# Haskell 98 Language and Libraries The Revised Report 

Simon Peyton Jones (editor)

## Copyright notice.

The authors and publisher intend this Report to belong to the entire Haskell community, and grant permission to copy and distribute it for any purpose, provided that it is reproduced in its entirety, including this Notice. Modified versions of this Report may also be copied and distributed for any purpose, provided that the modified version is clearly presented as such, and that it does not claim to be a definition of the language Haskell 98.

## Contents

I The Haskell 98 Language ..... 1
1 Introduction ..... 3
1.1 Program Structure ..... 3
1.2 The Haskell Kernel ..... 4
1.3 Values and Types ..... 4
1.4 Namespaces ..... 5
2 Lexical Structure ..... 7
2.1 Notational Conventions ..... 7
2.2 Lexical Program Structure ..... 8
2.3 Comments ..... 9
2.4 Identifiers and Operators ..... 9
2.5 Numeric Literals ..... 11
2.6 Character and String Literals ..... 12
2.7 Layout ..... 13
3 Expressions ..... 15
3.1 Errors ..... 17
3.2 Variables, Constructors, Operators, and Literals ..... 17
3.3 Curried Applications and Lambda Abstractions ..... 19
3.4 Operator Applications ..... 19
3.5 Sections ..... 20
3.6 Conditionals ..... 21
3.7 Lists ..... 21
3.8 Tuples ..... 22
3.9 Unit Expressions and Parenthesized Expressions ..... 22
3.10 Arithmetic Sequences ..... 23
3.11 List Comprehensions ..... 23
3.12 Let Expressions ..... 24
3.13 Case Expressions ..... 25
3.14 Do Expressions ..... 26
3.15 Datatypes with Field Labels ..... 27
3.15.1 Field Selection ..... 27
3.15.2 Construction Using Field Labels ..... 28
3.15.3 Updates Using Field Labels ..... 29
3.16 Expression Type-Signatures ..... 30
3.17 Pattern Matching ..... 30
3.17.1 Patterns ..... 30
3.17.2 Informal Semantics of Pattern Matching ..... 31
3.17.3 Formal Semantics of Pattern Matching ..... 34
4 Declarations and Bindings ..... 37
4.1 Overview of Types and Classes ..... 38
4.1.1 Kinds ..... 39
4.1.2 Syntax of Types ..... 39
4.1.3 Syntax of Class Assertions and Contexts ..... 41
4.1.4 Semantics of Types and Classes ..... 42
4.2 User-Defined Datatypes ..... 43
4.2.1 Algebraic Datatype Declarations ..... 43
4.2.2 Type Synonym Declarations ..... 45
4.2.3 Datatype Renamings ..... 46
4.3 Type Classes and Overloading ..... 47
4.3.1 Class Declarations ..... 47
4.3.2 Instance Declarations ..... 49
4.3.3 Derived Instances ..... 51
4.3.4 Ambiguous Types, and Defaults for Overloaded Numeric Operations ..... 51
4.4 Nested Declarations ..... 53
4.4.1 Type Signatures ..... 53
4.4.2 Fixity Declarations ..... 54
4.4.3 Function and Pattern Bindings ..... 55
4.4.3.1 Function bindings ..... 56
4.4.3.2 Pattern bindings ..... 57
4.5 Static Semantics of Function and Pattern Bindings ..... 58
4.5.1 Dependency Analysis ..... 58
4.5.2 Generalization ..... 59
4.5.3 Context Reduction Errors ..... 59
4.5.4 Monomorphism ..... 60
4.5.5 The Monomorphism Restriction ..... 61
4.6 Kind Inference ..... 64
5 Modules ..... 65
5.1 Module Structure ..... 66
5.2 Export Lists ..... 66
5.3 Import Declarations ..... 69
5.3.1 What is imported ..... 69
5.3.2 Qualified import ..... 70
5.3.3 Local aliases ..... 70
5.3.4 Examples ..... 71
5.4 Importing and Exporting Instance Declarations ..... 71
5.5 Name Clashes and Closure ..... 72
5.5.1 Qualified names ..... 72
5.5.2 Name clashes ..... 72
5.5.3 Closure ..... 74
5.6 Standard Prelude ..... 74
5.6.1 The Prelude Module ..... 75
5.6.2 Shadowing Prelude Names ..... 75
5.7 Separate Compilation ..... 76
5.8 Abstract Datatypes ..... 76
6 Predefined Types and Classes ..... 79
6.1 Standard Haskell Types ..... 79
6.1.1 Booleans ..... 79
6.1.2 Characters and Strings ..... 79
6.1.3 Lists ..... 80
6.1.4 Tuples ..... 80
6.1.5 The Unit Datatype ..... 81
6.1.6 Function Types ..... 81
6.1.7 The IO and IOError Types ..... 81
6.1.8 Other Types ..... 81
6.2 Strict Evaluation ..... 81
6.3 Standard Haskell Classes ..... 82
6.3.1 The Eq Class ..... 82
6.3.2 The Ord Class ..... 84
6.3.3 The Read and Show Classes ..... 85
6.3.4 The Enum Class ..... 86
6.3.5 The Functor Class ..... 87
6.3.6 The Monad Class ..... 88
6.3.7 The Bounded Class ..... 89
6.4 Numbers ..... 89
6.4.1 Numeric Literals ..... 90
6.4.2 Arithmetic and Number-Theoretic Operations ..... 90
6.4.3 Exponentiation and Logarithms ..... 91
6.4.4 Magnitude and Sign ..... 92
6.4.5 Trigonometric Functions ..... 93
6.4.6 Coercions and Component Extraction ..... 93
7 Basic Input/Output ..... 95
7.1 Standard I/O Functions ..... 95
7.2 Sequencing I/O Operations ..... 97
7.3 Exception Handling in the I/O Monad ..... 98
8 Standard Prelude ..... 101
8.1 Prelude PreludeList ..... 115
8.2 Prelude PreludeText ..... 121
8.3 Prelude PreludeIO ..... 125
9 Syntax Reference ..... 127
9.1 Notational Conventions ..... 127
9.2 Lexical Syntax ..... 128
9.3 Layout ..... 130
9.4 Literate comments ..... 134
9.5 Context-Free Syntax ..... 136
10 Specification of Derived Instances ..... 141
10.1 Derived instances of Eq and Ord ..... 142
10.2 Derived instances of Enum ..... 142
10.3 Derived instances of Bounded ..... 143
10.4 Derived instances of Read and Show ..... 143
10.5 An Example ..... 145
11 Compiler Pragmas ..... 147
11.1 Inlining ..... 147
11.2 Specialization ..... 147
II The Haskell 98 Libraries ..... 149
12 Rational Numbers ..... 151
12.1 Library Ratio ..... 153
13 Complex Numbers ..... 155
13.1 Library Complex ..... 156
14 Numeric ..... 159
14.1 Showing functions ..... 160
14.2 Reading functions ..... 161
14.3 Miscellaneous ..... 161
14.4 Library Numeric ..... 161
15 Indexing Operations ..... 169
15.1 Deriving Instances of Ix ..... 170
15.2 Library Ix ..... 172
16 Arrays ..... 173
16.1 Array Construction ..... 174
16.1.1 Accumulated Arrays ..... 174
16.2 Incremental Array Updates ..... 175
16.3 Derived Arrays ..... 176
16.4 Library Array ..... 176
17 List Utilities ..... 179
17.1 Indexing lists ..... 182
17.2 "Set" operations ..... 182
17.3 List transformations ..... 183
17.4 unfoldr ..... 183
17.5 Predicates ..... 184
17.6 The "By" operations ..... 184
17.7 The "generic" operations ..... 185
17.8 Further "zip" operations ..... 185
17.9 Library List ..... 186
18 Maybe Utilities ..... 193
18.1 Library Maybe ..... 194
19 Character Utilities ..... 195
19.1 Library Char ..... 197
20 Monad Utilities ..... 201
20.1 Naming conventions ..... 203
20.2 Class MonadPlus ..... 203
20.3 Functions ..... 204
20.4 Library Monad ..... 206
21 Input/Output ..... 209
21.1 I/O Errors ..... 212
21.2 Files and Handles ..... 213
21.2.1 Standard Handles ..... 213
21.2.2 Semi-Closed Handles ..... 214
21.2.3 File locking ..... 214
21.3 Opening and Closing Files ..... 214
21.3.1 Opening Files ..... 214
21.3.2 Closing Files ..... 215
21.4 Determining the Size of a File ..... 215
21.5 Detecting the End of Input ..... 215
21.6 Buffering Operations ..... 215
21.6.1 Flushing Buffers ..... 217
21.7 Repositioning Handles ..... 217
21.7.1 Revisiting an I/O Position ..... 217
21.7.2 Seeking to a new Position ..... 217
21.8 Handle Properties ..... 218
21.9 Text Input and Output ..... 218
21.9.1 Checking for Input ..... 218
21.9.2 Reading Input ..... 218
21.9.3 Reading Ahead ..... 219
21.9.4 Reading The Entire Input ..... 219
21.9.5 Text Output ..... 219
21.10Examples ..... 219
21.10.1 Summing Two Numbers ..... 219
21.10.2 Copying Files ..... 220
21.11Library IO ..... 221
22 Directory Functions ..... 223
23 System Functions ..... 229
24 Dates and Times ..... 231
24.1 Library Time ..... 234
25 Locale ..... 239
25.1 Library Locale ..... 240
26 CPU Time ..... 241
27 Random Numbers ..... 243
27.1 The RandomGen class, and the StdGen generator ..... 245
27.2 The Random class ..... 247
27.3 The global random number generator ..... 248
References ..... 249
Index ..... 251

## Preface

"Some half dozen persons have written technically on combinatory logic, and most of these, including ourselves, have published something erroneous. Since some of our fellow sinners are among the most careful and competent logicians on the contemporary scene, we regard this as evidence that the subject is refractory. Thus fullness of exposition is necessary for accuracy; and excessive condensation would be false economy here, even more than it is ordinarily."

## Haskell B. Curry and Robert Feys

in the Preface to Combinatory Logic [2], May 31, 1956

In September of 1987 a meeting was held at the conference on Functional Programming Languages and Computer Architecture (FPCA '87) in Portland, Oregon, to discuss an unfortunate situation in the functional programming community: there had come into being more than a dozen non-strict, purely functional programming languages, all similar in expressive power and semantic underpinnings. There was a strong consensus at this meeting that more widespread use of this class of functional languages was being hampered by the lack of a common language. It was decided that a committee should be formed to design such a language, providing faster communication of new ideas, a stable foundation for real applications development, and a vehicle through which others would be encouraged to use functional languages. This document describes the result of that committee's efforts: a purely functional programming language called Haskell, named after the logician Haskell B. Curry whose work provides the logical basis for much of ours.

## Goals

The committee's primary goal was to design a language that satisfied these constraints:

1. It should be suitable for teaching, research, and applications, including building large systems.
2. It should be completely described via the publication of a formal syntax and semantics.
3. It should be freely available. Anyone should be permitted to implement the language and distribute it to whomever they please.
4. It should be based on ideas that enjoy a wide consensus.
5. It should reduce unnecessary diversity in functional programming languages.

## Haskell 98: language and libraries

The committee intended that Haskell would serve as a basis for future research in language design, and hoped that extensions or variants of the language would appear, incorporating experimental features.

Haskell has indeed evolved continuously since its original publication. By the middle of 1997, there had been four iterations of the language design (the latest at that point being Haskell 1.4). At the 1997 Haskell Workshop in Amsterdam, it was decided that a stable variant of Haskell was needed; this stable language is the subject of this Report, and is called "Haskell 98 ".

Haskell 98 was conceived as a relatively minor tidy-up of Haskell 1.4, making some simplifications, and removing some pitfalls for the unwary. It is intended to be a "stable" language in sense the implementors are committed to supporting Haskell 98 exactly as specified, for the foreseeable future.

The original Haskell Report covered only the language, together with a standard library called the Prelude. By the time Haskell 98 was stabilised, it had become clear that many programs need access to a larger set of library functions (notably concerning input/output and simple interaction with the operating system). If these program were to be portable, a set of libraries would have to be standardised too. A separate effort was therefore begun by a distinct (but overlapping) committee to fix the Haskell 98 Libraries.

The Haskell 98 Language and Library Reports were published in February 1999.

## Revising the Haskell 98 Reports

After a year or two, many typographical errors and infelicities had been spotted. I took on the role of gathering and acting on these corrections, with the following goals:

- Correct typographical errors.
- Clarify obscure passages.
- Resolve ambiguities.
- With reluctance, make small changes to make the overall language more consistent.

This task turned out to be much, much larger than I had anticipated. As Haskell becomes more widely used, the Report has been scrutinised by more and more people, and I have adopted hundreds of (mostly small) changes as a result of their feedback. The original committees ceased to exist when the original Haskell 98 Reports were published, so every change was instead proposed to the entire Haskell mailing list.

This document is the outcome of this process of refinement. It includes both the Haskell 98 Language Report and the Libraries Report, and constitutes the official specification of both. It is not a
tutorial on programming in Haskell such as the 'Gentle Introduction' [6], and some familiarity with functional languages is assumed.

The entire text of both Reports is available online (see "Haskell resources" below).

## Extensions to Haskell 98

Haskell continues to evolve, going well beyond Haskell 98. For example, at the time of writing there are Haskell implementations that support:

Syntactic sugar, including:

- pattern guards;
- recursive do-notation;
- lexically scoped type variables;
- meta-programming facilities;

Type system innovations, including:

- multi-parameter type classes;
- functional dependencies;
- existential types;
- local universal polymorphism and arbitrary rank-types;

Control extensions, including:

- monadic state;
- exceptions;
- concurrency;

There is more besides. Haskell 98 does not impede these developments. Instead, it provides a stable point of reference, so that those who wish to write text books, or use Haskell for teaching, can do so in the knowledge that Haskell 98 will continue to exist.

## Haskell Resources

The Haskell web site
http://haskell.org
gives access to many useful resources, including:

- Online versions of the language and library definitions, including a complete list of all the differences between Haskell 98 as published in February 1999 and this revised version.
- Tutorial material on Haskell.
- Details of the Haskell mailing list.
- Implementations of Haskell.
- Contributed Haskell tools and libraries.
- Applications of Haskell.

You are welcome to comment on, suggest improvements to, and criticise the language or its presentation in the report, via the Haskell mailing list.

## Building the language

Haskell was created, and continues to be sustained, by an active community of researchers and application programmers. Those who served on the Language and Library committees, in particular, devoted a huge amount of time and energy to the language. Here they are, with their affiliation(s) for the relevant period:

Arvind (MIT)
Lennart Augustsson (Chalmers University) Dave Barton (Mitre Corp)
Brian Boutel (Victoria University of Wellington)
Warren Burton (Simon Fraser University)
Jon Fairbairn (University of Cambridge)
Joseph Fasel (Los Alamos National Laboratory)
Andy Gordon (University of Cambridge)
Maria Guzman (Yale University)
Kevin Hammond (Uniiversity of Glasgow)
Ralf Hinze (University of Bonn)
Paul Hudak [editor] (Yale University)
John Hughes [editor] (University of Glasgow; Chalmers University)
Thomas Johnsson (Chalmers University)
Mark Jones (Yale University, University of Nottingham, Oregon Graduate Institute)
Dick Kieburtz (Oregon Graduate Institute)
John Launchbury (University of Glasgow; Oregon Graduate Institute)
Erik Meijer (Utrecht University)
Rishiyur Nikhil (MIT)
John Peterson (Yale University)
Simon Peyton Jones [editor] (University of Glasgow; Microsoft Research Ltd)

Mike Reeve (Imperial College)<br>Alastair Reid (University of Glasgow)<br>Colin Runciman (University of York)<br>Philip Wadler [editor] (University of Glasgow)<br>David Wise (Indiana University)<br>Jonathan Young (Yale University)

Those marked [editor] served as the co-ordinating editor for one or more revisions of the language.
In addition, dozens of other people made helpful contributions, some small but many substantial. They are as follows: Kris Aerts, Hans Aberg, Sten Anderson, Richard Bird, Stephen Blott, Tom Blenko, Duke Briscoe, Paul Callaghan, Magnus Carlsson, Mark Carroll, Manuel Chakravarty, Franklin Chen, Olaf Chitil, Chris Clack, Guy Cousineau, Tony Davie, Craig Dickson, Chris Dornan, Laura Dutton, Chris Fasel, Pat Fasel, Sigbjorn Finne, Michael Fryers, Andy Gill, Mike Gunter, Cordy Hall, Mark Hall, Thomas Hallgren, Matt Harden, Klemens Hemm, Fergus Henderson, Dean Herington, Ralf Hinze, Bob Hiromoto, Nic Holt, Ian Holyer, Randy Hudson, Alexander Jacobson, Patrik Jansson, Robert Jeschofnik, Orjan Johansen, Simon B. Jones, Stef Joosten, Mike Joy, Stefan Kahrs, Antti-Juhani Kaijanaho, Jerzy Karczmarczuk, Wolfram Kahl, Kent Karlsson, Richard Kelsey, Siau-Cheng Khoo, Amir Kishon, Feliks Kluzniak, Jan Kort, Marcin Kowalczyk, Jose Labra, Jeff Lewis, Mark Lillibridge, Bjorn Lisper, Sandra Loosemore, Pablo Lopez, Olaf Lubeck, Ian Lynagh, Christian Maeder, Ketil Malde, Simon Marlow, Michael Marte, Jim Mattson, John Meacham, Sergey Mechveliani, Gary Memovich, Randy Michelsen, Rick Mohr, Andy Moran, Graeme Moss, Henrik Nilsson, Arthur Norman, Nick North, Chris Okasaki, Bjarte M. Østvold, Paul Otto, Sven Panne, Dave Parrott, Ross Paterson, Larne Pekowsky, Rinus Plasmeijer, Ian Poole, Stephen Price, John Robson, Andreas Rossberg, George Russell, Patrick Sansom, Felix Schroeter, Julian Seward, Nimish Shah, Christian Sievers, Libor Skarvada, Jan Skibinski, Lauren Smith, Raman Sundaresh, Josef Svenningsson, Ken Takusagawa, Satish Thatte, Simon Thompson, Tom Thomson, Tommy Thorn, Dylan Thurston, Mike Thyer, Mark Tullsen, David Tweed, Pradeep Varma, Malcolm Wallace, Keith Wansbrough, Tony Warnock, Michael Webber, Carl Witty, Stuart Wray, and Bonnie Yantis.

Finally, aside from the important foundational work laid by Church, Rosser, Curry, and others on the lambda calculus, it is right to acknowledge the influence of many noteworthy programming languages developed over the years. Although it is difficult to pinpoint the origin of many ideas, the following languages were particularly influential: Lisp (and its modern-day incarnations Common Lisp and Scheme); Landin's ISWIM; APL; Backus's FP [1]; ML and Standard ML; Hope and Hope $^{+}$; Clean; Id; Gofer; Sisal; and Turner's series of languages culminating in Miranda ${ }^{1}$. Without these forerunners Haskell would not have been possible.

[^0]Simon Peyton Jones
Cambridge, September 2002

## Part I

## The Haskell 98 Language

## Chapter 1

## Introduction

Haskell is a general purpose, purely functional programming language incorporating many recent innovations in programming language design. Haskell provides higher-order functions, non-strict semantics, static polymorphic typing, user-defined algebraic datatypes, pattern-matching, list comprehensions, a module system, a monadic I/O system, and a rich set of primitive datatypes, including lists, arrays, arbitrary and fixed precision integers, and floating-point numbers. Haskell is both the culmination and solidification of many years of research on non-strict functional languages.

This report defines the syntax for Haskell programs and an informal abstract semantics for the meaning of such programs. We leave as implementation dependent the ways in which Haskell programs are to be manipulated, interpreted, compiled, etc. This includes such issues as the nature of programming environments and the error messages returned for undefined programs (i.e. programs that formally evaluate to $\perp$ ).

### 1.1 Program Structure

In this section, we describe the abstract syntactic and semantic structure of Haskell, as well as how it relates to the organization of the rest of the report.

1. At the topmost level a Haskell program is a set of modules, described in Chapter 5. Modules provide a way to control namespaces and to re-use software in large programs.
2. The top level of a module consists of a collection of declarations, of which there are several kinds, all described in Chapter 4. Declarations define things such as ordinary values, datatypes, type classes, and fixity information.
3. At the next lower level are expressions, described in Chapter 3. An expression denotes a value and has a static type; expressions are at the heart of Haskell programming "in the small."
4. At the bottom level is Haskell's lexical structure, defined in Chapter 2. The lexical structure captures the concrete representation of Haskell programs in text files.

This report proceeds bottom-up with respect to Haskell's syntactic structure.
The chapters not mentioned above are Chapter 6, which describes the standard built-in datatypes and classes in Haskell, and Chapter 7, which discusses the I/O facility in Haskell (i.e. how Haskell programs communicate with the outside world). Also, there are several chapters describing the Prelude, the concrete syntax, literate programming, the specification of derived instances, and pragmas supported by most Haskell compilers.

Examples of Haskell program fragments in running text are given in typewriter font:

```
let x = 1
    z = x+y
in z+1
```

"Holes" in program fragments representing arbitrary pieces of Haskell code are written in italics, as in if $e_{1}$ then $e_{2}$ else $e_{3}$. Generally the italicized names are mnemonic, such as $e$ for expressions, $d$ for declarations, $t$ for types, etc.

### 1.2 The Haskell Kernel

Haskell has adopted many of the convenient syntactic structures that have become popular in functional programming. In this Report, the meaning of such syntactic sugar is given by translation into simpler constructs. If these translations are applied exhaustively, the result is a program written in a small subset of Haskell that we call the Haskell kernel.

Although the kernel is not formally specified, it is essentially a slightly sugared variant of the lambda calculus with a straightforward denotational semantics. The translation of each syntactic structure into the kernel is given as the syntax is introduced. This modular design facilitates reasoning about Haskell programs and provides useful guidelines for implementors of the language.

### 1.3 Values and Types

An expression evaluates to a value and has a static type. Values and types are not mixed in Haskell. However, the type system allows user-defined datatypes of various sorts, and permits not only parametric polymorphism (using a traditional Hindley-Milner type structure) but also ad hoc polymorphism, or overloading (using type classes).

Errors in Haskell are semantically equivalent to $\perp$. Technically, they are not distinguishable from nontermination, so the language includes no mechanism for detecting or acting upon errors. However, implementations will probably try to provide useful information about errors. See Section 3.1.

### 1.4 Namespaces

There are six kinds of names in Haskell: those for variables and constructors denote values; those for type variables, type constructors, and type classes refer to entities related to the type system; and module names refer to modules. There are two constraints on naming:

1. Names for variables and type variables are identifiers beginning with lowercase letters or underscore; the other four kinds of names are identifiers beginning with uppercase letters.
2. An identifier must not be used as the name of a type constructor and a class in the same scope.

These are the only constraints; for example, Int may simultaneously be the name of a module, class, and constructor within a single scope.

## Chapter 2

## Lexical Structure

In this chapter, we describe the low-level lexical structure of Haskell. Most of the details may be skipped in a first reading of the report.

### 2.1 Notational Conventions

These notational conventions are used for presenting syntax:

$$
\begin{array}{cl}
{[\text { pattern }]} & \text { optional } \\
\{\text { pattern }\} & \text { zero or more repetitions } \\
(\text { pattern }) & \text { grouping } \\
\text { pat }_{1} \mid \text { pat }_{2} & \text { choice } \\
\text { pat }_{\langle\text {pat' }\rangle} & \text { difference-elements generated by pat } \\
& \text { except those generated by pat } \\
\text { fibonacci } & \text { terminal syntax in typewriter font }
\end{array}
$$

Because the syntax in this section describes lexical syntax, all whitespace is expressed explicitly; there is no implicit space between juxtaposed symbols. BNF-like syntax is used throughout, with productions having the form:
nonterm $\rightarrow$ alt $_{1} \mid$ alt $_{2}|\ldots|$ alt $_{n}$
Care must be taken in distinguishing metalogical syntax such as | and [...] from concrete terminal syntax (given in typewriter font) such as | and [...], although usually the context makes the distinction clear.

Haskell uses the Unicode [11] character set. However, source programs are currently biased toward the ASCII character set used in earlier versions of Haskell.

This syntax depends on properties of the Unicode characters as defined by the Unicode consortium. Haskell compilers are expected to make use of new versions of Unicode as they are made available.

### 2.2 Lexical Program Structure

```
program \(\rightarrow \quad\{\) lexeme \(\mid\) whitespace \(\}\)
lexeme \(\rightarrow\) qvarid \(\mid\) qconid \(\mid\) qvarsym \(\mid\) qconsym
    | literal \(\mid\) special \(\mid\) reservedop \(\mid\) reservedid
literal \(\quad \rightarrow \quad\) integer \(\mid\) float \(\mid\) char \(\mid\) string
special \(\rightarrow(\mid)|,|;|[\mid]| `|\{\mid\}\)
whitespace \(\rightarrow\) whitestuff \{whitestuff \}
whitestuff \(\rightarrow\) whitechar \(\mid\) comment \(\mid\) ncomment
whitechar \(\rightarrow\) newline \(\mid\) vertab \(\mid\) space \(\mid\) tab \(\mid\) uniWhite
newline \(\rightarrow\) return linefeed \(\mid\) return \(\mid\) linefeed \(\mid\) formfeed
return \(\rightarrow\) a carriage return
linefeed \(\rightarrow\) a line feed
vertab \(\rightarrow\) a vertical tab
formfeed \(\rightarrow\) a form feed
space \(\quad \rightarrow \quad\) a space
\(t a b \quad \rightarrow \quad\) a horizontal tab
uniWhite \(\rightarrow\) any Unicode character defined as whitespace
comment \(\rightarrow\) dashes \(\left[a n y_{\langle s y m b o l\rangle}\{a n y\}\right]\) newline
dashes \(\rightarrow--\{-\}\)
opencom \(\rightarrow\{-\)
closecom \(\rightarrow \quad-\}\)
ncomment \(\rightarrow\) opencom ANYseq \{ncomment ANYseq\} closecom
ANYseq \(\rightarrow\{A N Y\}_{\langle\{A N Y\}}\) (opencom \(\mid\) closecom ) \{ANY\} \(\rangle\)
ANY \(\quad \rightarrow \quad\) graphic \(\mid\) whitechar
any \(\quad \rightarrow \quad\) graphic \(\mid\) space \(\mid\) tab
graphic \(\rightarrow\) small \(\mid\) large \(\mid\) symbol \(\mid\) digit \(\mid\) special \(|:|"| '\)
small \(\quad \rightarrow \quad\) ascSmall \(\mid\) uniSmall \(\mid-\)
ascSmall \(\rightarrow\) a \(|\mathrm{b}| \ldots \mid \mathrm{z}\)
uniSmall \(\rightarrow\) any Unicode lowercase letter
large \(\rightarrow\) ascLarge \(\mid\) uniLarge
ascLarge \(\rightarrow\) A \(|\mathrm{B}| \ldots \mid \mathrm{Z}\)
uniLarge \(\rightarrow\) any uppercase or titlecase Unicode letter
```



```
ascSymbol \(\rightarrow \quad!|\#| \$|\%| \&|*|+|.|/|<|=|>|?| @\)
    \(|\quad \backslash|^{\wedge}| ||-|^{\sim}\)
uniSymbol \(\rightarrow\) any Unicode symbol or punctuation
digit \(\quad \rightarrow \quad\) ascDigit \(\mid\) uniDigit
```

```
ascDigit }->0|1|\ldots|
uniDigit }->\mathrm{ any Unicode decimal digit
octit }->00|1|\ldots|
hexit }\quad->\quad\operatorname{digit}|\textrm{A}|\ldots|\textrm{F}|\textrm{a}|\ldots|\textrm{f
```

Lexical analysis should use the "maximal munch" rule: at each point, the longest possible lexeme satisfying the lexeme production is read. So, although case is a reserved word, cases is not. Similarly, although $=$ is reserved, $==$ and $\sim=$ are not.

