Disproving Erdös's conjecture on multiplicities of complete subgraphs using computer

F. Franek, V. Rödl

Technical Report No. 88-11

Department of Computer Science and Systems

McMaster University

LIMITED DISTRIBUTION NOTICE

Disproving Erdös's conjecture on multiplicities of complete subgraphs using computer

F. Franek, V. Rödl

Abstract

Denote by $k_t(G)$ the number of cliques of order t in the graph G. Let $k_t(n) = min\{k_t(G) + k_t(\bar{G}) : |G| = n\}$, where $\bar{G}$ denotes the complement of G, and $|G|$ denotes the order of G. Let $c_t(n) = \frac{k_t(n)}{\binom{n}{t}}$, and let $c_t = lim_{n\to\infty} c_t(n)$. A well known conjecture of Erdös [E], related to Ramsey's theorem, states that $c_t = 2^{1-\binom{t}{2}}$. It was shown false by Thomason [T]. We present an alternative proof of the falsity of Erdös conjecture for $t = 4$, using graphs generated by computer. These graphs, though of large orders (2^{10} - 2^{14}), are rather simple and highly regular. The smallest lower bound for c_4 0.976501 obtained by this method is given by the graph on 10 elements (and hence of order 2^{10}) determined by the configuration $\{1, 3, 4, 7, 8, 10\}$, and by the graph on 11 elements (and hence of order 2^{11}) determined by the configuration $\{1, 3, 4, 7, 8, 10, 11\}$ (see Def. 1).

1. Introduction.

Denote by $k_t(G)$ the number of cliques of order t in the graph G. Let $k_t(n) = min\{k_t(G)+k_t(\bar{G}) : |G| = n\}$, where $\bar{G}$ denotes the complement of G, and $|G|$ denotes the order of G. Let $c_t(n) = \frac{k_t(n)}{\binom{n}{t}}$, and let $c_t = lim_{n\to\infty} c_t(n)$. Thus $c_t(n)$ denotes the minimum proportion of monochromatic K_t's in a coloring of the edges of K_n with two colors. A well known conjecture of Erdös [E], related to Ramsey's theorem, states that $c_t = 2^{1-\binom{t}{2}}$. It follows from Goodman's work [G], that the conjecture is true for $t = 3$. Erdös and Moon showed in [EM] that a modified conjecture for complete bipartite subgraphs of bipartite graphs is true. Sidorenko [S] showed that a modified conjecture is true for cycles, and not true for certain incomplete subgraphs. Erdös's conjecture is obviously true for random graphs, and it follows from results of various people that it is also true for "pseudo-random" graphs (see [GS], [FRW], [T1]). Thomason [T] disproved the conjecture in general, producing an infinite sequence from a single underlying graph leading to a limit smaller than what the conjecture stipulates. He obtained the following results (upper bounds): for $t = 4$, $0.976 \times 2^{1-\binom{4}{2}} = 0.976 \times \frac{1}{32}$, for $t = 5$, $0.906 \times 2^{1-\binom{5}{2}}$, and for $t \geq 6$, $0.936 \times 2^{1-\binom{t}{2}}$. His underlying graphs are formed by vectors in orthogonal geometries. As far as the lower bound, Giraud

[Gi] showed that $c_4 > \frac{1}{46}$. On the other hand the authors showed (a manuscript) that the conjecture not only holds for "pseudo-random" graphs, but also for graphs obtained by perturbing less than ϵn^2 edges of the "pseudo-random" graph, and that the conjecture holds for subgraphs on 4 vertices with 5 edges (i.e. K_4 less one edge).

Improving the upper bound for c_t is interesting for its relation to Ramsey numbers. If $c_t < \frac{1}{(2+\epsilon)^{\binom{t}{2}}}$, then the lower bound of diagonal Ramsey number $r(t,t)$ improves exponentially $\big($i.e. $log_2\ r(t,t) \geq$
$\frac{t}{2}{\cdot}log_2\ (2+\epsilon)$ from the current $\geq \frac{t}{2}{\cdot}(1+o(1))\big)$, and if $c_t = \frac{1}{(2+o(1))^{\binom{t}{2}}}$, then the upper bound improves exponentially $\big($i.e. $log_2\ r(t,t) \leq t{\cdot}log_2\ \frac{4\sqrt{2}}{\sqrt{\epsilon}}$ from the current $\leq 2t{\cdot}(1-o(1))\big)$ (see [R], [T2]).

