulating, what is the maximum possible throughput at node 1?

56. You are setting up a server system that has a single queue. Service is FCFS and you need to decide the number of servers. For cost reasons, you would like to choose the smallest number of servers. Arrivals follow a Poisson process with rate 20 per hour. Processing times are exponentially distributed with mean 2 minutes. There is a requirement that there is at most one job in the system 75 percent of the time. How many servers would you choose?

57. A system is modelled as follows. There is space for at most two jobs in the system. Arrivals occur according to a Poisson process with rate 1 per minute. Arrivals finding

two in the system are lost. If there is one job in the system, it is processed at rate 2 per minute. If there are two in the system, due to contention, they are each processed at rate 0.75 per minute. Each job has its own processing time and processing times are exponentially distributed. Calculate the steady-state probability that there is exactly one job in the system.

58. Consider a closed queueing network with $N = 3$ nodes and $M = 2$ jobs. Processing times at the three nodes are exponentially distributed with rates $\mu_1 = 0.7$, $\mu_2 = 0.1$ and $\mu_3 = 1.5$. After processing at nodes 1 or 2, jobs always go back to node 3. After processing at node 3, jobs go to node 1 with probability 0.4 and node 2 with probability 0.6.
(a) Calculate the bottleneck node.
(b) Calculate the throughput at node 3.

59. An administrator is trying to calculate how many (identical) servers are required for a database server system. Suppose arrivals occur according to a Poisson process with rate 1 per second. The administrator estimates that the processing time of a request is exponentially distributed with mean 0.8 seconds. For the questions below, use appropriate models to justify your answers.
(a) If the SLA (Service Level Agreement) is that the mean response time be less than 3 seconds, how many servers are required?
(b) Does your answer in (a) change if it was discovered that the processing times are always *exactly* 0.8 seconds?

60. Jobs arrive to node one of a network according to a Poisson process with rate two per minute. With probability 0.4, jobs exiting node one go to node two, with probability 0.5, jobs exiting node one go to node three, otherwise they exit the system. Jobs exiting node two go to node three or node one with equal probability. Jobs exiting node three go to node two or node one with equal probability. Processing times at nodes one, two and three are exponentially distributed with means 3 seconds, 5 seconds and 4 seconds, respectively. Calculate
(a) the mean time a job spends in the network from arrival to exit
(b) the system bottleneck.

61. A system operates as follows. Arrivals occur to a single server according to a Poisson process with rate 2 per minute. If there is one job in the system, its processing time is exponentially distributed with mean 30 seconds. If there are two jobs in the system, job processing times are exponentially distributed with mean 20 seconds. If an arriving job finds two jobs in the system, it is lost.
(a) Calculate the steady-state probability that the server is idle.
(b) Suppose that the last arrival occurred twenty seconds ago. How long would you expect to wait until the next arrival occurs?

62. Consider a closed queueing network with $N = 3$ nodes and $M = 2$ jobs. Processing times at the three nodes are exponentially distributed with rates $\mu_1 = 0.7$, $\mu_2 = 0.1$ and

$\mu_3 = 1.5$. After processing at nodes 1 or 2, jobs always go back to node 3. After processing at node 3, jobs go to node 1 with probability 0.4 and node 2 with probability 0.6.
(a) Calculate the bottleneck node.
(b) Calculate the throughput at node 2.

63. The performance of a system with fault detection is evaluated as follows. When operating, faults occur at a rate of $\lambda = 1$ per hour, with fault times being exponentially distributed. With probability 0.99, a fault is detected and recovery takes an exponentially distributed period of time with mean 1 second. If the fault is not detected, recovery takes an exponentially distributed period of time with mean 5 minutes. Calculate the steady-state probability that the system is operating.

64. Consider the following open network. External arrivals occur to node 1 according to a Poisson process with rate 2 per second. External arrivals occur to node 2 according to a Poisson process with rate 1 per second. After processing at node 1, jobs immediately return to node 1 with probability 0.25, otherwise they go to node 3. After processing at node 2, jobs immediately return to node 2 with probability 0.2, otherwise they go to node 3. After processing at node 3, jobs leave the system. The processing times at each node are exponentially distributed with rate μ per second, 2 per second and 5 per second, respectively. There is one processor at each node.
(a) For what range of values of μ is node 1 the bottleneck (while still keeping the system stable)?
(b) Let the processing rate at node 1 be twice the maximum value in (a). What is the steady-state probability that all three processors are simultaneously busy?

65. Consider a system with arrivals following a Poisson process and having processing times that are exponentially distributed. Suppose that the state is the number of jobs in the system and the only non-zero rates are:

$$\begin{aligned} \lambda_0 &= \lambda \\ \lambda_n &= \frac{\lambda}{n^2}, \qquad n = 1, 2 \\ \mu_n &= \mu, \qquad n = 1, 2, 3. \end{aligned}$$

Calculate the steady-state probability that the system has less than 2 jobs present if $\lambda = 2$ per second and $\mu = 1$ per second.

66. Arrivals to a single server follow a Poisson process with rate 30 per hour. Service times are exponentially distributed with mean 60/35 minutes.
(a) What fraction of the arrival rate given in the question yields half the expected number in system? Explain why this fraction is more or less than one half.
(b) It is now 9:00. The last job arrived at 8:57. What is the probability that at least one job arrives between now and 9:02?

67. A system operates with a central server that processes incoming requests. The operating cost of the server is 20 dollars per hour, whether it is working or not. The cost of a waiting

user is 15 dollars per hour per user.
(a) If the processing time of a job is exponentially distributed with mean three minutes and the arriving requests follow a Poisson process with rate 15 per hour, should another server be purchased?
(b) If the processing times and interarrival times are both deterministic (the values are the same as the means in (a)), would your recommendation change?

68. A closed network operates as follows. It has 4 nodes, the processing rates being 5, 10, 10, 1 at the nodes, respectively. After finishing processing at node 1, jobs proceed to either node 2 or 3 with equal probability. After processing at either nodes 2 or 3, with probability 1/10 a job visits node 4, otherwise it returns to node 1. After processing at node 4, jobs return to node 1.

    Calculate the throughput of this system with 2 jobs circulating. If you were to improve one processing rate, which would it be? Would you expect your recommendation to change as we increase the number of jobs in the system?