Any kind of whitespace is also a proper delimiter for lexemes.
Characters not in the category $A N Y$ are not valid in Haskell programs and should result in a lexing error.

### 2.3 Comments

Comments are valid whitespace.
An ordinary comment begins with a sequence of two or more consecutive dashes (e.g. --) and extends to the following newline. The sequence of dashes must not form part of a legal lexeme. For example, "-->" or "|--" do not begin a comment, because both of these are legal lexemes; however "--foo" does start a comment.

A nested comment begins with " $\{-$ " and ends with " -$\}$ ". No legal lexeme starts with " $\{-$ "; hence, for example, " $\{---"$ starts a nested comment despite the trailing dashes.

The comment itself is not lexically analysed. Instead, the first unmatched occurrence of the string " -$\}$ " terminates the nested comment. Nested comments may be nested to any depth: any occurrence of the string " $\{-$ " within the nested comment starts a new nested comment, terminated by " -$\}$ ". Within a nested comment, each " $\{-$ " is matched by a corresponding occurrence of " -$\}$ ".

In an ordinary comment, the character sequences " $\{-$ " and " -$\}$ " have no special significance, and, in a nested comment, a sequence of dashes has no special significance.

Nested comments are also used for compiler pragmas, as explained in Chapter 11.
If some code is commented out using a nested comment, then any occurrence of $\{-$ or -$\}$ within a string or within an end-of-line comment in that code will interfere with the nested comments.

### 2.4 Identifiers and Operators

varid $\quad \rightarrow \quad\left(\text { small }\left\{\text { small } \mid \text { large } \mid \text { digit }\left.\right|^{\prime}\right\}\right)_{\langle\text {reservedid }\rangle}$

```
conid }->\quad\mathrm{ large {small | large | digit |'}
reservedid }->\mathrm{ case|class|data|default|deriving|do|else
    | if|import|in|infix|infixl|infixr|instance
    | let|module|newtype|of|then|type|where|_
```

An identifier consists of a letter followed by zero or more letters, digits, underscores, and single quotes. Identifiers are lexically distinguished into two namespaces (Section 1.4): those that begin with a lower-case letter (variable identifiers) and those that begin with an upper-case letter (constructor identifiers). Identifiers are case sensitive: name, naMe, and Name are three distinct identifiers (the first two are variable identifiers, the last is a constructor identifier).

Underscore, "-", is treated as a lower-case letter, and can occur wherever a lower-case letter can. However, "-" all by itself is a reserved identifier, used as wild card in patterns. Compilers that offer warnings for unused identifiers are encouraged to suppress such warnings for identifiers beginning with underscore. This allows programmers to use "_foo" for a parameter that they expect to be unused.

```
varsym }->\mathrm{ (symbol {symbol |:})}\mp@subsup{\}{\langlereservedop | dashes\rangle}{
consym }->\quad(:{symbol|:})\langlereservedop
reservedop -> .. |:|:: |=|\|||<- |-> @@ | | =>
```

Operator symbols are formed from one or more symbol characters, as defined above, and are lexically distinguished into two namespaces (Section 1.4):

- An operator symbol starting with a colon is a constructor.
- An operator symbol starting with any other character is an ordinary identifier.

Notice that a colon by itself, ":", is reserved solely for use as the Haskell list constructor; this makes its treatment uniform with other parts of list syntax, such as " [] " and " $[a, b]$ ".

Other than the special syntax for prefix negation, all operators are infix, although each infix operator can be used in a section to yield partially applied operators (see Section 3.5). All of the standard infix operators are just predefined symbols and may be rebound.

In the remainder of the report six different kinds of names will be used:

```
varid (variables)
conid (constructors)
tyvar }->\mathrm{ varid (type variables)
tycon 相 conid (type constructors)
tycls }->\mathrm{ conid (type classes)
modid }->\mathrm{ conid (modules)
```

Variables and type variables are represented by identifiers beginning with small letters, and the other four by identifiers beginning with capitals; also, variables and constructors have infix forms, the other four do not. Namespaces are also discussed in Section 1.4.

A name may optionally be qualified in certain circumstances by prepending them with a module identifier. This applies to variable, constructor, type constructor and type class names, but not type variables or module names. Qualified names are discussed in detail in Chapter 5.

```
qvarid }->\quad[modid .] varid
qconid }->\quad[modid .] conid
qtycon }->\quad[modid.]tyco
qtycls }->\quad[modid.]tycl
qvarsym }->\mathrm{ [modid .] varsym
qconsym }->\quad[modid .] consy
```

Since a qualified name is a lexeme, no spaces are allowed between the qualifier and the name. Sample lexical analyses are shown below.

| This | Lexes as this |
| :--- | :--- |
| f.g | f $\cdot$ g (three tokens) |
| F.g | F.g (qualified 'g') |
| f.. | f . . (two tokens) |
| F. . | F. . (qualified '.') |
| F. | F . (two tokens) |

The qualifier does not change the syntactic treatment of a name; for example, Prelude. + is an infix operator with the same fixity as the definition of + in the Prelude (Section 4.4.2).

### 2.5 Numeric Literals

```
decimal \(\rightarrow\) digit\{digit\}
octal \(\rightarrow\) octit\{octit\}
hexadecimal \(\rightarrow \quad\) hexit \(\{\) hexit \(\}\)
integer \(\quad \rightarrow \quad\) decimal
    | 00 octal \(\mid 00\) octal
    | 0 x hexadecimal \(\mid 0 \mathrm{X}\) hexadecimal
float \(\quad \rightarrow \quad\) decimal. decimal [exponent]
    | decimal exponent
exponent \(\rightarrow(\mathrm{e} \mid \mathrm{E})[+\mid-]\) decimal
```

There are two distinct kinds of numeric literals: integer and floating. Integer literals may be given in decimal (the default), octal (prefixed by 00 or 00 ) or hexadecimal notation (prefixed by 0 x or 0 x ). Floating literals are always decimal. A floating literal must contain digits both before and after the decimal point; this ensures that a decimal point cannot be mistaken for another use of the dot character. Negative numeric literals are discussed in Section 3.4. The typing of numeric literals is discussed in Section 6.4.1.

### 2.6 Character and String Literals

```
char \(\rightarrow \quad\) ( graphic \(_{\langle\prime}|\backslash\rangle \mid\) space \(\mid\) escape \(\left._{\langle\backslash \&\rangle}\right)^{\prime}\)
string \(\quad \rightarrow \quad\) " \(\{\) graphic \(\langle\mathbf{n} \mid \backslash\rangle\) space \(\mid\) escape \(\mid\) gap \(\}\) "
escape \(\rightarrow\) (charesc \(\mid\) ascii \(\mid\) decimal \(\mid \circ\) octal \(\mid \mathrm{x}\) hexadecimal \()\)
charesc \(\rightarrow \mathrm{a}|\mathrm{b}| \mathrm{f}|\mathrm{n}| \mathrm{r}|\mathrm{t}| \mathrm{v}|\backslash| "|r| \&\)
ascii \(\quad \rightarrow \quad{ }^{\wedge}\) cntrl \(\mid\) NUL \(\mid\) SOH \(\mid\) STX \(\mid\) ETX \(|E O T| E N Q \mid A C K\)
    | BEL|BS|HT|LF|VT|FF|CR|SO|SI|DLE
    | DC1|DC2|DC3|DC4|NAK|SYN|ETB|CAN
    | EM|SUB|ESC|FS|GS|RS|US|SP|DEL
cntrl \(\rightarrow\) ascLarge \(\mid\) @|[|\|]|^|_
gap \(\rightarrow\) whitechar \(\{\) whitechar \(\} \backslash\)
```

Character literals are written between single quotes, as in ' $\mathrm{a}^{\prime}$, and strings between double quotes, as in "Hello".

Escape codes may be used in characters and strings to represent special characters. Note that a single quote ' may be used in a string, but must be escaped in a character; similarly, a double quote " may be used in a character, but must be escaped in a string. \ must always be escaped. The category charesc also includes portable representations for the characters "alert" ( $\backslash \mathrm{a})$, "backspace" ( $\backslash \mathrm{b}$ ), "form feed" ( $\backslash \mathrm{f})$, "new line" ( $\backslash \mathrm{n}$ ), "carriage return" ( $\backslash \mathrm{r}$ ), "horizontal tab" $(\backslash t)$, and "vertical tab" ( $\backslash \mathrm{v}$ ).

Escape characters for the Unicode character set, including control characters such as $\backslash^{\wedge} \mathrm{X}$, are also provided. Numeric escapes such as $\backslash 137$ are used to designate the character with decimal representation 137; octal (e.g. \o137) and hexadecimal (e.g. $\backslash x 37$ ) representations are also allowed.

Consistent with the "maximal munch" rule, numeric escape characters in strings consist of all consecutive digits and may be of arbitrary length. Similarly, the one ambiguous ASCII escape code, " $\backslash \mathrm{SOH}$ ", is parsed as a string of length 1 . The escape character $\backslash \&$ is provided as a "null character" to allow strings such as " $\backslash 137 \backslash \& 9$ " and $" \backslash S O \backslash \& \mathrm{H}$ " to be constructed (both of length two). Thus " $\backslash \&$ " is equivalent to " " and the character ${ }^{\prime} \backslash \&$ ' is disallowed. Further equivalences of characters are defined in Section 6.1.2.

A string may include a "gap"-two backslants enclosing white characters-which is ignored. This allows one to write long strings on more than one line by writing a backslant at the end of one line and at the start of the next. For example,

```
"Here is a backslant \\ as well as \137, \
    \a numeric escape character, and \^X, a control character."
```

String literals are actually abbreviations for lists of characters (see Section 3.7).

### 2.7 Layout

Haskell permits the omission of the braces and semicolons used in several grammar productions, by using layout to convey the same information. This allows both layout-sensitive and layoutinsensitive styles of coding, which can be freely mixed within one program. Because layout in not required, Haskell programs can be straightforwardly produced by other programs.

The effect of layout on the meaning of a Haskell program can be completely specified by adding braces and semicolons in places determined by the layout. The meaning of this augmented program is now layout insensitive.

Informally stated, the braces and semicolons are inserted as follows. The layout (or "off-side") rule takes effect whenever the open brace is omitted after the keyword where, let, do, or of. When this happens, the indentation of the next lexeme (whether or not on a new line) is remembered and the omitted open brace is inserted (the whitespace preceding the lexeme may include comments). For each subsequent line, if it contains only whitespace or is indented more, then the previous item is continued (nothing is inserted); if it is indented the same amount, then a new item begins (a semicolon is inserted); and if it is indented less, then the layout list ends (a close brace is inserted). If the indentation of the non-brace lexeme immediately following a where, let, do or of is less than or equal to the current indentation level, then instead of starting a layout, an empty list " $\}$ " is inserted, and layout processing occurs for the current level (i.e. insert a semicolon or close brace). A close brace is also inserted whenever the syntactic category containing the layout list ends; that is, if an illegal lexeme is encountered at a point where a close brace would be legal, a close brace is inserted. The layout rule matches only those open braces that it has inserted; an explicit open brace must be matched by an explicit close brace. Within these explicit open braces, no layout processing is performed for constructs outside the braces, even if a line is indented to the left of an earlier implicit open brace.

Section 9.3 gives a more precise definition of the layout rules.
Given these rules, a single newline may actually terminate several layout lists. Also, these rules permit:

```
f x = let a = 1; b = 2
        g y = exp2
    in exp1
```

making $a, b$ and $g$ all part of the same layout list.
As an example, Figure 2.1 shows a (somewhat contrived) module and Figure 2.2 shows the result of applying the layout rule to it. Note in particular: (a) the line beginning \} \} ; pop, where the

```
module AStack( Stack, push, pop, top, size ) where
data Stack a = Empty
    | MkStack a (Stack a)
push :: a -> Stack a -> Stack a
push x s = MkStack x s
size :: Stack a -> Int
size s = length (stkToLst s) where
    stkToLst Empty = []
    stkToLst (MkStack x s) = x:xs where xs = stkToLst s
pop :: Stack a -> (a, Stack a)
pop (MkStack x s)
    =(x, case s of r -> i r where i x = x) -- (pop Empty) is an error
top :: Stack a -> a
top (MkStack x s) = x -- (top Empty) is an error
```

Figure 2.1: A sample program

```
module AStack( Stack, push, pop, top, size ) where
{data Stack a = Empty
    | MkStack a (Stack a)
; push :: a -> Stack a -> Stack a
; push x s = MkStack x s
;size :: Stack a -> Int
; size s = length (stkToLst s) where
    {stkToLst Empty = []
    ;stkToLst (MkStack x s) = x:xs where {xs = stkToLst s
}};pop :: Stack a -> (a, Stack a)
;pop (MkStack x s)
    =(x, case s of {r -> i r where {i x = x}}) -- (pop Empty) is an errpr
;top :: Stack a -> a
;top (MkStack x s) = x
}
```

Figure 2.2: Sample program with layout expanded
termination of the previous line invokes three applications of the layout rule, corresponding to the depth (3) of the nested where clauses, (b) the close braces in the where clause nested within the tuple and case expression, inserted because the end of the tuple was detected, and (c) the close brace at the very end, inserted because of the column 0 indentation of the end-of-file token.

## Chapter 3

## Expressions

In this chapter, we describe the syntax and informal semantics of Haskell expressions, including their translations into the Haskell kernel, where appropriate. Except in the case of let expressions, these translations preserve both the static and dynamic semantics. Free variables and constructors used in these translations always refer to entities defined by the Prelude. For example, "concatMap" used in the translation of list comprehensions (Section 3.11) means the concatMap defined by the Prelude, regardless of whether or not the identifier "concatMap" is in scope where the list comprehension is used, and (if it is in scope) what it is bound to.

In the syntax that follows, there are some families of nonterminals indexed by precedence levels (written as a superscript). Similarly, the nonterminals op, varop, and conop may have a double index: a letter $l, r$, or $n$ for left-, right- or non-associativity and a precedence level. A precedencelevel variable $i$ ranges from 0 to 9 ; an associativity variable $a$ varies over $\{l, r, n\}$. For example

$$
\operatorname{aexp} \quad \rightarrow \quad\left(\exp ^{i+1} q o p^{(a, i)}\right)
$$

actually stands for 30 productions, with 10 substitutions for $i$ and 3 for $a$.

| exp | $\overrightarrow{\mid}$ | $\begin{aligned} & \exp ^{0}::[\text { context }=>] \text { type } \\ & \exp ^{0} \end{aligned}$ | (expression type signature) |
| :---: | :---: | :---: | :---: |
| $e x p{ }^{i}$ | $\vec{i}$ | $\begin{aligned} & \exp ^{i+1}\left[q o p^{(\mathrm{n}, i)} \exp ^{i+1}\right] \\ & \operatorname{lexp}^{i} \\ & \operatorname{rexp}^{i} \end{aligned}$ |  |
| $l e x p{ }^{i}$ | $\rightarrow$ | $\left(\exp ^{i} \mid \exp ^{i+1}\right) q o p^{(1, i)} \exp ^{i+1}$ |  |
| $l e x p{ }^{6}$ | $\rightarrow$ | $-e^{\prime}{ }^{7}$ |  |
| rexp ${ }^{i}$ | $\rightarrow$ | $\exp ^{i+1} q o p^{(\mathrm{r}, i)}\left(\operatorname{rexp}^{i} \mid \exp ^{i+1}\right)$ |  |
| exp ${ }^{10}$ | $\rightarrow$ | \apat a $_{1} \ldots$ apat $_{n} \rightarrow$ exp | (lambda abstraction, $n \geq 1$ ) |
|  | \| | let decls in exp | (let expression) |
|  | \| | if $\exp$ then $\exp$ else $\exp$ | (conditional) |
|  | 1 | case exp of \{ alts \} | (case expression) |



Expressions involving infix operators are disambiguated by the operator's fixity (see Section 4.4.2). Consecutive unparenthesized operators with the same precedence must both be either left or right associative to avoid a syntax error. Given an unparenthesized expression " $x q o p^{(a, i)} y q o p{ }^{(b, j)} z$ ", parentheses must be added around either " $x q o p^{(a, i)} y$ " or " $y q o p^{(b, j)} z$ " when $i=j$ unless $a=b=1$ or $a=b=\mathrm{r}$.

Negation is the only prefix operator in Haskell; it has the same precedence as the infix - operator defined in the Prelude (see Section 4.4.2, Figure 4.1).

The grammar is ambiguous regarding the extent of lambda abstractions, let expressions, and conditionals. The ambiguity is resolved by the meta-rule that each of these constructs extends as far to the right as possible.

Sample parses are shown below.

| This | Parses as |
| :---: | :---: |
| $f \mathrm{x}+\mathrm{g} \mathrm{y}$ | ( f x ) $+(\mathrm{g} y)$ |
| - $\mathrm{f} x+\mathrm{y}$ | (- (f x) ) + y |
| let \{ ... \} in $x+y$ | let $\{\ldots .$. \} in ( $\mathrm{x}+\mathrm{y}$ ) |
| $z+$ let $\{$... \} in $x+y$ |  |
| f x y : : Int | (f x y) : : Int |
| \ x -> a+b : : Int | \ x -> ( $\mathrm{a}+\mathrm{b}$ ) : : Int) |

A note about parsing. Expressions that involve the interaction of fixities with the let/lambda metarule may be hard to parse. For example, the expression

```
let x = True in x == x == True
```

cannot possibly mean

```
let x = True in (x == x == True)
```

because ( $==$ ) is a non-associative operator; so the expression must parse thus:

```
(let \(x=\) True in \((x==x)\) ) \(==\) True
```

However, implementations may well use a post-parsing pass to deal with fixities, so they may well incorrectly deliver the former parse. Programmers are advised to avoid constructs whose parsing involves an interaction of (lack of) associativity with the let/lambda meta-rule.

For the sake of clarity, the rest of this section shows the syntax of expressions without their precedences.

### 3.1 Errors

Errors during expression evaluation, denoted by $\perp$, are indistinguishable by a Haskell program from non-termination. Since Haskell is a non-strict language, all Haskell types include $\perp$. That is, a value of any type may be bound to a computation that, when demanded, results in an error. When evaluated, errors cause immediate program termination and cannot be caught by the user. The Prelude provides two functions to directly cause such errors:

```
error :: String -> a
undefined :: a
```

A call to error terminates execution of the program and returns an appropriate error indication to the operating system. It should also display the string in some system-dependent manner. When undefined is used, the error message is created by the compiler.

Translations of Haskell expressions use error and undefined to explicitly indicate where execution time errors may occur. The actual program behavior when an error occurs is up to the implementation. The messages passed to the error function in these translations are only suggestions; implementations may choose to display more or less information when an error occurs.

### 3.2 Variables, Constructors, Operators, and Literals


(general constructor)

| gcon | $\overrightarrow{\mid}$ | () <br> [] <br> (, $\{$,$\} )$ <br> qcon |
| :---: | :---: | :---: |
| var | $\rightarrow$ | varid $\mid$ ( varsym) |
| quar | $\rightarrow$ | quarid \| ( quarsym ) |
| con | $\rightarrow$ | conid \| ( consym) |
| qcon | $\rightarrow$ | qconid \| ( gconsym ) |
| varop | $\rightarrow$ | varsym \|`varid` |
| quarop | $\rightarrow$ | qvarsym \| ¢ quarid` \\ \hline conop & \(\rightarrow\) & consym\|` conid |
| qconop | $\rightarrow$ | gconsym \|` qconid` |
| $o p$ | $\rightarrow$ | varop \| conop |
| qop | $\rightarrow$ | quarop \| qconop |
| gconsym | $\rightarrow$ | : \| qconsym |

(variable)
(qualified variable)
(constructor)
(qualified constructor)
(variable operator)
(qualified variable operator)
(constructor operator)
(qualified constructor operator)
(operator)
(qualified operator)

Haskell provides special syntax to support infix notation. An operator is a function that can be applied using infix syntax (Section 3.4), or partially applied using a section (Section 3.5).

An operator is either an operator symbol, such as + or $\$ \$$, or is an ordinary identifier enclosed in grave accents (backquotes), such as `op`. For example, instead of writing the prefix application $o p x y$, one can write the infix application $x$ `op` $y$. If no fixity declaration is given for `op` then it defaults to highest precedence and left associativity (see Section 4.4.2).

Dually, an operator symbol can be converted to an ordinary identifier by enclosing it in parentheses. For example, ( + ) $x$ y is equivalent to $x+y$, and foldr (*) 1 xs is equivalent to foldr ( $\backslash x$ y $->x^{*} y$ ) 1 xs.

Special syntax is used to name some constructors for some of the built-in types, as found in the production for gcon and literal. These are described in Section 6.1.

An integer literal represents the application of the function fromInteger to the appropriate value of type Integer. Similarly, a floating point literal stands for an application of fromRational to a value of type Rational (that is, Ratio Integer).

Translation: The integer literal $i$ is equivalent to fromInteger $i$, where fromInteger is a method in class Num (see Section 6.4.1).
The floating point literal $f$ is equivalent to fromRational ( $n$ Ratio. $\% d$ ), where fromRational is a method in class Fractional and Ratio.\% constructs a rational from two integers, as defined in the Ratio library. The integers $n$ and $d$ are chosen so that $n / d=f$.

### 3.3 Curried Applications and Lambda Abstractions

| fexp | $\rightarrow$ [fexp $]$ aexp | (function application) |
| :--- | :--- | :--- |
| $\exp$ | $\rightarrow \quad \backslash$ apat $_{1} \ldots$ apat $_{n}->\exp$ | (lambda abstraction, $n \geq 1$ ) |

Function application is written $e_{1} e_{2}$. Application associates to the left, so the parentheses may be omitted in ( f x ) y. Because $e_{1}$ could be a data constructor, partial applications of data constructors are allowed.

Lambda abstractions are written $\backslash p_{1} \ldots p_{n} \rightarrow e$, where the $p_{i}$ are patterns. An expression such as $\backslash \mathrm{x}: \mathrm{xs}->\mathrm{x}$ is syntactically incorrect; it may legally be written as $\backslash(\mathrm{x}: \mathrm{xs})->\mathrm{x}$.

The set of patterns must be linear-no variable may appear more than once in the set.

Translation: The following identity holds:

$$
\left.\backslash p_{1} \ldots p_{n}->e=\backslash x_{1} \ldots x_{n} \rightarrow\right\rangle \operatorname{case}\left(x_{1}, \ldots, x_{n}\right) \text { of }\left(p_{1}, \ldots, p_{n}\right) \rightarrow e
$$

where the $x_{i}$ are new identifiers.

Given this translation combined with the semantics of case expressions and pattern matching described in Section 3.17.3, if the pattern fails to match, then the result is $\perp$.

### 3.4 Operator Applications

| $\exp$ | $\rightarrow$ | $\exp _{1}$ qop $\exp _{2}$ |  |
| :--- | :--- | :--- | :--- |
| qop | $\mid$ | $-\exp$ | (prefix negation) |
|  | $\rightarrow$ quarop $\mid$ qconop | (qualified operator) |  |

The form $e_{1} q o p e_{2}$ is the infix application of binary operator $q o p$ to expressions $e_{1}$ and $e_{2}$.
The special form $-e$ denotes prefix negation, the only prefix operator in Haskell, and is syntax for negate $(e)$. The binary - operator does not necessarily refer to the definition of - in the Prelude; it may be rebound by the module system. However, unary - will always refer to the negate function defined in the Prelude. There is no link between the local meaning of the - operator and unary negation.

Prefix negation has the same precedence as the infix operator - defined in the Prelude (see Table 4.1, page 55 ). Because e1-e 2 parses as an infix application of the binary operator - , one must write e1 (-e2) for the alternative parsing. Similarly, ( - ) is syntax for ( $\backslash \mathrm{x} y->\mathrm{x}-\mathrm{y}$ ), as with any infix operator, and does not denote ( $\backslash x->-x$ ) -one must use negate for that.

Translation: The following identities hold:

$$
\begin{array}{ll}
e_{1} \text { op } e_{2} & =(o p) e_{1} e_{2} \\
-e & =\text { negate }(e)
\end{array}
$$

### 3.5 Sections

| aexp | $\rightarrow$ | $\left(\exp ^{i+1} q o p^{(a, i)}\right)$ | (left section) |
| :---: | :---: | :---: | :---: |
|  | \| | $\left(\right.$ exp $\left.^{i} q o p^{(l, i)}\right)$ | (left section) |
|  | \| | $\left(q o p_{\langle-\rangle}^{(a, i)} e x p^{i+1}\right)$ | (right section) |
|  | \| | $\left(q_{0} p_{\langle-\rangle}^{(r, i)} \operatorname{rexp}^{i}\right.$ ) | (right section) |

Sections are written as (ope) or ( $e o p$ ), where $o p$ is a binary operator and $e$ is an expression. Sections are a convenient syntax for partial application of binary operators.

Syntactic precedence rules apply to sections as follows. (ope) is legal if and only if ( $\mathrm{x} o p e$ ) parses in the same way as ( $\mathrm{x} o p(e)$ ) ; and similarly for ( $e o p$ ). For example, ( ${ }^{*} \mathrm{a}+\mathrm{b}$ ) is syntactically invalid, but $(+a * b)$ and (* $(a+b)$ ) are valid. Because ( + ) is left associative, $(a+b+$ ) is syntactically correct, but $(+a+b)$ is not; the latter may legally be written as $(+(a+b))$. As another example, the expression

```
(let n = 10 in n +)
```

is invalid because, by the let/lambda meta-rule (Section 3), the expression

```
(let n = 10 in n + x)
```

parses as

$$
\text { (let } \mathrm{n}=10 \text { in }(\mathrm{n}+\mathrm{x}) \text { ) }
$$

rather than

$$
((\text { let } \mathrm{n}=10 \text { in } \mathrm{n})+\mathrm{x})
$$

Because - is treated specially in the grammar, ( $-\exp$ ) is not a section, but an application of prefix negation, as described in the preceding section. However, there is a subtract function defined in the Prelude such that (subtract exp) is equivalent to the disallowed section. The expression $(+\quad(-\exp ))$ can serve the same purpose.

Translation: The following identities hold:

$$
\begin{aligned}
& (o p e)=\backslash x->x o p e \\
& (e o p)=\backslash x->e o p x
\end{aligned}
$$

where $o p$ is a binary operator, $e$ is an expression, and $x$ is a variable that does not occur free in $e$.

### 3.6 Conditionals

$\exp \quad \rightarrow \quad$ if $\exp _{1}$ then $\exp _{2}$ else $\exp _{3}$

A conditional expression has the form if $e_{1}$ then $e_{2}$ else $e_{3}$ and returns the value of $e_{2}$ if the value of $e_{1}$ is True, $e_{3}$ if $e_{1}$ is False, and $\perp$ otherwise.

Translation: The following identity holds:

$$
\text { if } e_{1} \text { then } e_{2} \text { else } e_{3}=\text { case } e_{1} \text { of }\left\{\text { True }->e_{2} ; \text { False }->e_{3}\right\}
$$

where True and False are the two nullary constructors from the type Bool, as defined in the Prelude. The type of $e_{1}$ must be Bool; $e_{2}$ and $e_{3}$ must have the same type, which is also the type of the entire conditional expression.