It is easy to realize that in order to obtain an infinite sequence $\{G_n\}_{n=0}^{\infty}$ of graphs with a given value of $lim_{n\to\infty}\frac{k_4(G_n)+k_4(\bar{G}_n)}{\binom{|G_n|}{4}}$ it suffices to find just one graph that satisfies certain conditions (see Lemma 6 here, Lemma 1 in [T]). We are going to present an alternative way of obtaining the underlying graphs to get counterexamples to the conjecture for $t = 4$. The graphs were found (generated) by computer. These graphs, though of large orders (2^{10} - 2^{14}), are rather simple and highly regular.

Since the relative count of monochromatic K_4's in the random 2-coloring $2^{1-\binom{4}{2}} = \frac{1}{32}$ was the lowest value for the upper bound for c_4 known before Thomason's result [T], some attempts had been made to disprove Erdös's conjecture by modifying the structure of random graphs slightly to lower the value below $\frac{1}{32}$. The reason why they were not successful lies in the fact that "pseudo-random" graphs are not a good "start" for such modifications as they represent a local minimum of relative counts of monochromatic K_4's in a certain space of "representatives" of graphs (see our manuscript mentioned above). Moreover "small" modifications do not allow for getting out of the "hole". This realization lead us to computer-generate our graphs in a different way, starting with graphs "far away" from "pseudo-random" graphs, and in each step of the search a rather big class of edges is removed or added. As good candidates the so-called Boolean graphs were considered (see Def. 1). The effort, after a few weeks of computing on the departmental VAX 11/780 panned out. We obtained a class of graphs which all leads to infinite sequences with c_4 smaller than $1 \times \frac{1}{32}$ (see the appendix). The lowest upper bound for c_4 0.976501 obtained by this method is given by the sequence built from a graph $G_{X,F}$, where $|X| = 10$, and $F = \{1,3,4,7,8,10\}$, and coincidentally for $|X| = 11$ and $\{1,3,4,7,8,10,11\}$ (see the appendix). Interestingly, as Thomason graphs, these graphs have slightly different number of edges than non-edges, which runs counter the received wisdom prior to Thomason's work. The actual programs were written in the programming language C, and the source code and the results are presented in the appendix. The results presented here were re-computed on the departmental SUN 4/280-S.

2. Methods.

Def.1: Let X be a finite set, and let $F \subset \{1,2,...,|X|\}$. Graph $G_{X,F} = \langle V_{X,F}, E_{X,F} \rangle$ is defined by $V_{X,F} = \{a : a \subset X\}$, and $\{a,b\} \in E_{X,F}$ iff $|a \triangle b| \in F$, where $a \triangle b$ denotes the symmetric difference of sets a and b.

Note 2: By $\bar{F}$ we shall denote $\{1,2,...,|X|\} - F$. It follows that $\bar{G}_{X,F} = G_{X,\bar{F}}$, where $\bar{G}_{X,F}$ denotes the complement of $G_{X,F}$.

Def.3: Let X be a finite set, and let $F \subset \{1,2,...,|X|\}$.

(3.1) An ordered triple $\langle f_0, f_1, f_2 \rangle$ is an <u>X,F-triple</u>, if $f_0, f_1, f_2 \subset X$, $|f_i| \in F$ for each $i \leq 2$, and $|f_i \triangle f_j| \in F$ for all $i \neq j \leq 2$.

(3.2) An ordered pair $\langle f_0, f_1 \rangle$ is an <u>X,F-pair</u>, if $f_0, f_1 \subset X$, $|f_i| \in F$ for each $i \leq 1$, and $|f_i \triangle f_j| \in F$ for all $i \neq j \leq 1$.