### 3.7 Lists

$\begin{array}{llll}\exp & \rightarrow & \exp _{1} \text { qop } \exp _{2} & \\ \text { aexp } & \rightarrow & {\left[\exp _{1}, \ldots, \exp _{k}\right]} & (k \geq 1) \\ & \mid & \text { gcon } & \\ \text { gcon } & \rightarrow & {[]} & \\ & \mid & q c o n & \\ \text { qcon } & \rightarrow & (\text { gconsym }) & \\ \text { qop } & \rightarrow & \text { qconop } & \\ \text { qconop } & \rightarrow & \text { gconsym } & \\ \text { gconsym } & \rightarrow & : & \end{array}$

Lists are written $\left[e_{1}, \ldots, e_{k}\right]$, where $k \geq 1$. The list constructor is :, and the empty list is denoted []. Standard operations on lists are given in the Prelude (see Section 6.1.3, and Chapter 8 notably Section 8.1).

Translation: The following identity holds:

$$
\left[e_{1}, \ldots, e_{k}\right]=e_{1}:\left(e_{2}:\left(\ldots\left(e_{k}:[]\right)\right)\right)
$$

where : and [] are constructors for lists, as defined in the Prelude (see Section 6.1.3). The types of $e_{1}$ through $e_{k}$ must all be the same (call it $t$ ), and the type of the overall expression is [ $t$ ] (see Section 4.1.2).

The constructor ":" is reserved solely for list construction; like [ ], it is considered part of the language syntax, and cannot be hidden or redefined. It is a right-associative operator, with precedence level 5 (Section 4.4.2).

### 3.8 Tuples



Tuples are written $\left(e_{1}, \ldots, e_{k}\right)$, and may be of arbitrary length $k \geq 2$. The constructor for an $n$ tuple is denoted by $(, \ldots$,$) , where there are n-1$ commas. Thus ( $\mathrm{a}, \mathrm{b}, \mathrm{c}$ ) and (, , ) a b c denote the same value. Standard operations on tuples are given in the Prelude (see Section 6.1.4 and Chapter 8).

Translation: $\left(e_{1}, \ldots, e_{k}\right)$ for $k \geq 2$ is an instance of a $k$-tuple as defined in the Prelude, and requires no translation. If $t_{1}$ through $t_{k}$ are the types of $e_{1}$ through $e_{k}$, respectively, then the type of the resulting tuple is $\left(t_{1}, \ldots, t_{k}\right)$ (see Section 4.1.2).

### 3.9 Unit Expressions and Parenthesized Expressions

| aexp | $\rightarrow$ | gcon |
| :--- | :--- | :--- |
|  | $\mid$ | $($ exp $)$ |
| gcon | $\rightarrow$ | () |

The form ( $e$ ) is simply a parenthesized expression, and is equivalent to $e$. The unit expression () has type () (see Section 4.1.2). It is the only member of that type apart from $\perp$, and can be thought of as the "nullary tuple" (see Section 6.1.5).

Translation: (e) is equivalent to $e$.

### 3.10 Arithmetic Sequences

$\operatorname{aexp} \quad \rightarrow \quad\left[\exp _{1}\left[, \exp _{2}\right] \ldots\left[\exp _{3}\right]\right]$

The arithmetic sequence $\left[e_{1}, e_{2} \ldots e_{3}\right.$ ] denotes a list of values of type $t$, where each of the $e_{i}$ has type $t$, and $t$ is an instance of class Enum.

Translation: Arithmetic sequences satisfy these identities:

$$
\begin{array}{ll}
{\left[e_{1} . .\right]} & \\
{\left[e_{1}, e_{2} .\right]} & =\text { enumFrom } e_{1} \\
{\left[e_{1} . . e_{3}\right]} & =\text { enumFromThen } e_{1} e_{2} \\
{\left[e_{1}, e_{2} \ldots e_{3}\right]} & =\text { enumFromTo } e_{1} e_{3} \\
\end{array}
$$

where enumFrom, enumFromThen, enumFromTo, and enumFromThenTo are class methods in the class Enum as defined in the Prelude (see Figure 6.1, page 83).

The semantics of arithmetic sequences therefore depends entirely on the instance declaration for the type $t$. See Section 6.3.4 for more details of which Prelude types are in Enum and their semantics.

### 3.11 List Comprehensions

| aexp | $\rightarrow \quad\left[\exp \mid\right.$ qual,$\ldots$, qual $\left._{n}\right]$ |  |
| :--- | :--- | :--- |
| qual | $\rightarrow$ pat $<-\exp$ |  |
|  | $\mid$ let decls comprehension, $n \geq 1$ ) |  |
|  | exp | (generator) |
|  | (local declaration) |  |

A list comprehension has the form $\left[e \mid q_{1}, \ldots, q_{n}\right], n \geq 1$, where the $q_{i}$ qualifiers are either

- generators of the form $p<-e$, where $p$ is a pattern (see Section 3.17) of type $t$ and $e$ is an expression of type [ $t$ ]
- guards, which are arbitrary expressions of type Bool
- local bindings that provide new definitions for use in the generated expression $e$ or subsequent guards and generators.

Such a list comprehension returns the list of elements produced by evaluating $e$ in the successive environments created by the nested, depth-first evaluation of the generators in the qualifier list.

Binding of variables occurs according to the normal pattern matching rules (see Section 3.17), and if a match fails then that element of the list is simply skipped over. Thus:

```
\([\mathrm{x} \mid \mathrm{xs}<-[[(1,2),(3,4)],[(5,4),(3,2)]]\),
    \((3, x)<-x s]\)
```

yields the list $[4,2]$. If a qualifier is a guard, it must evaluate to True for the previous pattern match to succeed. As usual, bindings in list comprehensions can shadow those in outer scopes; for example:

$$
[\mathrm{x} \mid \mathrm{x}<-\mathrm{x}, \mathrm{x}<-\mathrm{x}]=[\mathrm{z} \mid \mathrm{y}<-\mathrm{x}, \mathrm{z}<-\mathrm{y}]
$$

Translation: List comprehensions satisfy these identities, which may be used as a translation into the kernel:

```
\(\left[\begin{array}{l|l|l}e & \text { True }] \quad=[e]\end{array}\right.\)
\(\left[\begin{array}{l|l}e & q]\end{array}=\left[\begin{array}{l}e \mid q, ~ T r u e ~\end{array}\right]\right.\)
\(\left[\begin{array}{l|l|}{[ } & b\end{array}\right]=\) if \(b\) then \([e \mid Q\) ]else []
\([e \mid p<-l, Q]=\) let ok \(p=[e \mid Q]\)
    ok _ = []
    in concatMap ok \(l\)
[ \(e \mid\) let decls, \(Q\) ] \(=\) let decls in \([e \mid Q]\)
```

where $e$ ranges over expressions, $p$ over patterns, $l$ over list-valued expressions, $b$ over boolean expressions, decls over declaration lists, $q$ over qualifiers, and $Q$ over sequences of qualifiers. ok is a fresh variable. The function concatMap, and boolean value True, are defined in the Prelude.

As indicated by the translation of list comprehensions, variables bound by let have fully polymorphic types while those defined by <- are lambda bound and are thus monomorphic (see Section 4.5.4).

### 3.12 Let Expressions

$\exp \quad \rightarrow \quad$ let decls in $\exp$

Let expressions have the general form let $\left\{d_{1} ; \ldots ; d_{n}\right\}$ in $e$, and introduce a nested, lexically-scoped, mutually-recursive list of declarations (let is often called letrec in other languages). The scope of the declarations is the expression $e$ and the right hand side of the declarations. Declarations are described in Chapter 4. Pattern bindings are matched lazily; an implicit ~ makes these patterns irrefutable. For example,

```
let (x,y) = undefined in e
```

does not cause an execution-time error until x or y is evaluated.

Translation: The dynamic semantics of the expression let $\left\{d_{1} ; \ldots ; d_{n}\right\}$ in $e_{0}$ are captured by this translation: After removing all type signatures, each declaration $d_{i}$ is translated into an equation of the form $p_{i}=e_{i}$, where $p_{i}$ and $e_{i}$ are patterns and expressions respectively, using the translation in Section 4.4.3. Once done, these identities hold, which may be used as a translation into the kernel:

```
let \(\left\{p_{1}=e_{1} ; \ldots ; p_{n}=e_{n}\right\}\) in \(e_{0}=\operatorname{let}\left(\sim p_{1}, \ldots,{ }^{\sim} p_{n}\right)=\left(e_{1}, \ldots, e_{n}\right)\) in \(e_{0}\)
let \(p=e_{1}\) in \(e_{0} \quad=\) case \(e_{1}\) of \(\sim p \rightarrow e_{0}\)
        where no variable in \(p\) appears free in \(e_{1}\)
let \(p=e_{1}\) in \(e_{0} \quad=\) let \(p=\mathrm{fix}\left(\backslash \sim p \rightarrow e_{1}\right)\) in \(e_{0}\)
```

where fix is the least fixpoint operator. Note the use of the irrefutable patterns $\sim p$. This translation does not preserve the static semantics because the use of case precludes a fully polymorphic typing of the bound variables. The static semantics of the bindings in a let expression are described in Section 4.4.3.

### 3.13 Case Expressions

| exp | $\rightarrow$ case exp of $\{$ alts $\}$ |  |
| :--- | :--- | :--- | :--- |
| alts | $\rightarrow$ alt $; \ldots ;$ alt |  |
| alt | $\rightarrow$ pat $\rightarrow$ exp $[$ where decls $]$ | $(n \geq 1)$ |
|  | $\mid \quad$ pat gdpat $[$ where decls $]$ |  |
|  | $\mid$ |  |
| gdpat | $\rightarrow$ gd $\rightarrow \exp ^{2}[$ gdpat $]$ |  |
| gd | $\rightarrow \mid \exp ^{0}$ |  |

A case expression has the general form

$$
\text { case } e \text { of }\left\{p_{1} \text { match }_{1} ; \ldots ; p_{n} \text { match }_{n}\right\}
$$

where each $m a t c h_{i}$ is of the general form

$$
\begin{array}{ll}
\mid g_{i 1} & ->e_{i 1} \\
\ldots & \\
\because g_{i m_{i}} & ->e_{i m_{i}} \\
\text { where } \text { decls }_{i}
\end{array}
$$

(Notice that in the syntax rule for $g d$, the " $\mid$ " is a terminal symbol, not the syntactic metasymbol for alternation.) Each alternative $p_{i}$ matchi consists of a pattern $p_{i}$ and its matches, matchi. Each match in turn consists of a sequence of pairs of guards $g_{i j}$ and bodies $e_{i j}$ (expressions), followed by optional bindings ( $\operatorname{decls}_{i}$ ) that scope over all of the guards and expressions of the alternative. An alternative of the form
pat -> exp where decls
is treated as shorthand for:

$$
\begin{aligned}
& \text { pat | True -> exp } \\
& \text { where decls }
\end{aligned}
$$

A case expression must have at least one alternative and each alternative must have at least one body. Each body must have the same type, and the type of the whole expression is that type.

A case expression is evaluated by pattern matching the expression $e$ against the individual alternatives. The alternatives are tried sequentially, from top to bottom. If $e$ matches the pattern in the alternative, the guards for that alternative are tried sequentially from top to bottom, in the environment of the case expression extended first by the bindings created during the matching of the pattern, and then by the $d e c s_{i}$ in the where clause associated with that alternative. If one of the guards evaluates to True, the corresponding right-hand side is evaluated in the same environment as the guard. If all the guards evaluate to False, matching continues with the next alternative. If no match succeeds, the result is $\perp$. Pattern matching is described in Section 3.17, with the formal semantics of case expressions in Section 3.17.3.

A note about parsing. The expression

$$
\text { case } x \text { of }\left\{\left(a, \_\right) \mid \text {let } b=\text { not } a \text { in } b:: \text { Bool }->a\right\}
$$

is tricky to parse correctly. It has a single unambiguous parse, namely

$$
\text { case } x \text { of }\left\{\left(a, \_\right) \mid(l e t ~ b=n o t ~ a ~ i n ~ b: ~ B o o l) ~->~ a ~\right\} ~
$$

However, the phrase Bool -> a is syntactically valid as a type, and parsers with limited lookahead may incorrectly commit to this choice, and hence reject the program. Programmers are advised, therefore, to avoid guards that end with a type signature - indeed that is why a $g d$ contains an $e x p^{0}$ not an $\exp$.

### 3.14 Do Expressions

| exp | $\rightarrow$ do $\{$ stmts $\}$ | (do expression) |
| :--- | :--- | :--- |
| stmts | $\rightarrow$ stmt $\ldots$ stmt $t_{n} \exp [;]$ | $(n \geq 0)$ |
| stmt | $\rightarrow$ exp ; |  |
|  | $\|$pat $<-\exp ;$  <br>  let decls ; <br>  $;$ |  |
|  |  |  |

A do expression provides a more conventional syntax for monadic programming. It allows an expression such as

```
putStr "x: " >>
getLine >>= \l ->
return (words l)
```

to be written in a more traditional way as:

```
do putStr "x: "
    l <- getLine
    return (words l)
```

Translation: Do expressions satisfy these identities, which may be used as a translation into the kernel, after eliminating empty stmts:

```
do {e} =}
do {e;stmts} = e>> do {stmts}
do {p<- e; stmts}= let ok p = do {stmts }
    ok _ = fail "..."
    in e >>= ok
do {let decls; stmts} = let decls in do {stmts}
```

The ellipsis " . . ." stands for a compiler-generated error message, passed to fail, preferably giving some indication of the location of the pattern-match failure; the functions $\gg, \gg=$, and fail are operations in the class Monad, as defined in the Prelude; and ok is a fresh identifier.

As indicated by the translation of do, variables bound by let have fully polymorphic types while those defined by <- are lambda bound and are thus monomorphic.

### 3.15 Datatypes with Field Labels

A datatype declaration may optionally define field labels (see Section 4.2.1). These field labels can be used to construct, select from, and update fields in a manner that is independent of the overall structure of the datatype.

Different datatypes cannot share common field labels in the same scope. A field label can be used at most once in a constructor. Within a datatype, however, a field label can be used in more than one constructor provided the field has the same typing in all constructors. To illustrate the last point, consider:

```
data S = S1 { x :: Int } | S2 { x :: Int } -- OK
data T = T1 { y :: Int } | T2 { y :: Bool } -- BAD
```

Here $S$ is legal but $T$ is not, because $y$ is given inconsistent typings in the latter.

### 3.15.1 Field Selection

aexp $\rightarrow \quad$ quar

Field labels are used as selector functions. When used as a variable, a field label serves as a function that extracts the field from an object. Selectors are top level bindings and so they may be shadowed by local variables but cannot conflict with other top level bindings of the same name. This shadowing only affects selector functions; in record construction (Section 3.15.2) and update (Section 3.15.3), field labels cannot be confused with ordinary variables.

Translation: A field label $f$ introduces a selector function defined as:

$$
f \mathrm{x}=\text { case } \mathrm{x} \text { of }\left\{C_{1} p_{11} \ldots p_{1 k} \rightarrow e_{1} ; \ldots ; C_{n} p_{n 1} \ldots p_{n k}{ }^{->} e_{n}\right\}
$$

where $C_{1} \ldots C_{n}$ are all the constructors of the datatype containing a field labeled with $f, p_{i j}$ is y when $f$ labels the $j$ th component of $C_{i}$ or $\quad$ otherwise, and $e_{i}$ is y when some field in $C_{i}$ has a label of $f$ or undefined otherwise.

### 3.15.2 Construction Using Field Labels

```
\(\operatorname{aexp} \quad \rightarrow \quad q \operatorname{con}\left\{\right.\) fbind \(_{1}, \ldots\), fbind \(\left._{n}\right\} \quad\) (labeled construction, \(n \geq 0\) )
fbind \(\quad \rightarrow \quad\) qvar \(=\exp\)
```

A constructor with labeled fields may be used to construct a value in which the components are specified by name rather than by position. Unlike the braces used in declaration lists, these are not subject to layout; the \{ and \} characters must be explicit. (This is also true of field updates and field patterns.) Construction using field labels is subject to the following constraints:

- Only field labels declared with the specified constructor may be mentioned.
- A field label may not be mentioned more than once.
- Fields not mentioned are initialized to $\perp$.
- A compile-time error occurs when any strict fields (fields whose declared types are prefixed by !) are omitted during construction. Strict fields are discussed in Section 4.2.1.

The expression F \{\}, where F is a data constructor, is legal whether or not F was declared with record syntax (provided F has no strict fields - see the third bullet above); it denotes $\mathrm{F} \perp_{1} \ldots \perp_{n}$, where $n$ is the arity of F .

Translation: In the binding $f=v$, the field $f$ labels $v$.

$$
C\{b s\}=C\left(p i c k_{1}^{C} b s \text { undefined }\right) \ldots\left(p i c k_{k}^{C} b s \text { undefined }\right)
$$

where $k$ is the arity of $C$.
The auxiliary function $p i c k_{i}^{C} b s d$ is defined as follows:
If the $i$ th component of a constructor $C$ has the field label $f$, and if $f=v$ appears in the binding list $b s$, then $p i c k_{i}^{C} b s d$ is $v$. Otherwise, $p i c k_{i}^{C} b s d$ is the default value $d$.

### 3.15.3 Updates Using Field Labels

$\operatorname{aexp} \quad \rightarrow \quad \operatorname{eexp}_{\langle q c o n\rangle}\left\{\right.$ fbind $_{1}, \ldots$, fbind $\left._{n}\right\} \quad$ (labeled update, $n \geq 1$ )

Values belonging to a datatype with field labels may be non-destructively updated. This creates a new value in which the specified field values replace those in the existing value. Updates are restricted in the following ways:

- All labels must be taken from the same datatype.
- At least one constructor must define all of the labels mentioned in the update.
- No label may be mentioned more than once.
- An execution error occurs when the value being updated does not contain all of the specified labels.

Translation: Using the prior definition of pick,

$$
\begin{aligned}
& e\{b s\}=\text { case } e \text { of } \\
& C_{1} v_{1} \ldots v_{k_{1}}->C_{1}\left(\text { pick }_{1}^{C_{1}} \text { bs } v_{1}\right) \ldots\left(\text { pick }_{k_{1}}^{C_{1}} \text { bs } v_{k_{1}}\right) \\
& C_{j} v_{1} \ldots v_{k_{j}}->C_{j}\left(p i c k_{1}^{C_{j}} \text { bs } v_{1}\right) \ldots\left(p i c k_{k_{j}}^{C_{j}} \text { bs } v_{k_{j}}\right) \\
& \text { _ -> error "Update error" }
\end{aligned}
$$

where $\left\{C_{1}, \ldots, C_{j}\right\}$ is the set of constructors containing all labels in $b s$, and $k_{i}$ is the arity of $C_{i}$.

Here are some examples using labeled fields:

```
data T = C1 {f1,f2 :: Int}
    | C2 {f1 :: Int,
    f3,f4 :: Char}
```

| Expression | Translation |
| :---: | :---: |
| $\begin{aligned} & \mathrm{C} 1\{\mathrm{f} 1=3\} \\ & \mathrm{C} 2\left\{\mathrm{f} 1=1, \mathrm{f} 4=\prime^{\prime} \mathrm{A}^{\prime}, \mathrm{f} 3='^{\prime} \mathrm{B}^{\prime}\right\} \\ & \mathrm{x}\{\mathrm{f} 1=1\} \end{aligned}$ | ```C1 3 undefined C2 1 'B' 'A' case x of C1 _ f2 -> C1 1 f2 C2 _ f3 f4 -> C2 1 f3 f4``` |

The field $f 1$ is common to both constructors in T. This example translates expressions using constructors in field-label notation into equivalent expressions using the same constructors without field labels. A compile-time error will result if no single constructor defines the set of field labels used in an update, such as x \{f2 $=1$, $\left.\mathrm{f} 3=' \mathrm{x}^{\prime}\right\}$.

### 3.16 Expression Type-Signatures

$\exp \quad \rightarrow \quad \exp ::[$ context $=>]$ type

Expression type-signatures have the form $e:: t$, where $e$ is an expression and $t$ is a type (Section 4.1.2); they are used to type an expression explicitly and may be used to resolve ambiguous typings due to overloading (see Section 4.3.4). The value of the expression is just that of exp. As with normal type signatures (see Section 4.4.1), the declared type may be more specific than the principal type derivable from $\exp$, but it is an error to give a type that is more general than, or not comparable to, the principal type.

## Translation:

$$
e:: t=\operatorname{let}\{v:: t ; v=e\} \text { in } v
$$

### 3.17 Pattern Matching

Patterns appear in lambda abstractions, function definitions, pattern bindings, list comprehensions, do expressions, and case expressions. However, the first five of these ultimately translate into case expressions, so defining the semantics of pattern matching for case expressions is sufficient.

### 3.17.1 Patterns

Patterns have this syntax:

| pat | $\overrightarrow{\mid}$ | $\begin{aligned} & \text { var }+ \text { integer } \\ & \text { pat }^{0} \end{aligned}$ | (successor pattern) |
| :---: | :---: | :---: | :---: |
| pat ${ }^{i}$ | $\rightarrow$ | pat ${ }^{i+1}\left[\right.$ qconop $^{(\mathrm{n}, i)}$ pat $\left.^{\text {i+1 }}\right]$ |  |
|  | \| | lpat ${ }^{\text {i }}$ |  |
|  | \| | rpat ${ }^{i}$ |  |
| lpat ${ }^{\text {i }}$ | $\rightarrow$ | $\left(\right.$ lpat $^{i} \mid$ pat $^{\text {i+1 }}$ ) cconop $^{(1, i)}$ pat $^{\text {i+1 }}$ |  |
| lpat ${ }^{6}$ | $\rightarrow$ | - (integer $\mid$ float $)$ | (negative literal) |
| rpat ${ }^{i}$ | $\rightarrow$ | pat $^{\text {i+1 }}$ qconop $^{(\mathrm{r}, i)}\left(\right.$ rpat $^{\text {i }} \mid$ pat $\left.^{i+1}\right)$ |  |
| pat ${ }^{10}$ | $\rightarrow$ | apat |  |
|  | \| | gcon apat ${ }_{1} \ldots$ apat $^{\text {a }}$ | (arity gcon $=k, k \geq 1)$ |
| apat | $\rightarrow$ | var [ @ apat] | (as pattern) |
|  | \| | gcon | $($ arity gcon $=0$ ) |
|  |  | qcon $\left\{\right.$ fpat $_{1}, \ldots$, fpat $\left._{k}\right\}$ | (labeled pattern, $k \geq 0$ ) |
|  |  | literal |  |


|  | - | (wildcard) |
| :---: | :---: | :---: |
|  | ( pat ) | (parenthesized pattern) |
|  | $\left(\right.$ pat $_{1}, \ldots,{ }^{\text {a }}$, $t_{k}$ ) | (tuple pattern, $k \geq 2$ ) |
|  | [ $\mathrm{pat}_{1}, \ldots, \mathrm{pat}_{k}$ ] | (list pattern, $k \geq 1$ ) |
|  | ~ apat | (irrefutable pattern) |
| fpat | $q v a r=p a t$ |  |

The arity of a constructor must match the number of sub-patterns associated with it; one cannot match against a partially-applied constructor.

All patterns must be linear -no variable may appear more than once. For example, this definition is illegal: $f(x, x)=x$-- ILLEGAL; $x$ used twice in pattern

Patterns of the form var@pat are called as-patterns, and allow one to use var as a name for the value being matched by pat. For example,

```
case e of { xs@(x:rest) -> if x==0 then rest else xs }
```

is equivalent to:

```
let { xs = e } in
    case xs of { (x:rest) -> if x==0 then rest else xs }
```

Patterns of the form _ are wildcards and are useful when some part of a pattern is not referenced on the right-hand-side. It is as if an identifier not used elsewhere were put in its place. For example,

```
case e of { [x,_,_] -> if x==0 then True else False }
```

is equivalent to:

```
case e of { [x,y,z] -> if x==0 then True else False }
```


### 3.17.2 Informal Semantics of Pattern Matching

Patterns are matched against values. Attempting to match a pattern can have one of three results: it may fail; it may succeed, returning a binding for each variable in the pattern; or it may diverge (i.e. return $\perp$ ). Pattern matching proceeds from left to right, and outside to inside, according to the following rules:

1. Matching the pattern $v a r$ against a value $v$ always succeeds and binds $v a r$ to $v$.
2. Matching the pattern ~apat against a value $v$ always succeeds. The free variables in apat are bound to the appropriate values if matching apat against $v$ would otherwise succeed, and to $\perp$ if matching apat against $v$ fails or diverges. (Binding does not imply evaluation.)
Operationally, this means that no matching is done on a ~ apat pattern until one of the variables in apat is used. At that point the entire pattern is matched against the value, and if the match fails or diverges, so does the overall computation.
3. Matching the wildcard pattern _ against any value always succeeds, and no binding is done.
4. Matching the pattern con pat against a value, where con is a constructor defined by newtype, depends on the value:

- If the value is of the form con $v$, then pat is matched against $v$.
- If the value is $\perp$, then pat is matched against $\perp$.

That is, constructors associated with newt ype serve only to change the type of a value.
5. Matching the pattern con pat $\quad \ldots$ pat $_{n}$ against a value, where con is a constructor defined by data, depends on the value:

- If the value is of the form $\operatorname{con} v_{1} \ldots v_{n}$, sub-patterns are matched left-to-right against the components of the data value; if all matches succeed, the overall match succeeds; the first to fail or diverge causes the overall match to fail or diverge, respectively.
- If the value is of the form $\operatorname{con}^{\prime} v_{1} \ldots v_{m}$, where $c o n$ is a different constructor to $c o n^{\prime}$, the match fails.
- If the value is $\perp$, the match diverges.

6. Matching against a constructor using labeled fields is the same as matching ordinary constructor patterns except that the fields are matched in the order they are named in the field list. All fields listed must be declared by the constructor; fields may not be named more than once. Fields not named by the pattern are ignored (matched against _).
7. Matching a numeric, character, or string literal pattern $k$ against a value $v$ succeeds if $v==k$, where $==$ is overloaded based on the type of the pattern. The match diverges if this test diverges.
The interpretation of numeric literals is exactly as described in Section 3.2; that is, the overloaded function fromInteger or fromRational is applied to an Integer or Rational literal (resp) to convert it to the appropriate type.
8. Matching an $n+k$ pattern (where $n$ is a variable and $k$ is a positive integer literal) against a value $v$ succeeds if $x>=k$, resulting in the binding of $n$ to $x-k$, and fails otherwise. Again, the functions $>=$ and - are overloaded, depending on the type of the pattern. The match diverges if the comparison diverges.

The interpretation of the literal $k$ is the same as in numeric literal patterns, except that only integer literals are allowed.
9. Matching an as-pattern var@apat against a value $v$ is the result of matching apat against $v$, augmented with the binding of var to $v$. If the match of apat against $v$ fails or diverges, then so does the overall match.

Aside from the obvious static type constraints (for example, it is a static error to match a character against a boolean), the following static class constraints hold:

- An integer literal pattern can only be matched against a value in the class Num.
- A floating literal pattern can only be matched against a value in the class Fractional.
- An $n+k$ pattern can only be matched against a value in the class Integral.

Many people feel that $n+k$ patterns should not be used. These patterns may be removed or changed in future versions of Haskell.

It is sometimes helpful to distinguish two kinds of patterns. Matching an irrefutable pattern is nonstrict: the pattern matches even if the value to be matched is $\perp$. Matching a refutable pattern is strict: if the value to be matched is $\perp$ the match diverges. The irrefutable patterns are as follows: a variable, a wildcard, $N$ apat where $N$ is a constructor defined by newt ype and apat is irrefutable (see Section 4.2.3), var@apat where apat is irrefutable, or of the form ${ }^{\sim}$ apat (whether or not apat is irrefutable). All other patterns are refutable.

Here are some examples:

1. If the pattern [' $\left.a^{\prime}, b^{\prime}\right]$ is matched against $\left[' \mathrm{x}\right.$ ',$\perp$ ], then ' $\mathrm{a}^{\prime}$ fails to match against $' x^{\prime}$, and the result is a failed match. But if [ $\left.{ }^{\prime} a^{\prime},{ }^{\prime} b^{\prime}\right]$ is matched against $\left[\perp, x^{\prime}\right]$, then attempting to match ' $\mathrm{a}^{\prime}$ against $\perp$ causes the match to diverge.
2. These examples demonstrate refutable vs. irrefutable matching:

$$
\begin{aligned}
& (\backslash \sim(x, y)->0) \perp \quad \Rightarrow \quad 0 \\
& \text { ( } \backslash(x, y)->0) \perp \quad \Rightarrow \quad \perp \\
& \text { ( } \backslash \text { ~ [x] -> 0) [] } \quad \Rightarrow \quad 0 \\
& (\backslash \sim[x]->x)[] \quad \Rightarrow \quad \perp \\
& (\backslash \sim[x, \sim(a, b)] \quad->x) \quad[(0,1), \perp] \quad \Rightarrow \quad(0,1) \\
& (\backslash \sim[x,(a, b)] \quad->x)[(0,1), \perp] \quad \Rightarrow \quad \perp \\
& \text { ( } \backslash(x: x s) ~->~ x: x: x s) ~ \perp \quad \Rightarrow \quad \perp \\
& \text { ( } \left.{ }^{\sim}(x: x s) ~->~ x: x: x s\right) ~ \perp \quad \Rightarrow \quad \perp: \perp: \perp
\end{aligned}
$$

3. Consider the following declarations:
```
newtype N = N Bool
data D = D !Bool
```

These examples illustrate the difference in pattern matching between types defined by data and newtype:

$$
\begin{array}{lrllll}
(\backslash & \text { (N True) } & -> & \text { True }) & \perp & \Rightarrow \\
(\backslash & \text { (D True) } & -> & \text { True }) & \perp & \Rightarrow \\
(\backslash & \perp \\
(D & \text { (D True }) & -> & \text { True }) & \perp & \Rightarrow \\
\text { True }
\end{array}
$$

Additional examples may be found in Section 4.2.3.

Top level patterns in case expressions and the set of top level patterns in function or pattern bindings may have zero or more associated guards. A guard is a boolean expression that is evaluated only after all of the arguments have been successfully matched, and it must be true for the overall pattern match to succeed. The environment of the guard is the same as the right-hand-side of the caseexpression alternative, function definition, or pattern binding to which it is attached.

The guard semantics have an obvious influence on the strictness characteristics of a function or case expression. In particular, an otherwise irrefutable pattern may be evaluated because of a guard. For example, in

```
f :: (Int,Int,Int) -> [Int] -> Int
f ~ (x,y,z) [a] | (a == y) = 1
```

both $a$ and $y$ will be evaluated by $==$ in the guard.

### 3.17.3 Formal Semantics of Pattern Matching

The semantics of all pattern matching constructs other than case expressions are defined by giving identities that relate those constructs to case expressions. The semantics of case expressions themselves are in turn given as a series of identities, in Figures 3.1-3.2. Any implementation should behave so that these identities hold; it is not expected that it will use them directly, since that would generate rather inefficient code.

In Figures 3.1-3.2: $e, e^{\prime}$ and $e_{i}$ are expressions; $g$ and $g_{i}$ are boolean-valued expressions; $p$ and $p_{i}$ are patterns; $v, x$, and $x_{i}$ are variables; $K$ and $K^{\prime}$ are algebraic datatype (data) constructors (including tuple constructors); and $N$ is a newt ype constructor.

Rule (b) matches a general source-language case expression, regardless of whether it actually includes guards-if no guards are written, then True is substituted for the guards $g_{i, j}$ in the match ${ }_{i}$ forms. Subsequent identities manipulate the resulting case expression into simpler and simpler forms.

Rule (h) in Figure 3.2 involves the overloaded operator $==$; it is this rule that defines the meaning of pattern matching against overloaded constants.

These identities all preserve the static semantics. Rules (d), (e), (j), (q), and (s) use a lambda rather than a let; this indicates that variables bound by case are monomorphically typed (Section 4.1.4).
(a) case $e$ of $\{$ alts $\}=(\backslash v \rightarrow$ case $v$ of $\{$ alts $\}) e$ where $v$ is a new variable
(b) case $v$ of $\left\{p_{1}\right.$ match $_{1} ; \ldots \ldots$; $p_{n}$ match $\left._{n}\right\}$
$=$ case $v$ of $\left\{p_{1}\right.$ match $_{1}$;

```
_ -> ... case v of {
                        pn match 
                        _ -> error "No match" }...}
```

where each match $_{i}$ has the form:
$\left|g_{i, 1} \rightarrow e_{i, 1} ; \ldots ;\right| g_{i, m_{i}} \rightarrow e_{i, m_{i}}$ where $\left\{d e c l s_{i}\right\}$
(c) case $v$ of $\left\{p \left\lvert\, \begin{array}{lll}g_{1} & -> & e_{1} ; \ldots \\ g_{n} & -> & \ldots\end{array}\right.\right.$ $g_{n} \rightarrow e_{n}$ where $\{$ decls $\}$
$=$ case $e^{\prime}$ of
$\{y \rightarrow>\quad$ (where $y$ is a new variable)
case $v$ of \{
$p \rightarrow$ let $\{$ decls \} in if $g_{1}$ then $e_{1} \ldots$ else if $g_{n}$ then $e_{n}$ else $y$; $\left.-\quad \begin{array}{ll}y\end{array}\right\}$
(d) case $v$ of $\left\{{ }^{\sim} p->e ; \quad->e^{\prime}\right\}$ $=\left(\backslash x_{1} \ldots x_{n}->e\right) \quad\left(\right.$ case $v$ of $\left.\left\{p->x_{1}\right\}\right) \ldots\left(\right.$ case $v$ of $\left.\left\{p->x_{n}\right\}\right)$ where $x_{1}, \ldots, x_{n}$ are all the variables in $p$
(e) case $v$ of $\left\{x @ p->e ; \quad->e^{\prime}\right\}$
$=$ case $v$ of $\left\{p \rightarrow>(\backslash x->e) v ;->e^{\prime}\right\}$
(f) case $v$ of $\left\{\ldots->e ; \quad->e^{\prime}\right\}=e$

Figure 3.1: Semantics of Case Expressions, Part 1
(g) case $v$ of $\left\{K p_{1} \ldots p_{n} \rightarrow e_{;}\right.$_ $\left.^{->} e^{\prime}\right\}$
$=$ case $v$ of $\{$
$K x_{1} \ldots x_{n}$-> case $x_{1}$ of \{
$p_{1}->\ldots$ case $x_{n}$ of \{ $p_{n}$-> $e$; _ -> $\left.e^{\prime}\right\} \ldots$ _ $->e^{\prime}$ \}
_ -> $e^{\prime}$ \}
at least one of $p_{1}, \ldots, p_{n}$ is not a variable; $x_{1}, \ldots, x_{n}$ are new variables
(h) case $v$ of $\left\{k \rightarrow e ; ~->e^{\prime}\right\}=$ if $(v==k)$ then $e$ else $e^{\prime}$ where $k$ is a numeric, character, or string literal.
(i) case $v$ of $\left\{x \rightarrow e ; \quad->e^{\prime}\right\}=c a s e ~ v$ of $\{x$-> $e$ \}
(j) case $v$ of $\{x \rightarrow e\}=(\backslash x->e) v$
(k) case $N v$ of \{ $\left.N p \rightarrow e ; ~-->e^{\prime}\right\}$
$=$ case $v$ of $\left\{p\right.$-> $\left.e ; ~->e^{\prime}\right\}$
where $N$ is a newt ype constructor
(l) case $\perp$ of $\left\{N p \rightarrow e ; \quad->e^{\prime}\right\}=c a s e ~ \perp$ of $\{p \rightarrow e\}$ where $N$ is a newt ype constructor
(m) case $v$ of $\left\{K\left\{f_{1}=p_{1}, f_{2}=p_{2}, \ldots\right\}->e\right.$; _ -> $\left.e^{\prime}\right\}$ $=$ case $e^{\prime}$ of $\{$ $y$-> case $v$ of $\{$ $K\left\{f_{1}=p_{1}\right\}->$ case $v$ of $\left\{K\left\{f_{2}=p_{2}, \ldots\right.\right.$ \} -> $\left.e ;->y\right\} ;$ -> $y$ \}\}
where $f_{1}, f_{2}, \ldots$ are fields of constructor $K ; y$ is a new variable
(n) case $v$ of $\left\{K\{f=p\}->e ; ~->e^{\prime}\right\}$
= case $v$ of \{
$K p_{1} \ldots p_{n}$-> $e ; ~ ~->~_{l} e^{\prime}$
where $p_{i}$ is $p$ if $f$ labels the $i$ th component of $K,{ }_{C}$ otherwise
(o) case $v$ of $\left\{K\right.$ \{\} -> $e$; _ -> $\left.e^{\prime}\right\}$ = case $v$ of $\{$
$\left.K_{-}\right]_{-}$-> $\left.e ; ~->e^{\prime}\right\}$
(p) case ( $\bar{K}^{\prime} e_{1} \ldots e_{m}$ ) of $\left\{\begin{array}{llllll}K & x_{1} & \ldots & x_{n} \rightarrow> & \text {; } \quad \text {-> } & \left.e^{\prime}\right\}=e^{\prime}\end{array}\right.$ where $K$ and $K^{\prime}$ are distinct data constructors of arity $n$ and $m$, respectively
(q) case ( $K e_{1} \ldots e_{n}$ ) of $\left\{\begin{array}{llllll} & x_{1} & \ldots & x_{n} & \text {-> } e_{;} & \left.\text {-> } e^{\prime}\right\}\end{array}\right.$ $=\left(\backslash x_{1} \ldots x_{n}->e\right) e_{1} \ldots e_{n}$
where $K$ is a data constructor of arity $n$
(r) case $\perp$ of $\left\{\begin{array}{lllll} & x_{1} & \ldots & x_{n} & -> \\ e & \text { _ } & -> & \left.e^{\prime}\right\}=\perp\end{array}\right.$ where $K$ is a data constructor of arity $n$
(s) case $v$ of $\left\{x+k\right.$-> $e$; - -> $\left.e^{\prime}\right\}$
$=$ if $v>=k$ then ( $\backslash x->e$ ) $(v-k)$ else $e^{\prime}$
where $k$ is a numeric literal

Figure 3.2: Semantics of Case Expressions, Part 2

## Chapter 4

## Declarations and Bindings

In this chapter, we describe the syntax and informal semantics of Haskell declarations.


| gendecl | $\overrightarrow{\mid}$ | vars : : [context =>] type fixity [integer] ops | (type signature) <br> (fixity declaration) <br> (empty declaration) |
| :---: | :---: | :---: | :---: |
| ops | $\rightarrow$ | $o p_{1}, \ldots, o p_{n}$ | $(n \geq 1)$ |
| vars | $\rightarrow$ | $\operatorname{var}_{1}, \ldots, \operatorname{var}_{n}$ | $(n \geq 1)$ |
| fixity | $\rightarrow$ | infixl\|infixr|infix |  |

The declarations in the syntactic category topdecls are only allowed at the top level of a Haskell module (see Chapter 5), whereas decls may be used either at the top level or in nested scopes (i.e. those within a let or where construct).

For exposition, we divide the declarations into three groups: user-defined datatypes, consisting of type, newtype, and dat a declarations (Section 4.2); type classes and overloading, consisting of class, instance, and default declarations (Section 4.3); and nested declarations, consisting of value bindings, type signatures, and fixity declarations (Section 4.4).

Haskell has several primitive datatypes that are "hard-wired" (such as integers and floating-point numbers), but most "built-in" datatypes are defined with normal Haskell code, using normal type and data declarations. These "built-in" datatypes are described in detail in Section 6.1.

### 4.1 Overview of Types and Classes

Haskell uses a traditional Hindley-Milner polymorphic type system to provide a static type semantics [3, 5], but the type system has been extended with type classes (or just classes) that provide a structured way to introduce overloaded functions.

A class declaration (Section 4.3.1) introduces a new type class and the overloaded operations that must be supported by any type that is an instance of that class. An instance declaration (Section 4.3.2) declares that a type is an instance of a class and includes the definitions of the overloaded operations-called class methods-instantiated on the named type.

For example, suppose we wish to overload the operations (+) and negate on types Int and Float. We introduce a new type class called Num:

```
class Num a where -- simplified class declaration for Num
    (+) : : a -> a -> a -- (Num is defined in the Prelude)
    negate : : a -> a
```

This declaration may be read "a type a is an instance of the class Num if there are class methods $(+)$ and negate, of the given types, defined on it."

We may then declare Int and Float to be instances of this class:

```
instance Num Int where -- simplified instance of Num Int
    x + y = addInt x y
    negate x = negateInt x
instance Num Float where -- simplified instance of Num Float
    x + y = addFloat x y
    negate x = negateFloat x
```

where addInt, negateInt, addFloat, and negateFloat are assumed in this case to be primitive functions, but in general could be any user-defined function. The first declaration above may be read "Int is an instance of the class Num as witnessed by these definitions (i.e. class methods) for (+) and negate."

More examples of type classes can be found in the papers by Jones [7] or Wadler and Blott [12]. The term 'type class' was used to describe the original Haskell 1.0 type system; 'constructor class' was used to describe an extension to the original type classes. There is no longer any reason to use two different terms: in this report, 'type class' includes both the original Haskell type classes and the constructor classes introduced by Jones.

### 4.1.1 Kinds

To ensure that they are valid, type expressions are classified into different kinds, which take one of two possible forms:

- The symbol $*$ represents the kind of all nullary type constructors.
- If $\kappa_{1}$ and $\kappa_{2}$ are kinds, then $\kappa_{1} \rightarrow \kappa_{2}$ is the kind of types that take a type of kind $\kappa_{1}$ and return a type of kind $\kappa_{2}$.

Kind inference checks the validity of type expressions in a similar way that type inference checks the validity of value expressions. However, unlike types, kinds are entirely implicit and are not a visible part of the language. Kind inference is discussed in Section 4.6.

### 4.1.2 Syntax of Types

| type | $\rightarrow$ | btype [-> type] | (function type) |
| :---: | :---: | :---: | :---: |
| btype | $\rightarrow$ | [btype] atype | (type application) |
| atype | $\rightarrow$ | gtycon |  |
|  | - | tyvar |  |
|  |  | $\left(\right.$ type $_{1}, \ldots$, type $\left._{k}\right)$ | (tuple type, $k \geq 2$ 2) |
|  |  | [ type ] | (list type) |
|  | \| | ( type ) | (parenthesised constructor) |


| gtycon | $\rightarrow$ | qtycon |  |
| :---: | :---: | :---: | :---: |
|  |  | () | (unit type) |
|  |  | [] | (list constructor) |
|  |  | (->) | (function constructor) |
|  |  | (, $\{$,$\} )$ | (tupling constructors) |

The syntax for Haskell type expressions is given above. Just as data values are built using data constructors, type values are built from type constructors. As with data constructors, the names of type constructors start with uppercase letters. Unlike data constructors, infix type constructors are not allowed (other than (->)).

The main forms of type expression are as follows:

1. Type variables, written as identifiers beginning with a lowercase letter. The kind of a variable is determined implicitly by the context in which it appears.
2. Type constructors. Most type constructors are written as an identifier beginning with an uppercase letter. For example:

- Char, Int, Integer, Float, Double and Bool are type constants with kind $*$.
- Maybe and IO are unary type constructors, and treated as types with kind $* \rightarrow *$.
- The declarations data T . . . or newt ype T . . . add the type constructor T to the type vocabulary. The kind of T is determined by kind inference.

Special syntax is provided for certain built-in type constructors:

- The trivial type is written as () and has kind $*$. It denotes the "nullary tuple" type, and has exactly one value, also written () (see Sections 3.9 and 6.1.5).
- The function type is written as ( $->$ ) and has kind $* \rightarrow * \rightarrow *$.
- The list type is written as [] and has kind $* \rightarrow *$.
- The tuple types are written as $(),,(,$,$) , and so on. Their kinds are * \rightarrow * \rightarrow *$, $* \rightarrow * \rightarrow * \rightarrow *$, and so on.

Use of the ( $->$ ) and [] constants is described in more detail below.
3. Type application. If $t_{1}$ is a type of kind $\kappa_{1} \rightarrow \kappa_{2}$ and $t_{2}$ is a type of kind $\kappa_{1}$, then $t_{1} t_{2}$ is a type expression of kind $\kappa_{2}$.
4. A parenthesized type, having form $(t)$, is identical to the type $t$.

For example, the type expression IO a can be understood as the application of a constant, IO, to the variable a. Since the IO type constructor has kind $* \rightarrow *$, it follows that both the variable a and the whole expression, IO a, must have kind *. In general, a process of kind inference (see Section 4.6) is needed to determine appropriate kinds for user-defined datatypes, type synonyms, and classes.

Special syntax is provided to allow certain type expressions to be written in a more traditional style:

1. A function type has the form $t_{1} \rightarrow>t_{2}$, which is equivalent to the type $(->) t_{1} t_{2}$. Function arrows associate to the right. For example, Int -> Int -> Float means Int -> (Int -> Float).
2. A tuple type has the form $\left(t_{1}, \ldots, t_{k}\right)$ where $k \geq 2$, which is equivalent to the type $(, \ldots,) t_{1} \ldots t_{k}$ where there are $k-1$ commas between the parenthesis. It denotes the type of $k$-tuples with the first component of type $t_{1}$, the second component of type $t_{2}$, and so on (see Sections 3.8 and 6.1.4).
3. A list type has the form [ $t$ ], which is equivalent to the type [] $t$. It denotes the type of lists with elements of type $t$ (see Sections 3.7 and 6.1.3).

These special syntactic forms always denote the built-in type constructors for functions, tuples, and lists, regardless of what is in scope. In a similar way, the prefix type constructors (->), [ ], (), (, ), and so on, always denote the built-in type constructors; they cannot be qualified, nor mentioned in import or export lists (Chapter 5). (Hence the special production, "gtycon", above.)

Although the list and tuple types have special syntax, their semantics is the same as the equivalent user-defined algebraic data types.

Notice that expressions and types have a consistent syntax. If $t_{i}$ is the type of expression or pattern $e_{i}$, then the expressions $\left(\backslash e_{1}->e_{2}\right),\left[e_{1}\right]$, and $\left(e_{1}, e_{2}\right)$ have the types $\left(t_{1}->t_{2}\right),\left[t_{1}\right]$, and ( $t_{1}, t_{2}$ ), respectively.

With one exception (that of the distinguished type variable in a class declaration (Section 4.3.1)), the type variables in a Haskell type expression are all assumed to be universally quantified; there is no explicit syntax for universal quantification [3]. For example, the type expression a -> a denotes the type $\forall a . a \rightarrow a$. For clarity, however, we often write quantification explicitly when discussing the types of Haskell programs. When we write an explicitly quantified type, the scope of the $\forall$ extends as far to the right as possible; for example, $\forall a . a \rightarrow a$ means $\forall a .(a \rightarrow a)$.

### 4.1.3 Syntax of Class Assertions and Contexts

$\left.\begin{array}{llll}\text { context } & \rightarrow & \text { class } & \\ & \mid & \left(\text { class }_{1}, \ldots, \text { class }_{n}\right) & (n \geq 0) \\ \text { class } & \rightarrow & \text { qtycls tyvar }\end{array}\right)$

A class assertion has form qtycls tyvar, and indicates the membership of the type tyvar in the class qtycls. A class identifier begins with an uppercase letter. A context consists of zero or more class assertions, and has the general form

$$
\left(C_{1} u_{1}, \ldots, C_{n} u_{n}\right)
$$

where $C_{1}, \ldots, C_{n}$ are class identifiers, and each of the $u_{1}, \ldots, u_{n}$ is either a type variable, or the application of type variable to one or more types. The outer parentheses may be omitted when $n=1$. In general, we use $c x$ to denote a context and we write $c x=>t$ to indicate the type $t$ restricted by the context $c x$. The context $c x$ must only contain type variables referenced in $t$. For convenience, we write $c x=>t$ even if the context $c x$ is empty, although in this case the concrete syntax contains no $=>$.

### 4.1.4 Semantics of Types and Classes

In this section, we provide informal details of the type system. (Wadler and Blott [12] and Jones [7] discuss type and constructor classes, respectively, in more detail.)

The Haskell type system attributes a type to each expression in the program. In general, a type is of the form $\forall \bar{u} . c x \Rightarrow t$, where $\bar{u}$ is a set of type variables $u_{1}, \ldots, u_{n}$. In any such type, any of the universally-quantified type variables $u_{i}$ that are free in $c x$ must also be free in $t$. Furthermore, the context $c x$ must be of the form given above in Section 4.1.3. For example, here are some valid types:

```
Eq a => a -> a
(Eq a, Show a, Eq b) => [a] -> [b] -> String
(Eq (f a), Functor f) => (a -> b) -> f a -> f b -> Bool
```

In the third type, the constraint Eq ( f a) cannot be made simpler because f is universally quantified.

The type of an expression $e$ depends on a type environment that gives types for the free variables in $e$, and a class environment that declares which types are instances of which classes (a type becomes an instance of a class only via the presence of an instance declaration or a deriving clause).

Types are related by a generalization preorder (specified below); the most general type, up to the equivalence induced by the generalization preorder, that can be assigned to a particular expression (in a given environment) is called its principal type. Haskell's extended Hindley-Milner type system can infer the principal type of all expressions, including the proper use of overloaded class methods (although certain ambiguous overloadings could arise, as described in Section 4.3.4). Therefore, explicit typings (called type signatures) are usually optional (see Sections 3.16 and 4.4.1).

The type $\forall \bar{u} . c x_{1} \Rightarrow t_{1}$ is more general than the type $\forall \bar{w} . c x_{2} \Rightarrow t_{2}$ if and only if there is a substitution $S$ whose domain is $\bar{u}$ such that:

- $t_{2}$ is identical to $S\left(t_{1}\right)$.
- Whenever $c x_{2}$ holds in the class environment, $S\left(c x_{1}\right)$ also holds.

A value of type $\forall \bar{u} . c x \Rightarrow t$, may be instantiated at types $\bar{s}$ if and only if the context $c x[\bar{s} / \bar{u}]$ holds. For example, consider the function double:

```
double x = x + x
```

The most general type of double is $\forall a$. Num $a \Rightarrow a \rightarrow a$. double may be applied to values of type Int (instantiating $a$ to Int), since Num Int holds, because Int is an instance of the class Num. However, double may not normally be applied to values of type Char, because Char is not normally an instance of class Num. The user may choose to declare such an instance, in which case double may indeed be applied to a Char.

### 4.2 User-Defined Datatypes

In this section, we describe algebraic datatypes (data declarations), renamed datatypes (newt ype declarations), and type synonyms (type declarations). These declarations may only appear at the top level of a module.

### 4.2.1 Algebraic Datatype Declarations

```
topdecl \(\rightarrow\) data [context \(=>]\) simpletype \(=\) constrs \([\) deriving \(]\)
simpletype \(\rightarrow\) tycon tyvar \({ }_{1} \ldots\) tyvar \(_{k} \quad(k \geq 0)\)
constrs \(\rightarrow\) constr \(_{1}|\ldots|\) constr \(_{n} \quad(n \geq 1)\)
constr \(\rightarrow\) con [!] atype \(A_{1} \ldots[!]\) atype \(_{k} \quad(\) arity con \(=k, k \geq 0)\)
    | (btype |!atype) conop (btype |!atype) (infix conop)
    \(\mid \operatorname{con}\left\{\right.\) fielddecl \(_{1}, \ldots\), fielddecl \(\left._{n}\right\} \quad(n \geq 0)\)
fielddecl \(\rightarrow\) vars : : (type |! atype)
deriving \(\rightarrow\) deriving \(\left(\right.\) dclass \(\mid\left(\right.\) dclass \(_{1}, \ldots\), dclass \(\left.\left._{n}\right)\right)(n \geq 0)\)
dclass \(\rightarrow\) qtycls
```

The precedence for constr is the same as that for expressions-normal constructor application has higher precedence than infix constructor application (thus a : F○○ a parses as a : (F○○ a) ).

An algebraic datatype declaration has the form:

$$
\text { data } c x=>T u_{1} \ldots u_{k}=K_{1} t_{11} \ldots t_{1 k_{1}}|\cdots| \begin{array}{lll}
K_{n} & t_{n 1} \ldots & t_{n k_{n}}
\end{array}
$$

where $c x$ is a context. This declaration introduces a new type constructor $T$ with one or more constituent data constructors $K_{1}, \ldots, K_{n}$. In this Report, the unqualified term "constructor" always means "data constructor".

The types of the data constructors are given by:

$$
K_{i}:: \forall u_{1} \ldots u_{k} . c x_{i} \Rightarrow t_{i 1} \rightarrow \cdots \rightarrow t_{i k_{i}} \rightarrow\left(T u_{1} \ldots u_{k}\right)
$$

where $c x_{i}$ is the largest subset of $c x$ that constrains only those type variables free in the types $t_{i 1}, \ldots, t_{i k_{i}}$. The type variables $u_{1}$ through $u_{k}$ must be distinct and may appear in $c x$ and the $t_{i j}$; it is a static error for any other type variable to appear in $c x$ or on the right-hand-side. The new type constant $T$ has a kind of the form $\kappa_{1} \rightarrow \ldots \rightarrow \kappa_{k} \rightarrow *$ where the kinds $\kappa_{i}$ of the argument variables $u_{i}$ are determined by kind inference as described in Section 4.6. This means that $T$ may be used in type expressions with anywhere between 0 and $k$ arguments.

For example, the declaration

```
data Eq a => Set a = NilSet | ConsSet a (Set a)
```

introduces a type constructor Set of kind $* \rightarrow *$, and constructors NilSet and ConsSet with types

```
NilSet :: }\foralla.\mathrm{ Set }
ConsSet :: }\foralla.\mathrm{ Eq }a=>a->\mathrm{ Set }a->\mathrm{ Set }
```

In the example given, the overloaded type for ConsSet ensures that Consset can only be applied to values whose type is an instance of the class Eq. Pattern matching against ConsSet also gives rise to an Eq a constraint. For example:

```
f (ConsSet a s) = a
```

the function $f$ has inferred type Eq a $=>$ Set a -> a. The context in the data declaration has no other effect whatsoever.

The visibility of a datatype's constructors (i.e. the "abstractness" of the datatype) outside of the module in which the datatype is defined is controlled by the form of the datatype's name in the export list as described in Section 5.8.

The optional deriving part of a data declaration has to do with derived instances, and is described in Section 4.3.3.

Labelled Fields A data constructor of arity $k$ creates an object with $k$ components. These components are normally accessed positionally as arguments to the constructor in expressions or patterns. For large datatypes it is useful to assign field labels to the components of a data object. This allows a specific field to be referenced independently of its location within the constructor.