(3.3) A singleton $\langle f_0 \rangle$ is an <u>X,F-singleton</u>, if $f_0 \subset X$, an $|f_0| \in F$.

Lemma 4: Let X be a finite set, and let $F \subset \{1,2,...,|X|\}$. Let $k_t(G_{X,F})$ denote the number of cliques of size t in the graph $G_{X,F}$. Let $tc(X,F)$ denote the number of X,F-triples, $pc(X,F)$ the number of X,F-pairs, and $sc(X,F)$ the number of X,F-singletons. Then

(4.1) $k_4(G_{X,F}) = \frac{2^{|X|}}{24} tc(X,F)$,

(4.2) $k_3(G_{X,F}) = \frac{2^{|X|}}{6} pc(X,F)$,

(4.3) $k_2(G_{X,F}) = \frac{2^{|X|}}{2} sc(X,F)$.

Proof: (1) Let $\{a,b,c,d\}$ be a 4-clique in $G_{X,F}$. It is easy to verify that $\langle a \triangle b, a \triangle c, a \triangle d \rangle$ is an X,F-triple, and all elements in the triple are mutually distinct. But any permutation of the three elements in the triple forms an X,F-triple. Thus there are 6 distinct X,F- triples. We could have chosen any of the vertices of the clique to determine the 6 X,F-triples, not just a. Therefore the clique $\{a,b,c,d\}$ determines $4 \times 6 = 24$ distinct X,F-triples. On the other hand, it is easy to show that if $\langle f_0, f_1, f_2 \rangle$ is an X,F-triple, and if $a \subset X$, then $\{a, a \triangle f_0, a \triangle f_1, a \triangle f_2\}$ is a 4-clique in $G_{X,F}$. Thus there are exactly $\frac{2^{|X|}}{24} tc(X,F)$ 4-cliques in $G_{X,F}$.

(2) The proof for 3-cliques is practically identical to (1), except one has to realize that there are 2 permutations of a pair, and a 3-clique has 3 vertices, hence each 3-clique determines $2 \times 3 = 6$ distinct X, F-pairs.

(3) The proof for 2-cliques (edges) is again practically identical to (1), except one has to realize that there is only 1 permutation of a singleton, and a 2-clique has 2 vertices, hence each 2-clique determines $1 \times 2 = 2$ X, F-singletons.

Similarly as Thomason did, we shall produce an infinite sequence of graphs from a single graph:

Def. 5: Let $G = \langle V, E\rangle$ be a graph, and let n be a positive integer. The graph $G_n = \langle V_n, E_n\rangle$ is defined as follows: let $\{B_v : v{\in}V\}$ be a system of mutually disjoint sets of size n. Then $V_n = \bigcup\{B_v : v{\in}V\}$. If $a, b{\in}B_v$, then $\{a, b\}{\in}E_n$, and if $a{\in}B_u$, $b{\in}B_v$, $u \neq v$, then $\{a, b\}{\in}E_n$ iff $\{u, v\}{\in}E$. (In other words, each vertex of G is "blown up" to a set of vertices of size n, every two vertices in a "blown up" G-vertex form an edge in G_n, and two vertices from different "blown up" G-vertices form an edge in G_n only if the original G-vertices formed an edge in G.)

Lemma 6: If all graphs in an infinite sequence of graphs $\{G_n\}_{n=0}^{\infty}$ were obtained from a single graph G of size t as in Def. 4, then

$$lim_{n\to\infty}\frac{k_4(G_n)+k_4(\bar{G}_n)}{\binom{tn}{4}} = \frac{24(k_4(G)+k_4(\bar{G}))+36k_3(G)+14k_2(G)+t}{t^4}\ .$$

Proof: Fix an n. Let's calculate the number of all 4-cliques in G_n; there are 5 possible cases:

(1) Each of the four vertices of the 4-clique are from distinct "blown up" G-vertices.

Since each vertex of such a 4-clique can be chosen independently, there are $n^4 k_4(G)$ such 4-cliques in G_n. Denote this number as $H_1(n)$.