A constructor definition in a data declaration may assign labels to the fields of the constructor, using the record syntax ( C \{ . . . \}). Constructors using field labels may be freely mixed with constructors without them. A constructor with associated field labels may still be used as an ordinary constructor; features using labels are simply a shorthand for operations using an underlying positional constructor. The arguments to the positional constructor occur in the same order as the labeled fields. For example, the declaration

```
data C = F { f1,f2 :: Int, f3 :: Bool }
```

defines a type and constructor identical to the one produced by

```
data C = F Int Int Bool
```

Operations using field labels are described in Section 3.15. A data declaration may use the same field label in multiple constructors as long as the typing of the field is the same in all cases after type synonym expansion. A label cannot be shared by more than one type in scope. Field names share the top level namespace with ordinary variables and class methods and must not conflict with other top level names in scope.

The pattern F \{\} matches any value built with constructor F , whether or not F was declared with record syntax.

Strictness Flags Whenever a data constructor is applied, each argument to the constructor is evaluated if and only if the corresponding type in the algebraic datatype declaration has a strictness flag, denoted by an exclamation point, "!". Lexically, "!" is an ordinary varsym not a reservedop; it has special significance only in the context of the argument types of a data declaration.

Translation: A declaration of the form

$$
\text { data } c x=>T u_{1} \ldots u_{k}=\ldots\left|K s_{1} \ldots s_{n}\right| \ldots
$$

where each $s_{i}$ is either of the form $!t_{i}$ or $t_{i}$, replaces every occurrence of $K$ in an expression by

$$
\left(\backslash x_{1} \ldots x_{n}->\left(\left(\left(K o p_{1} x_{1}\right) o p_{2} x_{2}\right) \ldots\right) o p_{n} x_{n}\right)
$$

where $o p_{i}$ is the non-strict apply function $\$$ if $s_{i}$ is of the form $t_{i}$, and $o p_{i}$ is the strict apply function $\$!$ (see Section 6.2) if $s_{i}$ is of the form $!t_{i}$. Pattern matching on $K$ is not affected by strictness flags.

### 4.2.2 Type Synonym Declarations

topdecl $\rightarrow \quad$ type simpletype $=$ type
simpletype $\rightarrow$ tycon tyvar ${ }_{1} \ldots$ tyvar $_{k} \quad(k \geq 0)$

A type synonym declaration introduces a new type that is equivalent to an old type. It has the form

$$
\text { type } T u_{1} \ldots u_{k}=t
$$

which introduces a new type constructor, $T$. The type ( $T t_{1} \ldots t_{k}$ ) is equivalent to the type $t\left[t_{1} / u_{1}, \ldots, t_{k} / u_{k}\right]$. The type variables $u_{1}$ through $u_{k}$ must be distinct and are scoped only over $t$; it is a static error for any other type variable to appear in $t$. The kind of the new type constructor $T$ is of the form $\kappa_{1} \rightarrow \ldots \rightarrow \kappa_{k} \rightarrow \kappa$ where the kinds $\kappa_{i}$ of the arguments $u_{i}$ and $\kappa$ of the right hand side $t$ are determined by kind inference as described in Section 4.6. For example, the following definition can be used to provide an alternative way of writing the list type constructor:

```
type List = []
```

Type constructor symbols $T$ introduced by type synonym declarations cannot be partially applied; it is a static error to use $T$ without the full number of arguments.

Although recursive and mutually recursive datatypes are allowed, this is not so for type synonyms, unless an algebraic datatype intervenes. For example,

```
type Rec a = [Circ a]
data Circ a = Tag [Rec a]
```

is allowed, whereas

```
type Rec a = [Circ a] -- invalid
type Circ a = [Rec a] -- invalid
```

is not. Similarly, type $\operatorname{Rec} a=[\operatorname{Rec} a]$ is not allowed.
Type synonyms are a convenient, but strictly syntactic, mechanism to make type signatures more readable. A synonym and its definition are completely interchangeable, except in the instance type of an instance declaration (Section 4.3.2).

### 4.2.3 Datatype Renamings

```
topdecl \(\rightarrow\) newtype [context \(=>\) ] simpletype \(=\) newconstr \([\) deriving \(]\)
newconstr \(\rightarrow\) con atype
    | con \{ var : : type \}
simpletype \(\rightarrow\) tycon tyvar \({ }_{1} \ldots\) tyvar \(_{k} \quad(k \geq 0)\)
```

A declaration of the form

$$
\text { newtype } c x=>T u_{1} \ldots u_{k}=N t
$$

introduces a new type whose representation is the same as an existing type. The type ( $T u_{1} \ldots u_{k}$ ) renames the datatype $t$. It differs from a type synonym in that it creates a distinct type that must be explicitly coerced to or from the original type. Also, unlike type synonyms, newt ype may be used to define recursive types. The constructor $N$ in an expression coerces a value from type $t$ to type ( $T u_{1} \ldots u_{k}$ ). Using $N$ in a pattern coerces a value from type ( $T u_{1} \ldots u_{k}$ ) to type $t$. These coercions may be implemented without execution time overhead; newtype does not change the underlying representation of an object.

New instances (see Section 4.3.2) can be defined for a type defined by newtype but may not be defined for a type synonym. A type created by newtype differs from an algebraic datatype in that the representation of an algebraic datatype has an extra level of indirection. This difference may make access to the representation less efficient. The difference is reflected in different rules for pattern matching (see Section 3.17). Unlike algebraic datatypes, the newtype constructor $N$ is unlifted, so that $N \perp$ is the same as $\perp$.

The following examples clarify the differences between data (algebraic datatypes), type (type synonyms), and newt ype (renaming types.) Given the declarations

```
data D1 = D1 Int
data D2 = D2 !Int
type S = Int
newtype N = N Int
d1 (D1 i) = 42
d2 (D2 i) = 42
s i = 42
n (N i) = 42
```

the expressions $(\mathrm{d} 1 \perp),(\mathrm{d} 2 \perp)$ and $(\mathrm{d} 2(\mathrm{D} 2 \perp)$ ) are all equivalent to $\perp$, whereas $(\mathrm{n} \perp),(\mathrm{n}(\mathrm{N} \perp)),(\mathrm{d} 1(\mathrm{D} 1 \perp))$ and $(\mathrm{s} \perp)$ are all equivalent to 42. In particular, ( $\mathrm{N} \perp$ ) is equivalent to $\perp$ while ( $\mathrm{D} 1 \perp$ ) is not equivalent to $\perp$.

The optional deriving part of a newtype declaration is treated in the same way as the deriving component of a data declaration; see Section 4.3.3.

A newtype declaration may use field-naming syntax, though of course there may only be one field. Thus:

```
newtype Age = Age { unAge :: Int }
```

brings into scope both a constructor and a de-constructor:

```
Age :: Int -> Age
unAge :: Age -> Int
```


### 4.3 Type Classes and Overloading

### 4.3.1 Class Declarations

```
topdecl \(\quad \rightarrow \quad\) class [scontext \(=>\) ] tycls tyvar [where cdecls]
scontext \(\rightarrow\) simpleclass
    \(\mid \quad\left(\right.\) simpleclass \(_{1}, \ldots\), simpleclass \(\left._{n}\right) \quad(n \geq 0)\)
simpleclass \(\rightarrow\) qtycls tyvar
cdecls \(\rightarrow \quad\left\{\right.\) cdecl \(\left._{1} ; \ldots ; \operatorname{cdecl}_{n}\right\} \quad(n \geq 0)\)
cdecl \(\rightarrow\) gendecl
    | (funlhs \(\mid\) var) rhs
```

A class declaration introduces a new class and the operations (class methods) on it. A class declaration has the general form:

```
class cx => Cu where cdecls
```

This introduces a new class name $C$; the type variable $u$ is scoped only over the class method signatures in the class body. The context $c x$ specifies the superclasses of $C$, as described below; the only type variable that may be referred to in $c x$ is $u$.

The superclass relation must not be cyclic; i.e. it must form a directed acyclic graph.
The cdecls part of a class declaration contains three kinds of declarations:

- The class declaration introduces new class methods $v_{i}$, whose scope extends outside the class declaration. The class methods of a class declaration are precisely the $v_{i}$ for which there is an explicit type signature

$$
v_{i}:: c x_{i}=>t_{i}
$$

in cdecls. Class methods share the top level namespace with variable bindings and field names; they must not conflict with other top level bindings in scope. That is, a class method can not have the same name as a top level definition, a field name, or another class method.
The type of the top-level class method $v_{i}$ is:

$$
v_{i}:: \forall u, \bar{w} \cdot\left(C u, c x_{i}\right) \Rightarrow t_{i}
$$

The $t_{i}$ must mention $u$; it may mention type variables $\bar{w}$ other than $u$, in which case the type of $v_{i}$ is polymorphic in both $u$ and $\bar{w}$. The $c x_{i}$ may constrain only $\bar{w}$; in particular, the $c x_{i}$ may not constrain $u$. For example:

```
class Foo a where
    op :: Num b => a -> b -> a
```

Here the type of op is $\forall a, b$. (F○○ $a$, Num $b) \Rightarrow a \rightarrow b \rightarrow a$.

- The cdecls may also contain a fixity declaration for any of the class methods (but for no other values). However, since class methods declare top-level values, the fixity declaration for a class method may alternatively appear at top level, outside the class declaration.
- Lastly, the cdecls may contain a default class method for any of the $v_{i}$. The default class method for $v_{i}$ is used if no binding for it is given in a particular instance declaration (see Section 4.3.2). The default method declaration is a normal value definition, except that the left hand side may only be a variable or function definition. For example:

```
class Foo a where
    op1, op2 :: a -> a
    (op1, op2) = ...
```

is not permitted, because the left hand side of the default declaration is a pattern.

Other than these cases, no other declarations are permitted in cdecls.
A class declaration with no where part may be useful for combining a collection of classes into a larger one that inherits all of the class methods in the original ones. For example:

$$
\text { class (Read a, Show a) }=>\text { Textual a }
$$

In such a case, if a type is an instance of all superclasses, it is not automatically an instance of the subclass, even though the subclass has no immediate class methods. The instance declaration must be given explicitly with no where part.

### 4.3.2 Instance Declarations

| topdecl <br> inst | $\rightarrow$ | instance [scontext $=>$ ] qty gtycon |  |
| :---: | :---: | :---: | :---: |
|  |  | ( gtycon tyvar $_{1} \ldots$ tyvar $_{k}$ ) | ( $k \geq$ |
|  |  | ( tyvar ${ }_{1}, \ldots$, tyvar $_{k}$ ) | ( $k \geq$ |
|  |  | [ tyvar ] |  |
|  | \| | ( tyvar $_{1} \rightarrow$ tyvar $_{2}$ ) | (tyva |
| idecls | $\rightarrow$ | \{ decl $_{1} ; \ldots$; idecl $\left._{n}\right\}$ | ( $n \geq$ |
| idecl | $\rightarrow$ | (funlhs \| var) rhs |  |
|  |  |  | (emp |

An instance declaration introduces an instance of a class. Let

$$
\text { class } c x=>C u \text { where }\{\text { cbody }\}
$$

be a class declaration. The general form of the corresponding instance declaration is:

$$
\text { instance } c x^{\prime}=>C\left(T u_{1} \ldots u_{k}\right) \text { where }\{d\}
$$

where $k \geq 0$. The type ( $T u_{1} \ldots u_{k}$ ) must take the form of a type constructor $T$ applied to simple type variables $u_{1}, \ldots u_{k}$; furthermore, $T$ must not be a type synonym, and the $u_{i}$ must all be distinct.

This prohibits instance declarations such as:

```
instance C (a,a) where ...
instance C (Int,a) where ...
instance C [[a]] where ...
```

The declarations $d$ may contain bindings only for the class methods of $C$. It is illegal to give a binding for a class method that is not in scope, but the name under which it is in scope is immaterial; in particular, it may be a qualified name. (This rule is identical to that used for subordinate names in export lists - Section 5.2.) For example, this is legal, even though range is in scope only with the qualified name Ix.range.

```
module A where
    import qualified Ix
    instance Ix.Ix T where
        range = ...
```

The declarations may not contain any type signatures or fixity declarations, since these have already
been given in the class declaration. As in the case of default class methods (Section 4.3.1), the method declarations must take the form of a variable or function definition.

If no binding is given for some class method then the corresponding default class method in the class declaration is used (if present); if such a default does not exist then the class method of this instance is bound to undefined and no compile-time error results.

An instance declaration that makes the type $T$ to be an instance of class $C$ is called a $C-T$ instance declaration and is subject to these static restrictions:

- A type may not be declared as an instance of a particular class more than once in the program.
- The class and type must have the same kind; this can be determined using kind inference as described in Section 4.6.
- Assume that the type variables in the instance type ( $T u_{1} \ldots u_{k}$ ) satisfy the constraints in the instance context $c x^{\prime}$. Under this assumption, the following two conditions must also be satisfied:

1. The constraints expressed by the superclass context $c x[(T u 1 \ldots u k) / u]$ of $C$ must be satisfied. In other words, $T$ must be an instance of each of $C$ 's superclasses and the contexts of all superclass instances must be implied by $c x^{\prime}$.
2. Any constraints on the type variables in the instance type that are required for the class method declarations in $d$ to be well-typed must also be satisfied.

In fact, except in pathological cases it is possible to infer from the instance declaration the most general instance context $c x^{\prime}$ satisfying the above two constraints, but it is nevertheless mandatory to write an explicit instance context.

The following example illustrates the restrictions imposed by superclass instances:

```
class Foo a => Bar a where ...
instance (Eq a, Show a) => Foo [a] where ...
instance Num a => Bar [a] where ...
```

This example is valid Haskell. Since Foo is a superclass of Bar, the second instance declaration is only valid if [a] is an instance of Foo under the assumption Num a. The first instance declaration does indeed say that [a] is an instance of Foo under this assumption, because Eq and Show are superclasses of Num.

If the two instance declarations instead read like this:

```
instance Num a => Foo [a] where ...
instance (Eq a, Show a) => Bar [a] where ...
```

then the program would be invalid. The second instance declaration is valid only if [a] is an
instance of Foo under the assumptions (Eq a, Show a). But this does not hold, since [a] is only an instance of Foo under the stronger assumption Num $a$.

Further examples of instance declarations may be found in Chapter 8.

### 4.3.3 Derived Instances

As mentioned in Section 4.2.1, data and newtype declarations contain an optional deriving form. If the form is included, then derived instance declarations are automatically generated for the datatype in each of the named classes. These instances are subject to the same restrictions as user-defined instances. When deriving a class $C$ for a type $T$, instances for all superclasses of $C$ must exist for $T$, either via an explicit instance declaration or by including the superclass in the deriving clause.

Derived instances provide convenient commonly-used operations for user-defined datatypes. For example, derived instances for datatypes in the class Eq define the operations $==$ and $/=$, freeing the programmer from the need to define them.

The only classes in the Prelude for which derived instances are allowed are Eq, Ord, Enum, Bounded, Show, and Read, all mentioned in Figure 6.1, page 83. The precise details of how the derived instances are generated for each of these classes are provided in Chapter 10, including a specification of when such derived instances are possible. Classes defined by the standard libraries may also be derivable.

A static error results if it is not possible to derive an instance declaration over a class named in a deriving form. For example, not all datatypes can properly support class methods in Enum. It is also a static error to give an explicit instance declaration for a class that is also derived.

If the deriving form is omitted from a data or newtype declaration, then no instance declarations are derived for that datatype; that is, omitting a deriving form is equivalent to including an empty deriving form: deriving ().

### 4.3.4 Ambiguous Types, and Defaults for Overloaded Numeric Operations

topdecl $\rightarrow$ default $\left(\right.$ type $_{1}, \ldots$, type $\left._{n}\right) \quad(n \geq 0)$

A problem inherent with Haskell-style overloading is the possibility of an ambiguous type. For example, using the read and show functions defined in Chapter 10, and supposing that just Int and Bool are members of Read and Show, then the expression

$$
\text { let } x=\text { read "..." in show } x \text {-- invalid }
$$

is ambiguous, because the types for show and read,

$$
\begin{aligned}
& \text { show }:: \forall a . \text { Show } a \Rightarrow a \rightarrow \text { String } \\
& \text { read }:: \forall a . \text { Read } a \Rightarrow \text { String } \rightarrow a
\end{aligned}
$$

could be satisfied by instantiating a as either Int in both cases, or Bool. Such expressions are considered ill-typed, a static error.

We say that an expression e has an ambiguous type if, in its type $\forall \bar{u} . c x \Rightarrow t$, there is a type variable $u$ in $\bar{u}$ that occurs in $c x$ but not in $t$. Such types are invalid.

For example, the earlier expression involving show and read has an ambiguous type since its type is $\forall a$. Show $a$, Read $a \Rightarrow$ String.

Ambiguous types can only be circumvented by input from the user. One way is through the use of expression type-signatures as described in Section 3.16. For example, for the ambiguous expression given earlier, one could write:

```
let x = read "..." in show (x::Bool)
```

which disambiguates the type.
Occasionally, an otherwise ambiguous expression needs to be made the same type as some variable, rather than being given a fixed type with an expression type-signature. This is the purpose of the function asTypeOf (Chapter 8): $x$ 'asTypeOf $y$ has the value of $x$, but $x$ and $y$ are forced to have the same type. For example,

```
approxSqrt x = encodeFloat 1 (exponent x `div` 2) `asTypeOf` x
```

(See Section 6.4.6 for a description of encodeFloat and exponent.)
Ambiguities in the class Num are most common, so Haskell provides another way to resolve themwith a default declaration:

$$
\text { default }\left(t_{1}, \ldots, t_{n}\right)
$$

where $n \geq 0$, and each $t_{i}$ must be a type for which Num $t_{i}$ holds. In situations where an ambiguous type is discovered, an ambiguous type variable, $v$, is defaultable if:

- $v$ appears only in constraints of the form $C v$, where $C$ is a class, and
- at least one of these classes is a numeric class, (that is, Num or a subclass of Num), and
- all of these classes are defined in the Prelude or a standard library (Figures 6.2-6.3, pages 9192 show the numeric classes, and Figure 6.1, page 83, shows the classes defined in the Prelude.)

Each defaultable variable is replaced by the first type in the default list that is an instance of all the ambiguous variable's classes. It is a static error if no such type is found.

Only one default declaration is permitted per module, and its effect is limited to that module. If no default declaration is given in a module then it assumed to be:

```
default (Integer, Double)
```

The empty default declaration, default (), turns off all defaults in a module.

### 4.4 Nested Declarations

The following declarations may be used in any declaration list, including the top level of a module.

### 4.4.1 Type Signatures

```
gendecl \(\rightarrow \quad\) vars \(::[\) context \(=>]\) type
vars \(\quad \rightarrow \quad \operatorname{var}_{1}, \ldots, \operatorname{var}_{n} \quad(n \geq 1)\)
```

A type signature specifies types for variables, possibly with respect to a context. A type signature has the form:

$$
v_{1}, \ldots, v_{n}:: c x=>t
$$

which is equivalent to asserting $v_{i}:: c x=>t$ for each $i$ from 1 to $n$. Each $v_{i}$ must have a value binding in the same declaration list that contains the type signature; i.e. it is invalid to give a type signature for a variable bound in an outer scope. Moreover, it is invalid to give more than one type signature for one variable, even if the signatures are identical.

As mentioned in Section 4.1.2, every type variable appearing in a signature is universally quantified over that signature, and hence the scope of a type variable is limited to the type signature that contains it. For example, in the following declarations

```
f :: a -> a
f x = x :: a -- invalid
```

the a's in the two type signatures are quite distinct. Indeed, these declarations contain a static error, since x does not have type $\forall a . a$. (The type of x is dependent on the type of f ; there is currently no way in Haskell to specify a signature for a variable with a dependent type; this is explained in Section 4.5.4.)

If a given program includes a signature for a variable $f$, then each use of $f$ is treated as having the declared type. It is a static error if the same type cannot also be inferred for the defining occurrence of $f$.

If a variable $f$ is defined without providing a corresponding type signature declaration, then each use of $f$ outside its own declaration group (see Section 4.5) is treated as having the corresponding inferred, or principal type. However, to ensure that type inference is still possible, the defining occurrence, and all uses of $f$ within its declaration group must have the same monomorphic type (from which the principal type is obtained by generalization, as described in Section 4.5.2).

For example, if we define

$$
\operatorname{sqr} x=x^{\star} x
$$

then the principal type is sqr :: $\forall a$. Num $a \Rightarrow a \rightarrow a$, which allows applications such as sqr 5 or sqr 0.1 . It is also valid to declare a more specific type, such as

```
sqr :: Int -> Int
```

but now applications such as sqr 0.1 are invalid. Type signatures such as

```
sqr :: (Num a, Num b) => a -> b -- invalid
sqr :: a -> a -- invalid
```

are invalid, as they are more general than the principal type of sqr.
Type signatures can also be used to support polymorphic recursion. The following definition is pathological, but illustrates how a type signature can be used to specify a type more general than the one that would be inferred:

```
data T a = K (T Int) (T a)
f :: T a -> a
f (K x y) = if f x == 1 then f y else undefined
```

If we remove the signature declaration, the type of $f$ will be inferred as $T$ Int $\rightarrow>$ Int due to the first recursive call for which the argument to $f$ is $T$ Int. Polymorphic recursion allows the user to supply the more general type signature, $\mathrm{T} a->\mathrm{a}$.

### 4.4.2 Fixity Declarations

```
gendecl }->\mathrm{ fixity [integer] ops
fixity }->\mathrm{ infixl|infixr|infix
ops }->o\mp@subsup{p}{1}{},\ldots,o\mp@subsup{p}{n}{}\quad(n\geq1
op varop|conop
```

A fixity declaration gives the fixity and binding precedence of one or more operators. The integer in a fixity declaration must be in the range 0 to 9 . A fixity declaration may appear anywhere that a type signature appears and, like a type signature, declares a property of a particular operator. Also like a type signature, a fixity declaration can only occur in the same sequence of declarations as the declaration of the operator itself, and at most one fixity declaration may be given for any operator. (Class methods are a minor exception; their fixity declarations can occur either in the class declaration itself or at top level.)

There are three kinds of fixity, non-, left- and right-associativity (infix, infixl, and infixr, respectively), and ten precedence levels, 0 to 9 inclusive (level 0 binds least tightly, and level 9 binds most tightly). If the digit is omitted, level 9 is assumed. Any operator lacking a fixity declaration is assumed to be infixl 9 (See Section 3 for more on the use of fixities). Table 4.1 lists the fixities and precedences of the operators defined in the Prelude.

Fixity is a property of a particular entity (constructor or variable), just like its type; fixity is not a property of that entity's name. For example:

| Precedence | Left associative operators | Non-associative operators | Right associative operators |
| :---: | :---: | :---: | :---: |
| 9 | ! ! |  | - |
| 8 |  |  | ^, ^^, ** |
| 7 | $\begin{aligned} & \text { *, /, 'div', } \\ & \text { 'mod', 'rem', 'quot } \end{aligned}$ |  |  |
| 6 | +, - |  |  |
| 5 |  |  | : , ++ |
| 4 |  | $\begin{aligned} & ==, /=,<,<=,>,>=, \\ & \text { 'elem{fd79a9c36-b24c-499e-9d84-6cf0d0630b85} } \end{aligned}$ |  |
| 3 |  |  |  |
| 2 |  |  | 1 |
| 1 | >>, >>= |  |  |
| 0 |  |  | \$, \$!, 'seq' |

Table 4.1: Precedences and fixities of prelude operators

```
module Bar( op ) where
    infixr 7 `op'
    op = ...
module Foo where
    import qualified Bar
    infix 3 'op'
    a 'op' b = (a `Bar.op` b) + 1
    f x = let
        p `op` q = (p `Foo.op' q) * 2
        in ...
```

Here, 'Bar.op'is infixr 7, 'Foo.op'is infix 3, and the nested definition of op in f's right-hand side has the default fixity of infixl 9. (It would also be possible to give a fixity to the nested definition of `op ' with a nested fixity declaration.)

### 4.4.3 Function and Pattern Bindings

$$
\begin{aligned}
& \text { decl } \quad \rightarrow \quad\left(\text { funlhs } \mid \text { pat }{ }^{0}\right) \text { rhs } \\
& \text { funlhs } \rightarrow \text { var apat }\{\text { apat }\} \\
& \text { | pat }{ }^{i+1} \text { varop }^{(a, i)} \text { pat }^{i+1} \\
& \text { | } \text { lpat }^{i} \text { varop }^{(1, i)} \text { pat }^{i+1} \\
& \text { pat }{ }^{i+1} \text { varop }^{(\mathrm{r}, i)} \text { rpat }^{i}
\end{aligned}
$$

```
    | (funlhs) apat { apat }
rhs }->\quad=\operatorname{exp}\mathrm{ [where decls]
    | gdrhs [where decls]
gdrhs }\quad->\quadgd=exp[gdrhs
gd }\quad->\quad| expo
```

We distinguish two cases within this syntax: a pattern binding occurs when the left hand side is a pat ${ }^{0}$; otherwise, the binding is called a function binding. Either binding may appear at the top-level of a module or within a where or let construct.

### 4.4.3.1 Function bindings

A function binding binds a variable to a function value. The general form of a function binding for variable $x$ is:

$$
\begin{array}{lllll}
x & p_{11} & \ldots & p_{1 k} & \text { match }_{1} \\
\ldots & & & \\
x & p_{n 1} \ldots & p_{n k} & \text { match }_{n}
\end{array}
$$

where each $p_{i j}$ is a pattern, and where each match $_{i}$ is of the general form:

$$
=e_{i} \text { where }\left\{d e c l s_{i}\right\}
$$

or

$$
\begin{aligned}
\mid g_{i 1} & =e_{i 1} \\
\cdots & \\
\mid g_{i m_{i}} & =e_{i m_{i}} \\
& \text { where }\left\{d^{2} l_{s}\right\}
\end{aligned}
$$

and where $n \geq 1,1 \leq i \leq n, m_{i} \geq 1$. The former is treated as shorthand for a particular case of the latter, namely:

$$
\mid \text { True }=e_{i} \text { where }\left\{\operatorname{decls}_{i}\right\}
$$

Note that all clauses defining a function must be contiguous, and the number of patterns in each clause must be the same. The set of patterns corresponding to each match must be linear-no variable is allowed to appear more than once in the entire set.

Alternative syntax is provided for binding functional values to infix operators. For example, these three function definitions are all equivalent:

```
plus x y z = x+y+z
x `plus` y = \ z -> x+y+z
    (x `plus` y) z = x+y+z
```

Translation: The general binding form for functions is semantically equivalent to the equation (i.e. simple pattern binding):

$$
\begin{aligned}
& x=\backslash x_{1} \ldots x_{k}->\text { case }\left(x_{1}, \ldots, x_{k}\right) \text { of }\left(p_{11}, \ldots, p_{1 k}\right) \text { match }_{1} \\
& \ldots \\
&\left(p_{n 1}, \ldots, p_{n k}\right) \text { match }_{n}
\end{aligned}
$$

where the $x_{i}$ are new identifiers.

### 4.4.3.2 Pattern bindings

A pattern binding binds variables to values. A simple pattern binding has form $p=e$. The pattern $p$ is matched "lazily" as an irrefutable pattern, as if there were an implicit ~ in front of it. See the translation in Section 3.12.