(2) Two of the four vertices of the 4-clique are from the same "blown up" G-vertex, while the remaining two vertices are from distinct "blown-up" G-vertices.

There are 3 ways to choose the "blown up" G-vertex with two vertices, and $\binom{n}{2}$ ways to choose the vertices in it. The remaining two can be chosen independently, hence there are $3\binom{n}{2}n^2k_3(G)$ such

4-cliques in G_n. Denote this number as $H_2(n)$.

(3) Three of the four vertices of the 4-clique are from the same "blown up" G-vertex, while the remaining one vertex is from a different "blown-up" G-vertex.

There are 2 ways to choose the "blown up" G-vertex with three vertices, and $\binom{n}{3}$ ways to choose the vertices in it. The remaining one can be chosen independently, hence there are $2\binom{n}{3}nk_2(G)$ such 4-cliques in G_n. Denote this number as $H_3(n)$.

(4) Two of the four vertices of the 4-clique are from one "blown up" G-vertex, while the remaining two vertices are from a different "blown-up" G-vertex.

There are $\binom{n}{2}$ ways to choose the two vertices in each of the "blown up" G-vertices. Hence there are $\binom{n}{2}^2 k_2(G)$ such 4-cliques in G_n. Denote this number as $H_4(n)$.

(5 Finally, the four vertices of the 4-clique are from a single "blown up" G-vertex.

There are $\binom{n}{4}$ ways to choose the four vertices in the "blown up" G-vertex. Hence there are $\binom{n}{4}t$ such 4-cliques in G_n. Denote this number as $H_5(n)$.

$H_2(n) = \frac{3}{2}n^4k_3(G)O_2(n)$, where $O_2(n) = 1 - \frac{1}{n}$, and so $lim_{n\to\infty}O_2(n) = 1$.

$H_3(n) = \frac{1}{3}n^4k_2(G)O_3(n)$, where $O_3(n) = 1 - \frac{3}{n} + \frac{2}{n^2}$, and so $lim_{n\to\infty}O_3(n) = 1$.

$H_4(n) = \frac{1}{4}n^4k_2(G)O_4(n)$, where $O_4(n) = 1 - \frac{2}{n} + \frac{1}{n^2}$, and so $lim_{n\to\infty}O_4(n) = 1$.

$H_5(n) = \frac{1}{24}n^4tO_5(n)$, where $O_5(n) = 1 - \frac{6}{n} + \frac{11}{n^2} - \frac{6}{n^3}$, and so $lim_{n\to\infty}O_5(n) = 1$.

$\binom{tn}{4} = \frac{1}{24}t^4n^4O_6(n)$, where $O_6(n) = 1 - \frac{6}{nt} + \frac{11}{n^2t^2} - \frac{6}{n^3t^3}$, and so $lim_{n\to\infty}O_6(n) = 1$.

Hence $lim_{n\to\infty}\frac{k_4(G_n)}{\binom{tn}{4}} = lim_{n\to\infty}\frac{H_1(n)+H_2(n)+H_3(n)+H_4(n)+H_5(n)}{\binom{tn}{4}} = \frac{24k_4(G)+36k_3(G)+14k_2(G)+t}{t^4}$.

Since for $\bar{G}_n$ only case (1) can happen, $lim_{n\to\infty}\frac{k_4(\bar{G}_n)}{\binom{tn}{4}} = \frac{24k_4(\bar{G})}{t^4}$.

It follows that $lim_{n\to\infty}\frac{k_4(G_n)+k_4(\bar{G}_n)}{\binom{tn}{4}} = \frac{24(k_4(G)+k_4(\bar{G}))+36k_3(G)+14k_2(G)+t}{t^4}$. □

Def. 7: We shall call

$$32\frac{24(k_4(G)+k_4(\bar{G}))+36k_3(G)+14k_2(G)+t}{t^4}$$

Erdös's number of the graph G.

Lemma 8: Let X be a finite set, and let $F\subset\{1,2,...,|X|\}$. Let $tc(X,F)$ denote the number of X,F-triples, $tc(X,\bar{F})$ denote the number of $X,\bar{F}$-triples, $pc(X,F)$ the number of X,F-pairs, and $sc(X,F)$ the number of X,F-singletons. Then Erdös's number of $G_{X,F}$ =

$$\frac{tc(X,F)+tc(X,\bar{F})+6pc(X,F)+7sc(X,F)+1}{2^{3|X|-5}}\ .$$

Proof: Follows directly from Lemma 4 as $2^{|X|}$ is the size of the graph $G_{X,F}$.

Our task will be to find such a set X and such a family F so that Erdös's number of $G_{X,F}$ is less than 1. In the following we shall describe the algorithm to compute Erdös's number for given X and F.

Based on Lemma 8, it suffices to compute the number of X, F-triples, X, F-pairs, and X, F-singletons. (Note: the number of $X, \bar{F}$-triples can be computed by the same procedure having the family F as a parameter.)

How to compute the number of all X, F-triples:

Consider $\langle f_0, f_1, f_2 \rangle$, an ordered triple of mutually distinct subsets of X. Denote $|f_i|$ as a_i $(i \leq 2)$, $|f_0 \triangle f_1|$ as a_3, $|f_0 \triangle f_2|$ as a_4, and $|f_1 \triangle f_2|$ as a_5. Let $x_{012} = f_0 \cap f_1 \cap f_2$, let $x_{01} = (f_0 \cap f_1) - x_{012}$, let $x_{02} = (f_0 \cap f_2) - x_{012}$, let $x_{12} = (f_1 \cap f_2) - x_{012}$, let $x_0 = f_0 - (f_1 \cup f_2)$, let $x_1 = f_1 - (f_0 \cup f_2)$, and let $x_2 = f_2 - (f_0 \cup f_1)$. Then $x_0, x_1, x_2, x_{01}, x_{02}, x_{12}, x_{012}$ are mutually disjoint and $f_0 \cup f_1 \cup f_2 = x_0 \cup x_1 \cup x_2 \cup x_{01} \cup x_{02} \cup x_{12} \cup x_{012}$. Let $m_0 = |x_0|$, $m_1 = |x_1|$, $m_2 = |x_2|$, $m_{01} = |x_{01}|$, $m_{02} = |x_{02}|$, $m_{12} = |x_{12}|$, and $m_{012} = |x_{012}|$. Since f_0, f_1 and f_2 are mutually distinct, $2 \leq |f_0 \cup f_1 \cup f_2|$, and so $2 \leq m_0 + m_1 + m_2 + m_{01} + m_{02} + m_{12} + m_{012} \leq |X|$. Thus

$$m_0 + m_{01} + m_{02} + m_{012} = a_0,$$

$$m_1 + m_{01} + m_{12} + m_{012} = a_1,$$

$$m_2 + m_{02} + m_{12} + m_{012} = a_2,$$

$$m_0 + m_{02} + m_1 + m_{12} = a_3,$$

$$m_0 + m_{01} + m_2 + m_{12} = a_4,$$

$$m_1 + m_{01} + m_2 + m_{02} = a_5.$$

These equations lead to the following solutions:

$$m_0 = s_0 + m_{012}, \text{ where } s_0 = \tfrac{a_3 + a_4 - a_1 - a_2}{2},$$

$$m_1 = s_1 + m_{012}, \text{ where } s_1 = \tfrac{a_3 + a_5 - a_0 - a_2}{2},$$

$$m_2 = s_2 + m_{012}, \text{ where } s_2 = \tfrac{a_4 + a_5 - a_0 - a_1}{2},$$

$$m_{01} = s_{01} - m_{012}, \text{ where } s_{01} = \tfrac{a_0 + a_1 - a_3}{2},$$

$$m_{02} = s_{02} - m_{012}, \text{ where } s_{02} = \tfrac{a_0 + a_2 - a_4}{2},$$

$$m_{12} = s_{12} - m_{012}, \text{ where } s_{12} = \tfrac{a_1 + a_2 - a_5}{2}.$$