The general form of a pattern binding is $p$ match, where a match is the same structure as for function bindings above; in other words, a pattern binding is:

$$
\begin{aligned}
& p \quad \left\lvert\, \begin{array}{ll}
g_{1} & =e_{1} \\
g_{2} & =e_{2} \\
\ldots
\end{array}\right. \\
& \quad \mid g_{m}=e_{m} \\
& \text { where }\{\text { decls }\}
\end{aligned}
$$

Translation: The pattern binding above is semantically equivalent to this simple pattern binding:

```
p = let decls in
    if }\mp@subsup{g}{1}{}\mathrm{ then }\mp@subsup{e}{1}{}\mathrm{ else
    if }\mp@subsup{g}{2}{}\mathrm{ then }\mp@subsup{e}{2}{}\mathrm{ else
    if }\mp@subsup{g}{m}{}\mathrm{ then }\mp@subsup{e}{m}{}\mathrm{ else error "Unmatched pattern"
```

A note about syntax. It is usually straightforward to tell whether a binding is a pattern binding or a function binding, but the existence of $n+k$ patterns sometimes confuses the issue. Here are four examples:

```
x + 1 ... -- Function binding, defines (+)
    -- Equivalent to (+) x 1 = ...
(x + 1) = .. -- Pattern binding, defines x
(x + 1) * y = ... -- Function binding, defines (*)
    -- Equivalent to (*) (x+1) y = ...
(x + 1) y = ... -- Function binding, defines (+)
    -- Equivalent to (+) x 1 y = ...
```

The first two can be distinguished because a pattern binding has a pat ${ }^{0}$ on the left hand side, not a pat - the former cannot be an unparenthesised $\mathrm{n}+\mathrm{k}$ pattern.

### 4.5 Static Semantics of Function and Pattern Bindings

The static semantics of the function and pattern bindings of a let expression or where clause are discussed in this section.

### 4.5.1 Dependency Analysis

In general the static semantics are given by the normal Hindley-Milner inference rules. A dependency analysis transformation is first performed to increase polymorphism. Two variables bound by value declarations are in the same declaration group if either

1. they are bound by the same pattern binding, or
2. their bindings are mutually recursive (perhaps via some other declarations that are also part of the group).

Application of the following rules causes each let or where construct (including the where defining the top level bindings in a module) to bind only the variables of a single declaration group, thus capturing the required dependency analysis: ${ }^{1}$

1. The order of declarations in where/let constructs is irrelevant.
2. Let $\left\{d_{1} ; d_{2}\right\}$ in $e=$ let $\left\{d_{1}\right\}$ in (let $\left\{d_{2}\right\}$ in $e$ ) (when no identifier bound in $d_{2}$ appears free in $d_{1}$ )
[^1]
### 4.5.2 Generalization

The Hindley-Milner type system assigns types to a let-expression in two stages. First, the righthand side of the declaration is typed, giving a type with no universal quantification. Second, all type variables that occur in this type are universally quantified unless they are associated with bound variables in the type environment; this is called generalization. Finally, the body of the letexpression is typed.

For example, consider the declaration

```
f x = let g y = (y,y)
    in ...
```

The type of g's definition is $a \rightarrow(a, a)$. The generalization step attributes to g the polymorphic type $\forall a . a \rightarrow(a, a)$, after which the typing of the " . . " part can proceed.

When typing overloaded definitions, all the overloading constraints from a single declaration group are collected together, to form the context for the type of each variable declared in the group. For example, in the definition:

```
f x = let g1 x y = if x>y then show x else g2 y x
    g2 p q = g1 q p
    in ...
```

The types of the definitions of g 1 and g 2 are both $a \rightarrow a \rightarrow$ String, and the accumulated constraints are Ord $a$ (arising from the use of >), and Show $a$ (arising from the use of show). The type variables appearing in this collection of constraints are called the constrained type variables.

The generalization step attributes to both g 1 and g 2 the type

$$
\forall a .(\text { Ord } a, \text { Show } a) \Rightarrow a \rightarrow a \rightarrow \text { String }
$$

Notice that g 2 is overloaded in the same way as g 1 even though the occurrences of $>$ and show are in the definition of g 1 .

If the programmer supplies explicit type signatures for more than one variable in a declaration group, the contexts of these signatures must be identical up to renaming of the type variables.

### 4.5.3 Context Reduction Errors

As mentioned in Section 4.1.4, the context of a type may constrain only a type variable, or the application of a type variable to one or more types. Hence, types produced by generalization must be expressed in a form in which all context constraints have be reduced to this "head normal form". Consider, for example, the definition:

$$
\mathrm{f} x \mathrm{xS} y=\mathrm{xs}==[\mathrm{y}]
$$

Its type is given by
f :: Eq a => [a] -> a -> Bool
and not
f :: Eq [a] => [a] -> a -> Bool

Even though the equality is taken at the list type, the context must be simplified, using the instance declaration for Eq on lists, before generalization. If no such instance is in scope, a static error occurs.

Here is an example that shows the need for a constraint of the form $C(m t)$ where m is one of the type variables being generalized; that is, where the class $C$ applies to a type expression that is not a type variable or a type constructor. Consider:

```
f :: (Monad m, Eq (m a)) => a -> m a -> Bool
f x y = return x == y
```

The type of return is Monad m => a -> m a; the type of (==) is Eq a => a -> a -> Bool. The type of $f$ should be therefore (Monad m, Eq (m a)) => a -> m a -> Bool, and the context cannot be simplified further.

The instance declaration derived from a data type deriving clause (see Section 4.3.3) must, like any instance declaration, have a simple context; that is, all the constraints must be of the form $C a$, where $a$ is a type variable. For example, in the type

```
data Apply a b = App (a b) deriving Show
```

the derived Show instance will produce a context Show (a b), which cannot be reduced and is not simple; thus a static error results.

### 4.5.4 Monomorphism

Sometimes it is not possible to generalize over all the type variables used in the type of the definition. For example, consider the declaration

```
f x = let g y z = ([x,y], z)
    in ...
```

In an environment where x has type $a$, the type of g's definition is $a \rightarrow b \rightarrow([a], b)$. The generalization step attributes to g the type $\forall b . a \rightarrow b \rightarrow([a], b)$; only $b$ can be universally quantified because $a$ occurs in the type environment. We say that the type of g is monomorphic in the type variable $a$.

The effect of such monomorphism is that the first argument of all applications of $g$ must be of a single type. For example, it would be valid for the ". . ." to be
(g True, g False)
(which would, incidentally, force $x$ to have type Bool) but invalid for it to be
( $g$ True, $g^{\prime} C^{\prime}$ )
In general, a type $\forall \bar{u} . c x \Rightarrow t$ is said to be monomorphic in the type variable $a$ if $a$ is free in $\forall \bar{u} . c x \Rightarrow t$.

It is worth noting that the explicit type signatures provided by Haskell are not powerful enough to express types that include monomorphic type variables. For example, we cannot write

```
f \(x=\) let
    \(g:: a->b->([a], b)\)
    \(\mathrm{g} \mathrm{y} \mathrm{z}=([\mathrm{x}, \mathrm{y}], \mathrm{z})\)
    in ...
```

because that would claim that $g$ was polymorphic in both $a$ and $b$ (Section 4.4.1). In this program, $g$ can only be given a type signature if its first argument is restricted to a type not involving type variables; for example

$$
\mathrm{g}:: \text { Int }->\mathrm{b}->([\text { Int }], \mathrm{b})
$$

This signature would also cause x to have type Int.

### 4.5.5 The Monomorphism Restriction

Haskell places certain extra restrictions on the generalization step, beyond the standard HindleyMilner restriction described above, which further reduces polymorphism in particular cases.

The monomorphism restriction depends on the binding syntax of a variable. Recall that a variable is bound by either a function binding or a pattern binding, and that a simple pattern binding is a pattern binding in which the pattern consists of only a single variable (Section 4.4.3).

The following two rules define the monomorphism restriction:

## The monomorphism restriction

Rule 1. We say that a given declaration group is unrestricted if and only if:
(a): every variable in the group is bound by a function binding or a simple pattern binding (Section 4.4.3.2), and
(b): an explicit type signature is given for every variable in the group that is bound by simple pattern binding.

The usual Hindley-Milner restriction on polymorphism is that only type variables that do not occur free in the environment may be generalized. In addition, the constrained type variables of a restricted declaration group may not be generalized in the generalization step for that group. (Recall that a type variable is constrained if it must belong to some type class; see Section 4.5.2.)

Rule 2. Any monomorphic type variables that remain when type inference for an entire module is complete, are considered ambiguous, and are resolved to particular types using the defaulting rules (Section 4.3.4).

Motivation Rule 1 is required for two reasons, both of which are fairly subtle.

- Rule 1 prevents computations from being unexpectedly repeated. For example, genericLength is a standard function (in library List) whose type is given by
genericLength : : Num a => [b] -> a
Now consider the following expression:

```
let \(\{\) len \(=\) genericLength \(x s\) \} in (len, len)
```

It looks as if len should be computed only once, but without Rule 1 it might be computed twice, once at each of two different overloadings. If the programmer does actually wish the computation to be repeated, an explicit type signature may be added:

```
let { len :: Num a => a; len = genericLength xs } in (len, len)
```

- Rule 1 prevents ambiguity. For example, consider the declaration group
$[(n, s)]=r e a d s t$
Recall that reads is a standard function whose type is given by the signature

```
reads :: (Read a) => String -> [(a,String)]
```

Without Rule 1, n would be assigned the type $\forall a$. Read $a \Rightarrow a$ and sthe type $\forall a$. Read $a$ $\Rightarrow$ String. The latter is an invalid type, because it is inherently ambiguous. It is not possible to determine at what overloading to use s , nor can this be solved by adding a type signature for s . Hence, when non-simple pattern bindings are used (Section 4.4.3.2), the types inferred are always monomorphic in their constrained type variables, irrespective of whether a type signature is provided. In this case, both n and s are monomorphic in $a$.

The same constraint applies to pattern-bound functions. For example, in

$$
(f, g)=((+),(-))
$$

both $f$ and $g$ are monomorphic regardless of any type signatures supplied for $f$ or $g$.

Rule 2 is required because there is no way to enforce monomorphic use of an exported binding, except by performing type inference on modules outside the current module. Rule 2 states that the exact types of all the variables bound in a module must be determined by that module alone, and not by any modules that import it.

```
module M1(len1) where
    default( Int, Double )
    len1 = genericLength "Hello"
module M2 where
    import M1 (len1)
    len2 = (2*len1) :: Rational
```

When type inference on module M1 is complete, len 1 has the monomorphic type Num a => a (by Rule 1). Rule 2 now states that the monomorphic type variable a is ambiguous, and must be resolved using the defaulting rules of Section 4.3.4. Hence, len1 gets type Int, and its use in len2 is type-incorrect. (If the above code is actually what is wanted, a type signature on len1 would solve the problem.)

This issue does not arise for nested bindings, because their entire scope is visible to the compiler.

Consequences The monomorphism rule has a number of consequences for the programmer. Anything defined with function syntax usually generalizes as a function is expected to. Thus in

$$
f x y=x+y
$$

the function $f$ may be used at any overloading in class Num. There is no danger of recomputation here. However, the same function defined with pattern syntax:

$$
f=\backslash x->\backslash y \rightarrow x+y
$$

requires a type signature if $f$ is to be fully overloaded. Many functions are most naturally defined using simple pattern bindings; the user must be careful to affix these with type signatures to retain full overloading. The standard prelude contains many examples of this:

```
sum :: (Num a) => [a] -> a
sum = foldl (+) 0
```

Rule 1 applies to both top-level and nested definitions. Consider

```
module M where
    len1 = genericLength "Hello"
    len2 = (2*len1) :: Rational
```

Here, type inference finds that len1 has the monomorphic type (Num $a=>a$ ); and the type variable $a$ is resolved to Rational when performing type inference on len2.

### 4.6 Kind Inference

This section describes the rules that are used to perform kind inference, i.e. to calculate a suitable kind for each type constructor and class appearing in a given program.

The first step in the kind inference process is to arrange the set of datatype, synonym, and class definitions into dependency groups. This can be achieved in much the same way as the dependency analysis for value declarations that was described in Section 4.5. For example, the following program fragment includes the definition of a datatype constructor $D$, a synonym $S$ and a class $C$, all of which would be included in the same dependency group:

```
data C a => D a = Foo (S a)
type S a = [D a]
class C a where
    bar :: a -> D a -> Bool
```

The kinds of variables, constructors, and classes within each group are determined using standard techniques of type inference and kind-preserving unification [7]. For example, in the definitions above, the parameter a appears as an argument of the function constructor $(->)$ in the type of bar and hence must have kind $*$. It follows that both D and S must have kind $* \rightarrow *$ and that every instance of class C must have kind $*$.

It is possible that some parts of an inferred kind may not be fully determined by the corresponding definitions; in such cases, a default of $*$ is assumed. For example, we could assume an arbitrary kind $\kappa$ for the a parameter in each of the following examples:

```
data App f a = A (f a)
data Tree a = Leaf | Fork (Tree a) (Tree a)
```

This would give kinds $(\kappa \rightarrow *) \rightarrow \kappa \rightarrow *$ and $\kappa \rightarrow *$ for App and Tree, respectively, for any kind $\kappa$, and would require an extension to allow polymorphic kinds. Instead, using the default binding $\kappa=*$, the actual kinds for these two constructors are $(* \rightarrow *) \rightarrow * \rightarrow *$ and $* \rightarrow *$, respectively.

Defaults are applied to each dependency group without consideration of the ways in which particular type constructor constants or classes are used in later dependency groups or elsewhere in the program. For example, adding the following definition to those above does not influence the kind inferred for Tree (by changing it to $(* \rightarrow *) \rightarrow *$, for instance), and instead generates a static error because the kind of [ ], $* \rightarrow *$, does not match the kind $*$ that is expected for an argument of Tree:

```
type FunnyTree = Tree [] -- invalid
```

This is important because it ensures that each constructor and class are used consistently with the same kind whenever they are in scope.

## Chapter 5

## Modules

A module defines a collection of values, datatypes, type synonyms, classes, etc. (see Chapter 4), in an environment created by a set of imports (resources brought into scope from other modules). It exports some of these resources, making them available to other modules. We use the term entity to refer to a value, type, or class defined in, imported into, or perhaps exported from a module.

A Haskell program is a collection of modules, one of which, by convention, must be called Main and must export the value main. The value of the program is the value of the identifier main in module Main, which must be a computation of type IO $\tau$ for some type $\tau$ (see Chapter 7). When the program is executed, the computation main is performed, and its result (of type $\tau$ ) is discarded.

Modules may reference other modules via explicit import declarations, each giving the name of a module to be imported and specifying its entities to be imported. Modules may be mutually recursive.

Modules are used for name-space control, and are not first class values. A multi-module Haskell program can be converted into a single-module program by giving each entity a unique name, changing all occurrences to refer to the appropriate unique name, and then concatenating all the module bodies ${ }^{1}$. For example, here is a three-module program:

```
module Main where
    import A
    import B
    main = A.f >> B.f
module A where
    f = ...
module B where
    f = ...
```

It is equivalent to the following single-module program:

[^2]```
module Main where
    main = af >> bf
    af = ...
    bf = ...
```

Because they are allowed to be mutually recursive, modules allow a program to be partitioned freely without regard to dependencies.

The name-space for modules themselves is flat, with each module being associated with a unique module name (which are Haskell identifiers beginning with a capital letter; i.e. modid). There is one distinguished module, Prelude, which is imported into all modules by default (see Section 5.6), plus a set of standard library modules that may be imported as required (see Part II).

### 5.1 Module Structure

A module defines a mutually recursive scope containing declarations for value bindings, data types, type synonyms, classes, etc. (see Chapter 4).


A module begins with a header: the keyword module, the module name, and a list of entities (enclosed in round parentheses) to be exported. The header is followed by a possibly-empty list of import declarations (impdecls, Section 5.3) that specify modules to be imported, optionally restricting the imported bindings. This is followed by a possibly-empty list of top-level declarations (topdecls, Chapter 4).

An abbreviated form of module, consisting only of the module body, is permitted. If this is used, the header is assumed to be 'module Main (main) where'. If the first lexeme in the abbreviated module is not a $\{$, then the layout rule applies for the top level of the module.

### 5.2 Export Lists

$$
\text { exports } \quad \rightarrow \quad\left(\text { export }_{1}, \ldots, \text { export }_{n}[,]\right) \quad(n \geq 0)
$$

```
\(\begin{array}{lll}\text { export } & \rightarrow & \text { qvar } \\ & \mid \quad \text { qtycon }\left[(\ldots) \mid\left(\text { cname }_{1}, \ldots, \text { cname }_{n}\right)\right] \quad(n \geq 0)\end{array}\)
    | qtycls \(\left[(\ldots) \mid\left(\operatorname{var}_{1}, \ldots, \operatorname{var}_{n}\right)\right] \quad(n \geq 0)\)
    module modid
cname \(\quad \rightarrow \quad\) var \(\mid\) con
```

An export list identifies the entities to be exported by a module declaration. A module implementation may only export an entity that it declares, or that it imports from some other module. If the export list is omitted, all values, types and classes defined in the module are exported, but not those that are imported.

Entities in an export list may be named as follows:

1. A value, field name, or class method, whether declared in the module body or imported, may be named by giving the name of the value as a quarid, which must be in scope. Operators should be enclosed in parentheses to turn them into quarids.
2. An algebraic datatype $T$ declared by a data or newtype declaration may be named in one of three ways:

- The form $T$ names the type but not the constructors or field names. The ability to export a type without its constructors allows the construction of abstract datatypes (see Section 5.8).
- The form $T\left(c_{1}, \ldots, c_{n}\right)$, names the type and some or all of its constructors and field names.
- The abbreviated form $T$ (..) names the type and all its constructors and field names that are currently in scope (whether qualified or not).

In all cases, the (possibly-qualified) type constructor $T$ must be in scope. The constructor and field names $c_{i}$ in the second form are unqualified; one of these subordinate names is legal if and only if (a) it names a constructor or field of $T$, and (b) the constructor or field is in scope in the module body regardless of whether it is in scope under a qualified or unqualified name. For example, the following is legal

```
module A( Mb.Maybe( Nothing, Just ) ) where
    import qualified Maybe as Mb
```

Data constructors cannot be named in export lists except as subordinate names, because they cannot otherwise be distinguished from type constructors.
3. A type synonym $T$ declared by a type declaration may be named by the form $T$, where $T$ is in scope.
4. A class $C$ with operations $f_{1}, \ldots, f_{n}$ declared in a class declaration may be named in one of three ways:

- The form $C$ names the class but not the class methods.
- The form $C\left(f_{1}, \ldots, f_{n}\right)$, names the class and some or all of its methods.
- The abbreviated form $C$ (..) names the class and all its methods that are in scope (whether qualified or not).

In all cases, $C$ must be in scope. In the second form, one of the (unqualified) subordinate names $f_{i}$ is legal if and only if (a) it names a class method of $C$, and (b) the class method is in scope in the module body regardless of whether it is in scope under a qualified or unqualified name.
5. The form "module M" names the set of all entities that are in scope with both an unqualified name " $e$ " and a qualified name "M. e". This set may be empty. For example:

```
module Queue( module Stack, enqueue, dequeue ) where
    import Stack
    ...
```

Here the module Queue uses the module name Stack in its export list to abbreviate all the entities imported from Stack.

A module can name its own local definitions in its export list using its own name in the "module M" syntax, because a local declaration brings into scope both a qualified and unqualified name (Section 5.5.1). For example:

```
module Mod1( module Mod1, module Mod2 ) where
import Mod2
import Mod3
```

Here module Mod1 exports all local definitions as well as those imported from Mod2 but not those imported from Mod3.
It is an error to use module $M$ in an export list unless $M$ is the module bearing the export list, or $M$ is imported by at least one import declaration (qualified or unqualified).

Exports lists are cumulative: the set of entities exported by an export list is the union of the entities exported by the individual items of the list.

It makes no difference to an importing module how an entity was exported. For example, a field name $f$ from data type $T$ may be exported individually ( $f$, item (1) above); or as an explicitly-named member of its data type (T (f), item (2)); or as an implicitly-named member (T ( . . ), item(2)); or by exporting an entire module (module M , item (5)).

The unqualified names of the entities exported by a module must all be distinct (within their respective namespace). For example

```
module A ( C.f, C.g, g, module B ) where -- an invalid module
import B(f)
import qualified C(f,g)
g = f True
```

There are no name clashes within module $A$ itself, but there are name clashes in the export list
between C.g and $g$ (assuming C.g and $g$ are different entities - remember, modules can import each other recursively), and between module B and C.f (assuming B.f and C.f are different entities).

### 5.3 Import Declarations

| impdecl | $\rightarrow$ | import[qualified] modid [as modid] [impspec] |  |
| :--- | :--- | :--- | :--- |
|  | $\mid$ |  | (empty declaration $)$ |
| impspec | $\rightarrow$ | $\left(\right.$ import $_{1}, \ldots$, import $\left._{n}[],\right)$ | $(n \geq 0)$ |
|  | $\mid$ | hiding $\left(\right.$ import $_{1}, \ldots$, import $\left._{n}[],\right)$ | $(n \geq 0)$ |
| import | $\rightarrow$ | var |  |
|  | $\mid$ | tycon $\left[(\ldots) \mid\left(\right.\right.$ cname $_{1}, \ldots$, cname $\left.\left._{n}\right)\right]$ | $(n \geq 0)$ |
|  | $\mid$ | tycls $\left[(\ldots) \mid\left(\right.\right.$ var $_{1}, \ldots$, var $\left.\left._{n}\right)\right]$ | $(n \geq 0)$ |
| cname | $\rightarrow$ var $\mid$ con |  |  |

The entities exported by a module may be brought into scope in another module with an import declaration at the beginning of the module. The import declaration names the module to be imported and optionally specifies the entities to be imported. A single module may be imported by more than one import declaration. Imported names serve as top level declarations: they scope over the entire body of the module but may be shadowed by local non-top-level bindings.

The effect of multiple import declarations is strictly cumulative: an entity is in scope if it is imported by any of the import declarations in a module. The ordering of import declarations is irrelevant.

Lexically, the terminal symbols "as", "qualified" and "hiding" are each a varid rather than a reservedid. They have special significance only in the context of an import declaration; they may also be used as variables.

### 5.3.1 What is imported

Exactly which entities are to be imported can be specified in one of the following three ways:

1. The imported entities can be specified explicitly by listing them in parentheses. Items in the list have the same form as those in export lists, except qualifiers are not permitted and the 'module modid' entity is not permitted. When the (. . ) form of import is used for a type or class, the (. . ) refers to all of the constructors, methods, or field names exported from the module.

The list must name only entities exported by the imported module. The list may be empty, in which case nothing except the instances is imported.
2. Entities can be excluded by using the form hiding (import ${ }_{1}$, ..., import $_{n}$ ), which specifies that all entities exported by the named module should be imported except for those named in the list. Data constructors may be named directly in hiding lists without being prefixed by the associated type. Thus, in

```
import M hiding (C)
```

any constructor, class, or type named C is excluded. In contrast, using C in an import list names only a class or type.
It is an error to hide an entity that is not, in fact, exported by the imported module.
3. Finally, if impspec is omitted then all the entities exported by the specified module are imported.

### 5.3.2 Qualified import

For each entity imported under the rules of Section 5.3.1, the top-level environment is extended. If the import declaration used the qualified keyword, only the qualified name of the entity is brought into scope. If the qualified keyword is omitted, then both the qualified and unqualified name of the entity is brought into scope. Section 5.5.1 describes qualified names in more detail.

The qualifier on the imported name is either the name of the imported module, or the local alias given in the as clause (Section 5.3.3) on the import statement. Hence, the qualifier is not necessarily the name of the module in which the entity was originally declared.

The ability to exclude the unqualified names allows full programmer control of the unqualified namespace: a locally defined entity can share the same name as a qualified import:

```
module Ring where
import qualified Prelude -- All Prelude names must be qualified
import List( nub )
l1 + l2 = l1 Prelude.++ l2 -- This + differs from the one in the Prelude
l1 * l2 = nub (l1 + l2) -- This * differs from the one in the Prelude
succ = (Prelude.+ 1)
```


### 5.3.3 Local aliases

Imported modules may be assigned a local alias in the importing module using the as clause. For example, in
import qualified VeryLongModuleName as C
entities must be referenced using 'C.' as a qualifier instead of 'VeryLongModuleName.'. This also allows a different module to be substituted for VeryLongModuleName without changing the
qualifiers used for the imported module. It is legal for more than one module in scope to use the same qualifier, provided that all names can still be resolved unambiguously. For example:

```
module M where
    import qualified Foo as A
    import qualified Baz as A
    x = A.f
```

This module is legal provided only that Foo and Baz do not both export f .
An as clause may also be used on an un-qualified import statement:

```
import Foo as A(f)
```

This declaration brings into scope $f$ and A.f.

### 5.3.4 Examples

To clarify the above import rules, suppose the module $A$ exports x and y . Then this table shows what names are brought into scope by the specified import statement:

| Import declaration | Names brought into scope |
| :--- | :--- |
| import A | x, y, A.x, A.y |
| import A() | (nothing) |
| import A(x) | x, A.x |
| import qualified A | A.x, A.y |
| import qualified A() | (nothing) |
| import qualified A(x) | A.x |
| import A hiding () | x, y, A.x, A.y |
| import A hiding (x) | y, A.y |
| import qualified A hiding () | A.x, A.y |
| import qualified A hiding (x) | A.y |
| import A as B | x, y, B.x, B.y |
| import A as B(x) | x, B.x |
| import qualified A as B | B.x, B.y |

In all cases, all instance declarations in scope in module A are imported (Section 5.4).

### 5.4 Importing and Exporting Instance Declarations

Instance declarations cannot be explicitly named on import or export lists. All instances in scope within a module are always exported and any import brings all instances in from the imported
module. Thus, an instance declaration is in scope if and only if a chain of import declarations leads to the module containing the instance declaration.

For example, import M() does not bring any new names in scope from module M , but does bring in any instances visible in M. A module whose only purpose is to provide instance declarations can have an empty export list. For example

```
module MyInstances() where
    instance Show (a -> b) where
        show fn = "<<function>>"
    instance Show (IO a) where
        show io = "<<IO action>>"
```


### 5.5 Name Clashes and Closure

### 5.5.1 Qualified names

A qualified name is written as modid . name (Section 2.4). A qualified name is brought into scope:

- By a top level declaration. A top-level declaration brings into scope both the unqualified and the qualified name of the entity being defined. Thus:

```
module M where
    f x = ...
    g x = M.f x x
```

is legal. The defining occurrence must mention the unqualified name; therefore, it is illegal to write

```
module M where
    M.f x = ... -- ILLEGAL
    g x = let M.y = x+1 in ... -- ILLEGAL
```

- By an import declaration. An import declaration, whether qualified or not, always brings into scope the qualified name of the imported entity (Section 5.3). This allows a qualified import to be replaced with an unqualified one without forcing changes in the references to the imported names.


### 5.5.2 Name clashes

If a module contains a bound occurrence of a name, such as $f$ or A.f, it must be possible unambiguously to resolve which entity is thereby referred to; that is, there must be only one binding for f or A. f respectively.

It is not an error for there to exist names that cannot be so resolved, provided that the program does not mention those names. For example:

```
module A where
    import B
    import C
    tup = (b, c, d, x)
module B( d, b, x, y ) where
    import D
    x = ...
    y = ...
    b = ...
module C( d, c, x, y ) where
    import D
    x = ...
    y = ...
    C = ...
module D( d ) where
    d = ...
```

Consider the definition of tup.

- The references to b and c can be unambiguously resolved to b declared in B , and c declared in C respectively.
- The reference to $d$ is unambiguously resolved to d declared in D . In this case the same entity is brought into scope by two routes (the import of B and the import of C ), and can be referred to in A by the names $\mathrm{d}, \mathrm{B} . \mathrm{d}$, and C.d.
- The reference to x is ambiguous: it could mean x declared in $B$, or x declared in C. The ambiguity could be fixed by replacing the reference to x by $\mathrm{B} . \mathrm{x}$ or $\mathrm{C} . \mathrm{x}$.
- There is no reference to $y$, so it is not erroneous that distinct entities called $y$ are exported by both $B$ and $C$. An error is only reported if $y$ is actually mentioned.

The name occurring in a type signature or fixity declarations is always unqualified, and unambiguously refers to another declaration in the same declaration list (except that the fixity declaration for a class method can occur at top level - Section 4.4.2). For example, the following module is legal:

```
module F where
    sin :: Float -> Float
    sin x = (x::Float)
    f x = Prelude.sin (F.sin x)
```

The local declaration for sin is legal, even though the Prelude function sin is implicitly in scope.

The references to Prelude.sin and F.sin must both be qualified to make it unambiguous which sin is meant. However, the unqualified name sin in the type signature in the first line of $F$ unambiguously refers to the local declaration for sin.

### 5.5.3 Closure

Every module in a Haskell program must be closed. That is, every name explicitly mentioned by the source code must be either defined locally or imported from another module. However, entities that the compiler requires for type checking or other compile time analysis need not be imported if they are not mentioned by name. The Haskell compilation system is responsible for finding any information needed for compilation without the help of the programmer. That is, the import of a variable x does not require that the datatypes and classes in the signature of x be brought into the module along with x unless these entities are referenced by name in the user program. The Haskell system silently imports any information that must accompany an entity for type checking or any other purposes. Such entities need not even be explicitly exported: the following program is valid even though $T$ does not escape M1:

```
module M1(x) where
    data T = T
    x = T
module M2 where
    import M1(x)
    y = x
```

In this example, there is no way to supply an explicit type signature for y since T is not in scope. Whether or not T is explicitly exported, module M 2 knows enough about T to correctly type check the program.

The type of an exported entity is unaffected by non-exported type synonyms. For example, in

```
module M(x) where
    type T = Int
    x :: T
    x = 1
```

the type of x is both T and Int; these are interchangeable even when T is not in scope. That is, the definition of $T$ is available to any module that encounters it whether or not the name $T$ is in scope. The only reason to export $T$ is to allow other modules to refer it by name; the type checker finds the definition of $T$ if needed whether or not it is exported.

### 5.6 Standard Prelude

Many of the features of Haskell are defined in Haskell itself as a library of standard datatypes, classes, and functions, called the "Standard Prelude." In Haskell, the Prelude is contained in the
module Prelude. There are also many predefined library modules, which provide less frequently used functions and types. For example, complex numberss, arrays, and most of the input/output are all part of the standard libraries. These are defined in Part II Separating libraries from the Prelude has the advantage of reducing the size and complexity of the Prelude, allowing it to be more easily assimilated, and increasing the space of useful names available to the programmer.

Prelude and library modules differ from other modules in that their semantics (but not their implementation) are a fixed part of the Haskell language definition. This means, for example, that a compiler may optimize calls to functions in the Prelude without consulting the source code of the Prelude.

### 5.6.1 The Prelude Module

The Prelude module is imported automatically into all modules as if by the statement 'import Prelude', if and only if it is not imported with an explicit import declaration. This provision for explicit import allows entities defined in the Prelude to be selectively imported, just like those from any other module.

The semantics of the entities in Prelude is specified by a reference implementation of Prelude written in Haskell, given in Chapter 8. Some datatypes (such as Int) and functions (such as Int addition) cannot be specified directly in Haskell. Since the treatment of such entities depends on the implementation, they are not formally defined in Chapter 8. The implementation of Prelude is also incomplete in its treatment of tuples: there should be an infinite family of tuples and their instance declarations, but the implementation only gives a scheme.

Chapter 8 defines the module Prelude using several other modules: PreludeList, PreludeIO, and so on. These modules are not part of Haskell 98, and they cannot be imported separately. They are simply there to help explain the structure of the Prelude module; they should be considered part of its implementation, not part of the language definition.

### 5.6.2 Shadowing Prelude Names

The rules about the Prelude have been cast so that it is possible to use Prelude names for nonstandard purposes; however, every module that does so must have an import declaration that makes this nonstandard usage explicit. For example:

```
module A( null, nonNull ) where
    import Prelude hiding( null )
    null, nonNull :: Int -> Bool
    null }\textrm{x}=\textrm{x}==
    nonNull x = not (null x)
```

Module A redefines null, and contains an unqualified reference to null on the right hand side of nonNull. The latter would be ambiguous without the hiding (null) on the
import Prelude statement. Every module that imports A unqualified, and then makes an unqualified reference to null must also resolve the ambiguous use of null just as A does. Thus there is little danger of accidentally shadowing Prelude names.

It is possible to construct and use a different module to serve in place of the Prelude. Other than the fact that it is implicitly imported, the Prelude is an ordinary Haskell module; it is special only in that some objects in the Prelude are referenced by special syntactic constructs. Redefining names used by the Prelude does not affect the meaning of these special constructs. For example, in

```
module B where
    import Prelude()
    import MyPrelude
    f x = (x,x)
    g x = (, ) x x
    h x = [x] ++ []
```

the explicit import Prelude() declaration prevents the automatic import of Prelude, while the declaration import MyPrelude brings the non-standard prelude into scope. The special syntax for tuples (such as $(\mathrm{x}, \mathrm{x})$ and $($,$) ) and lists (such as [\mathrm{x}]$ and $[$ ]) continues to refer to the tuples and lists defined by the standard Prelude; there is no way to redefine the meaning of [x], for example, in terms of a different implementation of lists. On the other hand, the use of ++ is not special syntax, so it refers to ++ imported from MyPrelude.

It is not possible, however, to hide instance declarations in the Prelude. For example, one cannot define a new instance for Show Char.

### 5.7 Separate Compilation

Depending on the Haskell implementation used, separate compilation of mutually recursive modules may require that imported modules contain additional information so that they may be referenced before they are compiled. Explicit type signatures for all exported values may be necessary to deal with mutual recursion. The precise details of separate compilation are not defined by this report.

### 5.8 Abstract Datatypes

The ability to export a datatype without its constructors allows the construction of abstract datatypes (ADTs). For example, an ADT for stacks could be defined as:

```
module Stack( StkType, push, pop, empty ) where
    data StkType a = EmptyStk | Stk a (StkType a)
    push x s = Stk x s
    pop (Stk _ s) = s
    empty = EmptyStk
```

Modules importing Stack cannot construct values of type StkType because they do not have access to the constructors of the type. Instead, they must use push, pop, and empty to construct such values.

It is also possible to build an ADT on top of an existing type by using a newt ype declaration. For example, stacks can be defined with lists:

```
module Stack( StkType, push, pop, empty ) where
    newtype StkType a = Stk [a]
    push x (Stk s) = Stk (x:s)
    pop (Stk (_:s)) = Stk s
    empty = Stk []
```


## Chapter 6

## Predefined Types and Classes

The Haskell Prelude contains predefined classes, types, and functions that are implicitly imported into every Haskell program. In this chapter, we describe the types and classes found in the Prelude. Most functions are not described in detail here as they can easily be understood from their definitions as given in Chapter 8. Other predefined types such as arrays, complex numbers, and rationals are defined in Part II.

### 6.1 Standard Haskell Types

These types are defined by the Haskell Prelude. Numeric types are described in Section 6.4. When appropriate, the Haskell definition of the type is given. Some definitions may not be completely valid on syntactic grounds but they faithfully convey the meaning of the underlying type.

### 6.1.1 Booleans

```
data Bool = False | True deriving
    (Read, Show, Eq, Ord, Enum, Bounded)
```

The boolean type Bool is an enumeration. The basic boolean functions are $\& \&$ (and), || (or), and not. The name otherwise is defined as True to make guarded expressions more readable.

### 6.1.2 Characters and Strings

The character type Char is an enumeration whose values represent Unicode characters [11]. The lexical syntax for characters is defined in Section 2.6; character literals are nullary constructors in the datatype Char. Type Char is an instance of the classes Read, Show, Eq, Ord, Enum, and

Bounded. The toEnum and fromEnum functions, standard functions from class Enum, map characters to and from the Int type.

Note that ASCII control characters each have several representations in character literals: numeric escapes, ASCII mnemonic escapes, and the $\backslash^{\wedge} X$ notation. In addition, there are the following equivalences: \a and $\backslash B E L$, $\backslash \mathrm{b}$ and $\backslash \mathrm{BS}$, $\backslash \mathrm{f}$ and $\backslash F F, \backslash r$ and $\backslash C R$, $\backslash t$ and $\backslash H T$, $\backslash \mathrm{v}$ and $\backslash V T$, and $\backslash \mathrm{n}$ and $\backslash \mathrm{LF}$.

A string is a list of characters:
type String $=$ [Char]
Strings may be abbreviated using the lexical syntax described in Section 2.6. For example, "A string" abbreviates
[ 'A',' ','s','t','r', 'i','n','g']

### 6.1.3 Lists

$$
\text { data }[a]=[] \mid a:[a] \text { deriving (Eq, Ord) }
$$

Lists are an algebraic datatype of two constructors, although with special syntax, as described in Section 3.7. The first constructor is the null list, written ' [ ]' ("nil"), and the second is ':' ("cons"). The module PreludeList (see Section 8.1) defines many standard list functions. Arithmetic sequences and list comprehensions, two convenient syntaxes for special kinds of lists, are described in Sections 3.10 and 3.11, respectively. Lists are an instance of classes Read, Show, Eq, Ord, Monad, Functor, and MonadPlus.

### 6.1.4 Tuples

Tuples are algebraic datatypes with special syntax, as defined in Section 3.8. Each tuple type has a single constructor. All tuples are instances of Eq, Ord, Bounded, Read, and Show (provided, of course, that all their component types are).

There is no upper bound on the size of a tuple, but some Haskell implementations may restrict the size of tuples, and limit the instances associated with larger tuples. However, every Haskell implementation must support tuples up to size 15, together with the instances for Eq, Ord, Bounded, Read, and Show. The Prelude and libraries define tuple functions such as zip for tuples up to a size of 7 .

The constructor for a tuple is written by omitting the expressions surrounding the commas; thus $(\mathrm{x}, \mathrm{y})$ and (, ) x y produce the same value. The same holds for tuple type constructors; thus, (Int, Bool, Int) and (, , ) Int Bool Int denote the same type.

The following functions are defined for pairs (2-tuples): fst, snd, curry, and uncurry. Similar functions are not predefined for larger tuples.

### 6.1.5 The Unit Datatype

```
data () = () deriving (Eq, Ord, Bounded, Enum, Read, Show)
```

The unit datatype () has one non- $\perp$ member, the nullary constructor (). See also Section 3.9.

### 6.1.6 Function Types

Functions are an abstract type: no constructors directly create functional values. The following simple functions are found in the Prelude: id, const, (.), flip, (\$), and until.

### 6.1.7 The IO and IOError Types

The IO type serves as a tag for operations (actions) that interact with the outside world. The IO type is abstract: no constructors are visible to the user. IO is an instance of the Monad and Functor classes. Chapter 7 describes I/O operations.

IOError is an abstract type representing errors raised by I/O operations. It is an instance of Show and Eq. Values of this type are constructed by the various I/O functions and are not presented in any further detail in this report. The Prelude contains a few I/O functions (defined in Section 8.3), and Part II contains many more.

### 6.1.8 Other Types

```
data Maybe a = Nothing | Just a deriving (Eq, Ord, Read, Show)
data Either a b = Left a | Right b deriving (Eq, Ord, Read, Show)
data Ordering = LT | EQ | GT deriving
    (Eq, Ord, Bounded, Enum, Read, Show)
```

The Maybe type is an instance of classes Functor, Monad, and MonadPlus. The Ordering type is used by compare in the class Ord. The functions maybe and either are found in the Prelude.

### 6.2 Strict Evaluation

Function application in Haskell is non-strict; that is, a function argument is evaluated only when required. Sometimes it is desirable to force the evaluation of a value, using the seq function:

```
seq :: a -> b -> b
```

The function seq is defined by the equations:

$$
\begin{aligned}
& \text { seq } \perp b=\perp \\
& \text { seq } a b=b, \text { if } a \neq \perp
\end{aligned}
$$

seq is usually introduced to improve performance by avoiding unneeded laziness. Strict datatypes (see Section 4.2.1) are defined in terms of the $\$$ ! operator. However, the provision of seq has important semantic consequences, because it is available at every type. As a consequence, $\perp$ is not the same as $\backslash x \rightarrow \perp$, since seq can be used to distinguish them. For the same reason, the existence of seq weakens Haskell's parametricity properties.

The operator $\$!$ is strict (call-by-value) application, and is defined in terms of seq. The Prelude also defines the $\$$ operator to perform non-strict application.

```
infixr 0 $, $!
($), ($!) :: (a -> b) -> a -> b
f $ x = f x
f $! x = x `seq` f x
```

The non-strict application operator \$ may appear redundant, since ordinary application ( $\mathrm{f} x$ ) means the same as ( $f \$ x$ ). However, $\$$ has low, right-associative binding precedence, so it sometimes allows parentheses to be omitted; for example:

$$
f \$ g \$ h x=f(g(h x))
$$

It is also useful in higher-order situations, such as map (\$0) xs, or zipWith (\$) fs xs.

### 6.3 Standard Haskell Classes

Figure 6.1 shows the hierarchy of Haskell classes defined in the Prelude and the Prelude types that are instances of these classes.

Default class method declarations (Section 4.3) are provided for many of the methods in standard classes. A comment with each class declaration in Chapter 8 specifies the smallest collection of method definitions that, together with the default declarations, provide a reasonable definition for all the class methods. If there is no such comment, then all class methods must be given to fully specify an instance.

### 6.3.1 The Eq Class

$$
\begin{aligned}
& \text { class Eq a where } \\
& \begin{array}{l}
(==), \quad(/=):: a->a->\text { Bool } \\
x /=y=\operatorname{not}(x==y) \\
x==y=\operatorname{not}(x /=y)
\end{array}
\end{aligned}
$$



Figure 6.1: Standard Haskell Classes

The Eq class provides equality $(==)$ and inequality ( $/=$ ) methods. All basic datatypes except for functions and IO are instances of this class. Instances of Eq can be derived for any user-defined datatype whose constituents are also instances of Eq.

This declaration gives default method declarations for both $/=$ and $==$, each being defined in terms of the other. If an instance declaration for Eq defines neither $==$ nor $/=$, then both will loop. If one is defined, the default method for the other will make use of the one that is defined. If both are defined, neither default method is used.

### 6.3.2 The Ord Class

```
class (Eq a) => Ord a where
    compare :: a -> a -> Ordering
    \((<), \quad(<=), \quad(>=),(>):: ~ a ~->~ a ~->~ B o o l ~\)
```



```
    compare \(x y \mid x==y \quad=E Q\)
                | \(\mathrm{x}<=\mathrm{y}=\mathrm{LT}\)
                otherwise \(=\) GT
    \(\mathrm{x}<=\mathrm{y}=\) compare \(\mathrm{x} \mathrm{y} /=\mathrm{GT}\)
    \(x<y=\) compare \(x y==L T\)
    \(\mathrm{x}>=\mathrm{y}=\) compare \(\mathrm{x} y /=\mathrm{LT}\)
    \(\mathrm{x}>\mathrm{y}=\) compare \(\mathrm{x} \mathrm{y}==\mathrm{GT}\)
    -- Note that (min \(x\) y, max \(x y)=(x, y)\) or \((y, x)\)
    \(\max x y \mid x<=y=y\)
        otherwise \(=x\)
    \(\min x y \mid x<=y=x\)
        | otherwise \(=\mathrm{y}\)
```

The Ord class is used for totally ordered datatypes. All basic datatypes except for functions, IO, and IOError, are instances of this class. Instances of Ord can be derived for any user-defined datatype whose constituent types are in Ord. The declared order of the constructors in the data declaration determines the ordering in derived Ord instances. The Ordering datatype allows a single comparison to determine the precise ordering of two objects.

The default declarations allow a user to create an Ord instance either with a type-specific compare function or with type-specific $==$ and $<=$ functions.

### 6.3.3 The Read and Show Classes

```
type ReadS a = String -> [(a,String)]
type ShowS = String -> String
class Read a where
    readsPrec :: Int -> ReadS a
    readList :: ReadS [a]
    -- ... default decl for readList given in Prelude
class Show a where
    showsPrec :: Int -> a -> ShowS
    show :: a -> String
    showList :: [a] -> ShowS
    showsPrec _ x s = show x ++ s
    show x = showsPrec 0 x ""
    -- ... default decl for showList given in Prelude
```

The Read and Show classes are used to convert values to or from strings. The Int argument to showsPrec and readsPrec gives the operator precedence of the enclosing context (see Section 10.4).
showsPrec and showList return a String-to-String function, to allow constant-time concatenation of its results using function composition. A specialised variant, show, is also provided, which uses precedence context zero, and returns an ordinary String. The method showList is provided to allow the programmer to give a specialised way of showing lists of values. This is particularly useful for the Char type, where values of type String should be shown in double quotes, rather than between square brackets.

Derived instances of Read and Show replicate the style in which a constructor is declared: infix constructors and field names are used on input and output. Strings produced by showsPrec are usually readable by readsPrec.

All Prelude types, except function types and IO types, are instances of Show and Read. (If desired, a programmer can easily make functions and IO types into (vacuous) instances of Show, by providing an instance declaration.)

For convenience, the Prelude provides the following auxiliary functions:

```
reads :: (Read a) => ReadS a
reads = readsPrec 0
shows :: (Show a) => a -> ShowS
shows = showsPrec 0
read :: (Read a) => String -> a
read s = case [x | (x,t) <- reads s, ("","") <- lex t] of
    [x] -> x
    [] -> error "PreludeText.read: no parse"
    _ -> error "PreludeText.read: ambiguous parse"
```

shows and reads use a default precedence of 0 . The read function reads input from a string, which must be completely consumed by the input process.

The function lex : : ReadS String, used by read, is also part of the Prelude. It reads a single lexeme from the input, discarding initial white space, and returning the characters that constitute the lexeme. If the input string contains only white space, lex returns a single successful "lexeme" consisting of the empty string. (Thus lex $" "=[(" \|, ")]$.) If there is no legal lexeme at the beginning of the input string, lex fails (i.e. returns []).

### 6.3.4 The Enum Class

```
class Enum a where
    succ, pred : : a -> a
    toEnum : : Int -> a
    fromEnum : : a \(->\) Int
    enumFrom : : a \(->\) [a] -- [n..]
    enumFromThen : : a \(->\) a \(->\) [a] \(--\left[n, n^{\prime} ..\right]\)
    enumFromTo : : a \(->\) a \(->\) [a] -- [n..m]
    enumFromThenTo : : a \(->\) a \(->\) a \(->\) [a] -- [n, \(n^{\prime} . . m\) ]
    -- Default declarations given in Prelude
```

Class Enum defines operations on sequentially ordered types. The functions succ and pred return the successor and predecessor, respectively, of a value. The functions fromEnum and toEnum map values from a type in Enum to and from Int. The enumFrom... methods are used when translating arithmetic sequences (Section 3.10).

Instances of Enum may be derived for any enumeration type (types whose constructors have no fields); see Chapter 10.

For any type that is an instance of class Bounded as well as Enum, the following should hold:

- The calls succ maxBound and pred minBound should result in a runtime error.
- fromEnum and toEnum should give a runtime error if the result value is not representable in the result type. For example, toEnum 7 : : Bool is an error.
- enumFrom and enumFromThen should be defined with an implicit bound, thus:

```
enumFrom x = enumFromTo }\textrm{x}\mathrm{ maxBound
enumFromThen x y = enumFromThenTo x y bound
            where
            bound | fromEnum y >= fromEnum x = maxBound
            | otherwise = minBound
```

The following Prelude types are instances of Enum:

- Enumeration types: (), Bool, and Ordering. The semantics of these instances is given by Chapter 10. For example, [LT. . ] is the list [LT, EQ, GT].
- Char: the instance is given in Chapter 8, based on the primitive functions that convert between a Char and an Int. For example, enumFromTo ' $a^{\prime}$ ' $z$ ' denotes the list of lowercase letters in alphabetical order.
- Numeric types: Int, Integer, Float, Double. The semantics of these instances is given next.

For all four numeric types, succ adds 1, and pred subtracts 1. The conversions fromEnum and toEnum convert between the type and Int. In the case of Float and Double, the digits after the decimal point may be lost. It is implementation-dependent what fromEnum returns when applied to a value that is too large to fit in an Int.

For the types Int and Integer, the enumeration functions have the following meaning:

- The sequence enumFrom $e_{1}$ is the list $\left[e_{1}, e_{1}+1, e_{1}+2, \ldots\right]$.
- The sequence enumFromThen $e_{1} e_{2}$ is the list [ $\left.e_{1}, e_{1}+i, e_{1}+2 i, \ldots\right]$, where the increment, $i$, is $e_{2}-e_{1}$. The increment may be zero or negative. If the increment is zero, all the list elements are the same.
- The sequence enumFromTo $e_{1} e_{3}$ is the list $\left[e_{1}, e_{1}+1, e_{1}+2, \ldots e_{3}\right]$. The list is empty if $e_{1}>e_{3}$.
- The sequence enumFromThenTo $e_{1} e_{2} e_{3}$ is the list $\left[e_{1}, e_{1}+i, e_{1}+2 i, \ldots e_{3}\right.$ ], where the increment, $i$, is $e_{2}-e_{1}$. If the increment is positive or zero, the list terminates when the next element would be greater than $e_{3}$; the list is empty if $e_{1}>e_{3}$. If the increment is negative, the list terminates when the next element would be less than $e_{3}$; the list is empty if $e 1<e_{3}$.

For Float and Double, the semantics of the enumFrom family is given by the rules for Int above, except that the list terminates when the elements become greater than $e_{3}+i / 2$ for positive increment $i$, or when they become less than $e_{3}+i / 2$ for negative $i$.

For all four of these Prelude numeric types, all of the enumFrom family of functions are strict in all their arguments.

### 6.3.5 The Functor Class

```
class Functor f where
    fmap :: (a -> b) -> f a -> f b
```

The Functor class is used for types that can be mapped over. Lists, IO, and Maybe are in this class.

Instances of Functor should satisfy the following laws:

$$
\begin{array}{ll}
\text { fmap id } & =\text { id } \\
\text { fmap (f . g) } & =\text { fmap f. fmap } g
\end{array}
$$

All instances of Functor defined in the Prelude satisfy these laws.

### 6.3.6 The Monad Class

```
class Monad m where
    ( \(\gg=\) ) : : m a \(->(\mathrm{a}->\mathrm{m} \mathrm{b})\)-> m b
    ( \(\gg\) ) : : m a \(->\mathrm{m}\) b \(->\mathrm{m}\) b
    return :: a \(->\mathrm{m}\) a
    fail : : String -> m a
    \(\mathrm{m} \gg \mathrm{k}=\mathrm{m} \gg=\_{-}->\mathrm{k}\)
    fail \(s=\) error \(s\)
```

The Monad class defines the basic operations over a monad. See Chapter 7 for more information about monads.
"do" expressions provide a convenient syntax for writing monadic expressions (see Section 3.14). The fail method is invoked on pattern-match failure in a do expression.

In the Prelude, lists, Maybe, and IO are all instances of Monad. The fail method for lists returns the empty list [ ], for Maybe returns Nothing, and for IO raises a user exception in the IO monad (see Section 7.3).

Instances of Monad should satisfy the following laws:

$$
\begin{array}{ll}
\text { return a } \gg=k & =k a \\
m \gg=\text { return } & =m \\
m \gg=(\backslash x->k x \gg=h) & =(m \gg=k) \gg=h
\end{array}
$$

Instances of both Monad and Functor should additionally satisfy the law:

```
fmap f xs = xs >>= return . f
```

All instances of Monad defined in the Prelude satisfy these laws.
The Prelude provides the following auxiliary functions:

```
sequence :: Monad m => [m a] -> m [a]
sequence_ :: Monad m => [m a] -> m ()
mapM :: Monad m => (a -> m b) -> [a] -> m [b]
mapM_ :: Monad m => (a -> m b) -> [a] -> m ()
(=<<) :: Monad m => (a -> m b) -> m a -> m b
```


### 6.3.7 The Bounded Class

```
class Bounded a where
    minBound, maxBound :: a
```

The Bounded class is used to name the upper and lower limits of a type. Ord is not a superclass of Bounded since types that are not totally ordered may also have upper and lower bounds. The types Int, Char, Bool, (), Ordering, and all tuples are instances of Bounded. The Bounded class may be derived for any enumeration type; minBound is the first constructor listed in the data declaration and maxBound is the last. Bounded may also be derived for single-constructor datatypes whose constituent types are in Bounded.

### 6.4 Numbers

Haskell provides several kinds of numbers; the numeric types and the operations upon them have been heavily influenced by Common Lisp and Scheme. Numeric function names and operators are usually overloaded, using several type classes with an inclusion relation shown in Figure 6.1, page 83. The class Num of numeric types is a subclass of Eq, since all numbers may be compared for equality; its subclass Real is also a subclass of Ord, since the other comparison operations apply to all but complex numbers (defined in the Complex library). The class Integral contains integers of both limited and unlimited range; the class Fractional contains all non-integral types; and the class Floating contains all floating-point types, both real and complex.

The Prelude defines only the most basic numeric types: fixed sized integers (Int), arbitrary precision integers (Integer), single precision floating (Float), and double precision floating (Double). Other numeric types such as rationals and complex numbers are defined in libraries. In particular, the type Rational is a ratio of two Integer values, as defined in the Ratio library.

The default floating point operations defined by the Haskell Prelude do not conform to current language independent arithmetic (LIA) standards. These standards require considerably more complexity in the numeric structure and have thus been relegated to a library. Some, but not all, aspects of the IEEE floating point standard have been accounted for in Prelude class RealFloat.

The standard numeric types are listed in Table 6.1. The finite-precision integer type Int covers at least the range $\left[-2^{29}, 2^{29}-1\right]$. As Int is an instance of the Bounded class, maxBound and minBound can be used to determine the exact Int range defined by an implementation. Float is implementation-defined; it is desirable that this type be at least equal in range and precision to the IEEE single-precision type. Similarly, Double should cover IEEE double-precision. The results of exceptional conditions (such as overflow or underflow) on the fixed-precision numeric types are undefined; an implementation may choose error ( $\perp$, semantically), a truncated value, or a special value such as infinity, indefinite, etc.

The standard numeric classes and other numeric functions defined in the Prelude are shown in Figures 6.2-6.3. Figure 6.1 shows the class dependencies and built-in types that are instances of the numeric classes.

| Type | Class | Description |
| :--- | :--- | :--- |
| Integer | Integral | Arbitrary-precision integers |
| Int | Integral | Fixed-precision integers |
| (Integral a) $=>$ Ratio a | RealFrac | Rational numbers |
| Float | RealFloat | Real floating-point, single precision |
| Double | RealFloat | Real floating-point, double precision |
| (RealFloat a) $=>$ Complex $a$ | Floating | Complex floating-point |

Table 6.1: Standard Numeric Types

### 6.4.1 Numeric Literals

The syntax of numeric literals is given in Section 2.5. An integer literal represents the application of the function fromInteger to the appropriate value of type Integer. Similarly, a floating literal stands for an application of fromRational to a value of type Rational (that is, Ratio Integer). Given the typings:

```
fromInteger :: (Num a) => Integer -> a
fromRational :: (Fractional a) => Rational -> a
```

integer and floating literals have the typings (Num a) $=>$ a and (Fractional a) $=>$ a, respectively. Numeric literals are defined in this indirect way so that they may be interpreted as values of any appropriate numeric type. See Section 4.3 .4 for a discussion of overloading ambiguity.

### 6.4.2 Arithmetic and Number-Theoretic Operations

The infix class methods $(+),(*),(-)$, and the unary function negate (which can also be written as a prefix minus sign; see section 3.4) apply to all numbers. The class methods quot, rem, div, and mod apply only to integral numbers, while the class method (/) applies only to fractional ones. The quot, rem, div, and mod class methods satisfy these laws if $y$ is non-zero:

$$
\begin{aligned}
& (x \text { `quot` } y) * y+(x \text { `rem } y)==x \\
& (x \text { `div` } y) * y+(x \text { `mod } y)==x
\end{aligned}
$$

`quot ' is integer division truncated toward zero, while the result of `div` is truncated toward negative infinity. The quotRem class method takes a dividend and a divisor as arguments and returns a (quotient, remainder) pair; divMod is defined similarly:

```
quotRem x y = (x `quot` y, x `rem` y)
divMod x y = (x `div` y, x `mod` y)
```

Also available on integral numbers are the even and odd predicates:

```
even x = x `rem` 2 == 0
odd = not . even
```

```
class (Eq a, Show a) => Num a where
    (+), (-), (*) :: a -> a -> a
    negate :: a -> a
    abs, signum :: a -> a
    fromInteger :: Integer -> a
class (Num a, Ord a) => Real a where
    toRational :: a -> Rational
class (Real a, Enum a) => Integral a where
    quot, rem, div, mod :: a -> a -> a
    quotRem, divMod :: a -> a -> (a,a)
    toInteger :: a -> Integer
class (Num a) => Fractional a where
        (/) :: a -> a -> a
    recip :: a -> a
    fromRational :: Rational -> a
class (Fractional a) => Floating a where
    pi :: a
    exp, log, sqrt :: a -> a
    (**), logBase :: a -> a -> a
    sin, cos, tan :: a -> a
    asin, acos, atan :: a -> a
    sinh, cosh, tanh :: a -> a
    asinh, acosh, atanh :: a -> a
```

Figure 6.2: Standard Numeric Classes and Related Operations, Part 1

Finally, there are the greatest common divisor and least common multiple functions. gcd $x$ $y$ is the greatest (positive) integer that divides both $x$ and $y$; for example gcd ( -3 ) $6=3$, $\operatorname{gcd}(-3)(-6)=3$, gcd $04=4$. gcd 00 raises a runtime error.
$\operatorname{lcm} x y$ is the smallest positive integer that both $x$ and $y$ divide.

### 6.4.3 Exponentiation and Logarithms

The one-argument exponential function exp and the logarithm function log act on floating-point numbers and use base $e$. logBase $a x$ returns the logarithm of $x$ in base $a$. sqrt returns the principal square root of a floating-point number. There are three two-argument exponentiation operations: ( ${ }^{\wedge}$ ) raises any number to a nonnegative integer power, ( ${ }^{\wedge}$ ) raises a fractional number to any integer power, and (**) takes two floating-point arguments. The value of $x^{\wedge} 0$ or $x^{\wedge} \wedge^{\wedge} 0$ is 1 for any $x$, including zero; $0 * * y$ is undefined.