Thus $0 \leq m_{012} \leq |X|$, $-s_0 \leq m_{012} \leq |X| - s_0$, $-s_1 \leq m_{012} \leq |X| - s_1$, $-s_2 \leq m_{012} \leq |X| - s_2$, $-s_{01} - |X| \leq m_{012} \leq s_{01}$, $-s_{02} - |X| \leq m_{012} \leq s_{02}$, $-s_{12} - X| \leq m_{012} \leq s_{12}$, and $2 - s \leq m_{012} \leq |X| - s$, where $s = m_0 + m_1 + m_2 + m_{01} + m_{02} + m_{12} + m_{012}$. Let $s_{min} = max(0, -s_0, -s_1, -s_2, s_{01} - |X|, s_{02} - |X|, s_{12} - |X|, 2 - s)$, and let $s_{max} = min(|X|, |X| - s_0, |X| - s_1, |X| - s_2, s_{01}, s_{02}, s_{12}, |X| - s)$. Then $s_{min} \leq m_{012} \leq s_{max}$.

Generate all possible ordered 5-tuples $\langle a_0, a_1, a_2, a_3, a_4, a_5\rangle$ so that each $a_i \in F$. For each ordered 5-tuple $\langle a_0, a_1, a_2, a_3, a_4, a_5\rangle$ compute s_0, s_1, s_2, s_{01}, s_{02}, s_{12}, s, s_{min}, and s_{max}. If any of these are not integers, the 5-tuple has no solution, if $s_{max} < s_{min}$, then again the 5-tuple has no solution. If everything is all right, generate all possible m_{012} so that $s_{min} \leq m_{012} \leq s_{max}$. For each generated m_{012} compute $m_0 = s_0 + m_{012}$, $m_1 = s_1 + m_{012}$, $m_2 = s_2 + m_{012}$, $m_{01} = s_{01} - m_{012}$, $m_{02} = s_{02} - m_{012}$, and $m_{12} = s_{12} - m_{012}$. For each solution $m_0, m_1, m_2, m_{01}, m_{02}, m_{12}, m_{012}$ calculate

$\binom{|X|}{m_0} \cdot \binom{|X|-m_0}{m_1} \cdot \binom{|X|-m_0-m_1}{m_2} \cdot \binom{|X|-m_0-m_1}{m_2} \cdot \binom{|X|-m_0-m_1-m_2}{m_{01}} \cdot \binom{|X|-m_0-m_1-m_2-m_{01}}{m_{02}} \cdot$
$\cdot \binom{|X|-m_0-m_1-m_2-m_{01}-m_{02}}{m_{12}} \cdot \binom{|X|-m_0-m_1-m_2-m_{01}-m_{02}-m_{12}}{m_{012}}$.

Cumulate these numbers, and when done with all 5-tuples $\langle a_0, a_1, a_2, a_3, a_4, a_5\rangle$, we have the number of all X, F-triples.

How to compute the number of all X, F-pairs:

Consider $\langle f_0, f_1\rangle$, an ordered pair of mutually distinct subsets of X. Denote $|f_i|$ as a_i $(i \leq 1)$, $|f_0 \triangle f_1|$ as a_2. Let $x_{01} = f_0 \cap f_1$, let $x_0 = f_0 - f_1$, and let $x_1 = f_1 - f_0$. Then x_0, x_1, x_{01} are mutually disjoint and $f_0 \cup f_1 = x_0 \cup x_1 \cup x_{01}$. Let $m_0 = |x_0|$, $m_1 = |x_1|$, and $m_{01} = |x_{01}|$. Since f_0 and f_1 are mutually distinct, $2 \leq |f_0 \cup f_1|$, and so $2 \leq m_0 + m_1 + m_{01} \leq |X|$. Thus

$$m_0 + m_{01} = a_0,$$

$$m_1 + m_{01} = a_1,$$

$$m_0 + m_1 = a_2.$$

These equations lead to the following solutions:

$$m_0 = \tfrac{a_0+a_1-a_2}{2},$$

$$m_1 = \tfrac{a_0+a_2-a_1}{2}, \text{ and}$$

$$m_{01} = \tfrac{a_1+a_2-a_0}{2}.$$

Generate all possible ordered 3-tuples $\langle a_0, a_1, a_2\rangle$ so that each $a_i \in F$. For each ordered 3-tuple $\langle a_0, a_1, a_2\rangle$ compute m_0, m_1, and m_{01}. If any of these are not integers, the 3-tuple has no solution, if any of these are less than 0, or bigger than $|X|$, then again the 3-tuple has no solution. If $m_0 + m_1 + m_{01}$ is < 2 or $> |X|$, then again the 3-tuple has no solution. If everything is all right, calculate $\binom{|X|}{m_0} \cdot \binom{|X|-m_0}{m_1} \cdot$
$\binom{|X|-m_0-m_1}{m_2} \cdot \binom{|X|-m_0-m_1}{m_{01}}$.

Cumulate these numbers, and when done with all 3-tuples $\langle a_0, a_1, a_2\rangle$, we have the number of all X, F-pairs.

How to compute the number of all X, F-singletons:

Consider $\langle f_0 \rangle$, where f_0 is a subset of X. Denote $|f_0|$ as a_0. Generate all possible $a_0 \in F$. For each a_0 compute $\binom{|X|}{a_0}$. Cumulate these numbers, and when done with all $a_0 \in F$, we have the number of all X, F-singletons.

The computer program calculates for a given $|X|$ Erdös's number of $G_{X,F}$ for all possible families F in lexicographical order starting with the complete family F. Note that when F is compelte every subset of 4 vertices is a 4-clique, and no subset of 4 vertices is a 4-clique in the complement of any graph "blown up" from $G_{X,F}$. Hence Erdös's number of such a sequence must be 32. This can be used as one of criteria to test the credibility of the computer program. Also it is easy to verify by hand the number of F-singletons. For small X, even the number of F-pairs can be computed by hand. For the case of $|X|$ = 4 the complete generation of F-triples was checked by hand. All these tests were performed to our satisfaction and so our confidence in the program's results is high. The programs outputs the results of its calculations only when a new minimal value of Erdös's number is found. In the appendix the outputs for $|X|$ = 9, 10, 11, 12, 13 are presented. The results for $|X| < 10$ are not interesting as they do not lead to counterexamples to the conjecture. We also computed all configurations for $|X|$ = 14, but the number of configurations with Erdös's number less than 1 is too big to reproduce it here, but the least one obtained is 0.993069 .

References

[E] P.Erdös, On the number of complete subgraphs contained in certain graphs, *Publ. Math. Inst. Hung. Acad. Sci., VII, ser. A* 3 (1962), 459-464.

[G] A.W.Goodman, On sets of acquaintances and strangers at any party, *Amer.Math.Monthly*, 66 (1959), 778- 783.

[EM] P. Erdös and J.W.Moon, On subgraphs on the complete bipartite graph, *Canad.Math.Bull.* 7 (1964), 35-39.

[FRW] P.Frankl, V.Rödl, R.M.Wilson, The number of submatrices of given type in a Hadamard matrix and related results, *J.Comb.Theory*, 44 (1988), 317-328.

[Gi] G.Giraud, Sur le probleme de Goodman pour les quadrangles et la majoration des nombres de Ramsey, *J.Combin.Theory Ser. B* 30 (1979),237-253.

[GS] R.L.Graham, J.H.Spencer, A constructive solution to a tournament problem, *Canad.Math.Bull.* 14 (1971), 45-48.

[R] V.Rödl, Note on Ramsey numbers, *unpublished manuscript.*

[S] A.F.Sidorenko, Cycles in Graphs and Functional inequalities, *manuscript in Russian*, and also private communication.

[T] A.Thomason , A disproof of a conjecture of Erdös in Ramsey theory, *preprint.*

[T1] A.Thomason , Pseudo-random graphs, in "Proceedings of Random Graphs, Poznan, '85", (M. Karonski, ed.), *Discrete Applied Math.*, 307-331.

[T2] A.Thomason , On finite ramsey numbers, *European J. Combinatorics* 3 (1982) 263-273.