```
class (Real a, Fractional a) => RealFrac a where
    properFraction :: (Integral b) => a -> (b,a)
    truncate, round :: (Integral b) => a -> b
    ceiling, floor :: (Integral b) => a -> b
class (RealFrac a, Floating a) => RealFloat a where
    floatRadix :: a -> Integer
    floatDigits :: a -> Int
    floatRange :: a -> (Int,Int)
    decodeFloat :: a -> (Integer,Int)
    encodeFloat :: Integer -> Int -> a
    exponent :: a -> Int
    significand :: a -> a
    scaleFloat :: Int -> a -> a
    isNaN, isInfinite, isDenormalized, isNegativeZero, isIEEE
        :: a -> Bool
    atan2 :: a -> a -> a
gcd, lcm :: (Integral a) => a -> a-> a
(^) :: (Num a, Integral b) => a -> b -> a
(^^) :: (Fractional a, Integral b) => a -> b -> a
fromIntegral :: (Integral a, Num b) => a -> b
realToFrac :: (Real a, Fractional b) => a -> b
```

Figure 6.3: Standard Numeric Classes and Related Operations, Part 2

### 6.4.4 Magnitude and Sign

A number has a magnitude and a sign. The functions abs and signum apply to any number and satisfy the law:

```
abs x * signum x == x
```

For real numbers, these functions are defined by:

```
abs x { | | | = 0 = x
signum x | x > 0 = 1
    | == 0 = 0
    x<0=-1
```


### 6.4.5 Trigonometric Functions

Class Floating provides the circular and hyperbolic sine, cosine, and tangent functions and their inverses. Default implementations of tan, tanh, logBase, **, and sqrt are provided, but implementors are free to provide more accurate implementations.

Class RealFloat provides a version of arctangent taking two real floating-point arguments. For real floating $x$ and $y$, atan $2 y x$ computes the angle (from the positive x -axis) of the vector from the origin to the point $(x, y)$. atan2 $y x$ returns a value in the range $[-$ pi, pi]. It follows the Common Lisp semantics for the origin when signed zeroes are supported. atan $2 y 1$, with $y$ in a type that is RealFloat, should return the same value as atan $y$. A default definition of atan2 is provided, but implementors can provide a more accurate implementation.

The precise definition of the above functions is as in Common Lisp, which in turn follows Penfield's proposal for APL [9]. See these references for discussions of branch cuts, discontinuities, and implementation.

### 6.4.6 Coercions and Component Extraction

The ceiling, floor, truncate, and round functions each take a real fractional argument and return an integral result. ceiling $x$ returns the least integer not less than $x$, and floor $x$, the greatest integer not greater than $x$. truncate $x$ yields the integer nearest $x$ between 0 and $x$, inclusive. round $x$ returns the nearest integer to $x$, the even integer if $x$ is equidistant between two integers.

The function properFraction takes a real fractional number $x$ and returns a pair $(n, f)$ such that $x=n+f$, and: $n$ is an integral number with the same sign as $x$; and $f$ is a fraction $f$ with the same type and sign as $x$, and with absolute value less than 1 . The ceiling, floor, truncate, and round functions can be defined in terms of properFraction.

Two functions convert numbers to type Rational: toRational returns the rational equivalent of its real argument with full precision; approxRational takes two real fractional arguments $x$ and $\epsilon$ and returns the simplest rational number within $\epsilon$ of $x$, where a rational $p / q$ in reduced form is simpler than another $p^{\prime} / q^{\prime}$ if $|p| \leq\left|p^{\prime}\right|$ and $q \leq q^{\prime}$. Every real interval contains a unique simplest rational; in particular, note that $0 / 1$ is the simplest rational of all.

The class methods of class RealFloat allow efficient, machine-independent access to the components of a floating-point number. The functions floatRadix, floatDigits, and floatRange give the parameters of a floating-point type: the radix of the representation, the number of digits of this radix in the significand, and the lowest and highest values the exponent may assume, respectively. The function decodefloat applied to a real floating-point number returns the significand expressed as an Integer and an appropriately scaled exponent (an Int). If decodefloat x yields $(m, n)$, then x is equal in value to $m b^{n}$, where $b$ is the floating-point radix, and furthermore, either $m$ and $n$ are both zero or else $b^{d-1} \leq m<b^{d}$, where $d$ is the value of floatDigits x. encodeFloat performs the inverse of this transformation. The functions
significand and exponent together provide the same information as decodeFloat, but rather than an Integer, significand $x$ yields a value of the same type as $x$, scaled to lie in the open interval $(-1,1)$. exponent 0 is zero. scaleFloat multiplies a floating-point number by an integer power of the radix.

The functions isNaN, isInfinite, isDenormalized, isNegativeZero, and isIEEE all support numbers represented using the IEEE standard. For non-IEEE floating point numbers, these may all return false.

Also available are the following coercion functions:

```
fromIntegral :: (Integral a, Num b) => a -> b
realToFrac :: (Real a, Fractional b) => a -> b
```


## Chapter 7

## Basic Input/Output

The I/O system in Haskell is purely functional, yet has all of the expressive power found in conventional programming languages. To achieve this, Haskell uses a monad to integrate I/O operations into a purely functional context.

The I/O monad used by Haskell mediates between the values natural to a functional language and the actions that characterize I/O operations and imperative programming in general. The order of evaluation of expressions in Haskell is constrained only by data dependencies; an implementation has a great deal of freedom in choosing this order. Actions, however, must be ordered in a welldefined manner for program execution - and I/O in particular - to be meaningful. Haskell's I/O monad provides the user with a way to specify the sequential chaining of actions, and an implementation is obliged to preserve this order.

The term monad comes from a branch of mathematics known as category theory. From the perspective of a Haskell programmer, however, it is best to think of a monad as an abstract datatype. In the case of the I/O monad, the abstract values are the actions mentioned above. Some operations are primitive actions, corresponding to conventional I/O operations. Special operations (methods in the class Monad, see Section 6.3.6) sequentially compose actions, corresponding to sequencing operators (such as the semicolon) in imperative languages.

### 7.1 Standard I/O Functions

Although Haskell provides fairly sophisticated I/O facilities, as defined in the IO library, it is possible to write many Haskell programs using only the few simple functions that are exported from the Prelude, and which are described in this section.

All I/O functions defined here are character oriented. The treatment of the newline character will vary on different systems. For example, two characters of input, return and linefeed, may read as a single newline character. These functions cannot be used portably for binary I/O.

In the following, recall that String is a synonym for [Char] (Section 6.1.2).

Output Functions These functions write to the standard output device (this is normally the user's terminal).

```
putChar :: Char -> IO ()
putStr :: String -> IO ()
putStrLn :: String -> IO () -- adds a newline
print :: Show a => a -> IO ()
```

The print function outputs a value of any printable type to the standard output device. Printable types are those that are instances of class Show; print converts values to strings for output using the show operation and adds a newline.

For example, a program to print the first 20 integers and their powers of 2 could be written as:

```
main = print ([(n, 2^n) | n <- [0..19]])
```

Input Functions These functions read input from the standard input device (normally the user's terminal).

```
getChar :: IO Char
getLine :: IO String
getContents :: IO String
interact :: (String -> String) -> IO ()
readIO :: Read a => String -> IO a
readLn :: Read a => IO a
```

The get Char operation raises an exception (Section 7.3) on end-of-file; a predicate isEOFError that identifies this exception is defined in the IO library. The getLine operation raises an exception under the same circumstances as hGetLine, defined the IO library.

The get Contents operation returns all user input as a single string, which is read lazily as it is needed. The interact function takes a function of type String->String as its argument. The entire input from the standard input device is passed to this function as its argument, and the resulting string is output on the standard output device.

Typically, the read operation from class Read is used to convert the string to a value. The readIO function is similar to read except that it signals parse failure to the I/O monad instead of terminating the program. The readLn function combines getLine and readIO.

The following program simply removes all non-ASCII characters from its standard input and echoes the result on its standard output. (The isAscii function is defined in a library.)

```
main = interact (filter isAscii)
```

Files These functions operate on files of characters. Files are named by strings using some implementation-specific method to resolve strings as file names.

The writeFile and appendFile functions write or append the string, their second argument, to the file, their first argument. The readFile function reads a file and returns the contents of the file as a string. The file is read lazily, on demand, as with getContents.

```
type FilePath = String
writeFile :: FilePath -> String -> IO ()
appendFile :: FilePath -> String -> IO ()
readFile :: FilePath -> IO String
```

Note that writeFile and appendFile write a literal string to a file. To write a value of any printable type, as with print, use the show function to convert the value to a string first.

```
main = appendFile "squares" (show [(x,x*x) | x <- [0,0.1..2]])
```


### 7.2 Sequencing I/O Operations

The type constructor IO is an instance of the Monad class. The two monadic binding functions, methods in the Monad class, are used to compose a series of I/O operations. The >> function is used where the result of the first operation is uninteresting, for example when it is (). The >>= operation passes the result of the first operation as an argument to the second operation.

```
(>>=) :: IO a -> (a -> IO b) -> IO b
(>>) :: IO a -> IO b -> IO b
```

For example,

```
main = readFile "input-file" >>= \ s ->
    writeFile "output-file" (filter isAscii s) >>
    putStr "Filtering successful\n"
```

is similar to the previous example using interact, but takes its input from "input-file" and writes its output to "output-file". A message is printed on the standard output before the program completes.

The do notation allows programming in a more imperative syntactic style. A slightly more elaborate version of the previous example would be:

```
main = do
putStr "Input file: "
ifile <- getLine
putStr "Output file: "
ofile <- getLine
s <- readFile ifile
writeFile ofile (filter isAscii s)
putStr "Filtering successful\n"
```

The return function is used to define the result of an I/O operation. For example, get Line is defined in terms of getChar, using return to define the result:

```
getLine :: IO String
getLine = do c <- getChar
    if c == '\n' then return ""
        else do s <- getLine
                        return (c:s)
```


### 7.3 Exception Handling in the I/O Monad

The I/O monad includes a simple exception handling system. Any I/O operation may raise an exception instead of returning a result.

Exceptions in the I/O monad are represented by values of type IOError. This is an abstract type: its constructors are hidden from the user. The IO library defines functions that construct and examine IOError values. The only Prelude function that creates an IOError value is userError. User error values include a string describing the error.

```
userError :: String -> IOError
```

Exceptions are raised and caught using the following functions:

```
ioError :: IOError -> IO a
catch :: IO a -> (IOError -> IO a) -> IO a
```

The ioError function raises an exception; the catch function establishes a handler that receives any exception raised in the action protected by catch. An exception is caught by the most recent handler established by catch. These handlers are not selective: all exceptions are caught. Exception propagation must be explicitly provided in a handler by re-raising any unwanted exceptions. For example, in

```
f = catch g (\e -> if IO.isEOFError e then return [] else ioError e)
```

the function $f$ returns [] when an end-of-file exception occurs in $g$; otherwise, the exception is propagated to the next outer handler. The isEOFError function is part of IO library.

When an exception propagates outside the main program, the Haskell system prints the associated IOError value and exits the program.

The fail method of the IO instance of the Monad class (Section 6.3.6) raises a userError, thus:

```
instance Monad IO where
    ...bindings for return, (>>=), (>>)
    fail s = ioError (userError s)
```

The exceptions raised by the I/O functions in the Prelude are defined in Chapter 21.

## Chapter 8

## Standard Prelude

In this chapter the entire Haskell Prelude is given. It constitutes a specification for the Prelude. Many of the definitions are written with clarity rather than efficiency in mind, and it is not required that the specification be implemented as shown here.

The default method definitions, given with class declarations, constitute a specification only of the default method. They do not constitute a specification of the meaning of the method in all instances. To take one particular example, the default method for enumFrom in class Enum will not work properly for types whose range exceeds that of Int (because fromEnum cannot map all values in the type to distinct Int values).

The Prelude shown here is organized into a root module, Prelude, and three sub-modules, PreludeList, PreludeText, and PreludeIO. This structure is purely presentational. An implementation is not required to use this organisation for the Prelude, nor are these three modules available for import separately. Only the exports of module Prelude are significant.

Some of these modules import Library modules, such as Char, Monad, IO, and Numeric. These modules are described fully in Part II. These imports are not, of course, part of the specification of the Prelude. That is, an implementation is free to import more, or less, of the Library modules, as it pleases.

Primitives that are not definable in Haskell, indicated by names starting with "prim", are defined in a system dependent manner in module PreludeBuiltin and are not shown here. Instance declarations that simply bind primitives to class methods are omitted. Some of the more verbose instances with obvious functionality have been left out for the sake of brevity.

Declarations for special types such as Integer, or () are included in the Prelude for completeness even though the declaration may be incomplete or syntactically invalid. An ellipsis ". . ." is often used in places where the remainder of a definition cannot be given in Haskell.

To reduce the occurrence of unexpected ambiguity errors, and to improve efficiency, a number of commonly-used functions over lists use the Int type rather than using a more general numeric type, such as Integral a or Num a. These functions are: take, drop, !!, length,
splitAt, and replicate. The more general versions are given in the List library, with the prefix "generic"; for example genericLength.

```
module Prelude (
    module PreludeList, module PreludeText, module PreludeIO,
    Bool(False, True),
    Maybe (Nothing, Just),
    Either(Left, Right),
    Ordering(LT, EQ, GT),
    Char, String, Int, Integer, Float, Double, Rational, IO,
-- These built-in types are defined in the Prelude, but
-- are denoted by built-in syntax, and cannot legally
-- appear in an export list.
-- List type: []((:), [])
-- Tuple types: (,)((,)), (,,)((,,)), etc.
-- Trivial type: ()(())
-- Functions: (->)
    Eq((==), (/=)),
    Ord(compare, (<), (<=), (>=), (>), max, min),
    Enum(succ, pred, toEnum, fromEnum, enumFrom, enumFromThen,
        enumFromTo, enumFromThenTo),
    Bounded(minBound, maxBound),
    Num((+), (-), (*), negate, abs, signum, fromInteger),
    Real(toRational),
    Integral(quot, rem, div, mod, quotRem, divMod, toInteger),
    Fractional((/), recip, fromRational),
    Floating(pi, exp, log, sqrt, (**), logBase, sin, cos, tan,
                asin, acos, atan, sinh, cosh, tanh, asinh, acosh, atanh),
    RealFrac(properFraction, truncate, round, ceiling, floor),
    RealFloat(floatRadix, floatDigits, floatRange, decodeFloat,
                    encodeFloat, exponent, significand, scaleFloat, isNaN,
                            isInfinite, isDenormalized, isIEEE, isNegativeZero, atan2),
    Monad((>>=), (>>), return, fail),
    Functor(fmap),
    mapM, mapM_, sequence, sequence_, (=<<),
    maybe, either,
    (&&), (||), not, otherwise,
    subtract, even, odd, gcd, lcm, (^), (^^),
    fromIntegral, realToFrac,
    fst, snd, curry, uncurry, id, const, (.), flip, ($), until,
    asTypeOf, error, undefined,
    seq, ($!)
    ) where
import PreludeBuiltin -- Contains all 'prim' values
import UnicodePrims( primUnicodeMaxChar ) -- Unicode primitives
import PreludeList
import PreludeText
import PreludeIO
import Ratio( Rational )
```

```
infixr 9 .
infixr 8 ^, ^^, **
infixl 7 *, /, `quot`, `rem`, `div`, `mod`
infixl 6 +, -
-- The (:) operator is built-in syntax, and cannot legally be given
-- a fixity declaration; but its fixity is given by:
-- infixr 5 :
infix 4 ==, /=, <, <=, >=, >
infixr 3 &&
infixr 2 ||
infixl 1 >>, >>=
infixr 1 =<<
infixr 0 $, $!, `seq`
-- Standard types, classes, instances and related functions
-- Equality and Ordered classes
class Eq a where
    (==), (/=) :: a -> a -> Bool
            -- Minimal complete definition:
            -- (==) or (/=)
    x /= y = not ( }\textrm{x}==\textrm{y}
    x == y = not (x /= y)
class (Eq a) => Ord a where
    compare :: a -> a -> Ordering
    (<), (<=), (>=), (>) :: a -> a -> Bool
    max, min :: a -> a -> a
            -- Minimal complete definition:
            -- (<=) or compare
            -- Using compare can be more efficient for complex types.
    compare x y
            x == y = EQ
            x<= y = LT
            otherwise = GT
    x <= y = compare x y /= GT
    x < y = compare x y == LT
    x >= y = compare x y /= LT
    x > y = compare x y == GT
-- note that (min x y, max x y) = (x,y) or (y,x)
    max x y
                x<= y = y 
    min x y
                x<= y = x 
```

```
-- Enumeration and Bounded classes
class Enum a where
    succ, pred :: a -> a
    toEnum :: Int -> a
    fromEnum :: a -> Int
    enumFrom :: a -> [a] -- [n..]
    enumFromThen :: a -> a -> [a] -- [n,n'..]
    enumFromTo :: a -> a -> [a] -- [n..m]
    enumFromThenTo :: a -> a -> a -> [a] -- [n,n'..m]
            -- Minimal complete definition:
            -- toEnum, fromEnum
            --
            -- NOTE: these default methods only make sense for types
            -- that map injectively into Int using fromEnum
            -- and toEnum.
    succ = toEnum . (+1) . fromEnum
    pred = toEnum . (subtract 1) . fromEnum
    enumFrom x = map toEnum [fromEnum x ..]
    enumFromTo x y = map toEnum [fromEnum x .. fromEnum y]
    enumFromThen x y = map toEnum [fromEnum x, fromEnum y ..]
    enumFromThenTo x y z =
                                    map toEnum [fromEnum x, fromEnum y .. fromEnum z]
class Bounded a where
    minBound :: a
    maxBound :: a
-- Numeric classes
class (Eq a, Show a) => Num a where
    (+), (-), (*) :: a -> a -> a
    negate :: a -> a
    abs, signum :: a -> a
    fromInteger :: Integer -> a
            -- Minimal complete definition:
            -- All, except negate or (-)
    x - y = x + negate y
    negate x = 0 - x
class (Num a, Ord a) => Real a where
    toRational :: a -> Rational
```

```
class (Real a, Enum a) => Integral a where
    quot, rem :: a -> a -> a
    div, mod :: a -> a -> a
    quotRem, divMod :: a -> a -> (a,a)
    toInteger :: a -> Integer
        -- Minimal complete definition:
        -- quotRem, toInteger
    n `quot` d = q where (q,r) = quotRem n d
    n `rem` d = r where (q,r) = quotRem n d
    n `div` d = q where (q,r) = divMod n d
    n `mod` d = r where (q,r) = divMod n d
    divMod n d = if signum r == - signum d then (q-1, r+d) else qr
    where qr@(q,r) = quotRem n d
class (Num a) => Fractional a where
    (/) :: a -> a -> a
    recip :: a -> a
    fromRational :: Rational -> a
        -- Minimal complete definition:
        -- fromRational and (recip or (/))
    recip x = 1 / x
    x / y = x * recip y
class (Fractional a) => Floating a where
    pi :: a
    exp, log, sqrt :: a -> a
    (**), logBase :: a -> a -> a
    sin, cos, tan :: a -> a
    asin, acos, atan :: a -> a
    sinh, cosh, tanh :: a -> a
    asinh, acosh, atanh :: a -> a
        -- Minimal complete definition:
        -- pi, exp, log, sin, cos, sinh, cosh
        -- asin, acos, atan
        -- asinh, acosh, atanh
    x ** y = exp (log x * y)
    logBase x y = log y / log x
    sqrt x = x ** 0.5
    tan x = sin x / cos x
    tanh x = sinh x / cosh x
```

```
class (Real a, Fractional a) => RealFrac a where
    properFraction :: (Integral b) => a -> (b,a)
    truncate, round :: (Integral b) => a -> b
    ceiling, floor :: (Integral b) => a -> b
        -- Minimal complete definition:
        -- properFraction
    truncate x = m where (m,_) = properFraction x
    round x = let (n,r) = properFraction x
            m = if r < 0 then n - 1 else n + 1
        in case signum (abs r - 0.5) of
                        -1 -> n
        0 -> if even n then n else m
        1 -> m
    ceiling x = if r > 0 then n + 1 else n
    where (n,r) = properFraction x
    floor x = if r < 0 then n - 1 else n
        where (n,r) = properFraction x
```

```
class (RealFrac a, Floating a) \(=>\) RealFloat a where
    floatRadix : : a \(->\) Integer
    floatDigits : : a -> Int
    floatRange : : a \(->\) (Int, Int)
    decodeFloat : a \(->\) (Integer, Int)
    encodefloat : : Integer \(->\) Int \(->\) a
    exponent : : a \(->\) Int
    significand : : a \(->\) a
    scaleFloat : : Int \(->\) a \(->\) a
    isNaN, isInfinite, isDenormalized, isNegativeZero, isIEEE
    :: a -> Bool
    atan2 :: a \(->\) a \(->\) a
        -- Minimal complete definition:
        -- All except exponent, significand,
        -- scaleFloat, atan2
    exponent \(x \quad=\quad\) if \(m==0\) then 0 else \(n+f l o a t D i g i t s x\)
        where \((m, n)=\) decodeFloat \(x\)
    significand \(x=\) encodeFloat \(m\) (- floatDigits \(x\) )
        where \((\mathrm{m}, \ldots)=\) decodeFloat x
    scalefloat \(k \quad=\) encodeFloat \(m(n+k)\)
        where \((m, n)=\) decodeFloat \(x\)
    \(\operatorname{atan} 2 \mathrm{y} x\)
        \(x>0=\operatorname{atan}(y / x)\)
        \(x==0\) \& \& \(y>0=p i / 2\)
        \(\mathrm{x}<0 \quad \& \& \mathrm{y}>0=\mathrm{pi}+\operatorname{atan}(\mathrm{y} / \mathrm{x})\)
        \(\mid(x<=0 \quad \& \& \quad y<0)\)
            ( \(x<0\) \&\& isNegativeZero \(y)\)
            (isNegativeZero \(x\) \&\& isNegativeZero y)
                        \(=-\operatorname{atan} 2(-y) x\)
        \(\mid y==0 \& \&(x<0| |\) isNegativeZero \(x)\)
            \(=\) pi \(\quad--\) must be after the previous test on zero \(y\)
        \(x==0 \& \& \quad y=0 \quad-\quad\) must be after the other double zero tests
        otherwise \(=x+y--x\) or \(y\) is a NaN, return a NaN (via +)
-- Numeric functions
subtract : (Num a) \(=>\) a \(->\) a \(->\) a
subtract \(=\) flip (-)
even, odd : (Integral a) \(=>\) a \(->\) Bool
even \(n \quad=n\) 'rem' \(2==0\)
odd \(=\) not \(\cdot\) even
\(\operatorname{gcd} \quad::(\) Integral \(a)=>a \rightarrow a \operatorname{a}->a\)
gcd \(00=\) error "Prelude.gcd: gcd 00 is undefined"
\(\operatorname{gcd} x y \quad=\operatorname{gcd}^{\prime}(\operatorname{abs} x)(a b s y)\)
        where \(\operatorname{gcd}^{\prime} \mathrm{x} 0=\mathrm{x}\)
                        \(\operatorname{gcd}^{\prime} x y=\operatorname{gcd}^{\prime} y(x\) 'rem' \(y)\)
```

```
lcm :: (Integral a) => a -> a -> a
lcm_0 = 0
lcm 0_ = 0
lcm x y = abs ((x `quot` (gcd x y)) * y)
(^) :: (Num a, Integral b) => a -> b -> a
x ^ 0 = 1
x^n n n > 0 = f x (n-1) x
    where f - 0 y = y
                        f x n y = g x n where
                            g x n | even n = g ( }\mp@subsup{\textrm{x}}{}{*}\textrm{x}) (\textrm{n}\mathrm{ `quot` 2)
        = error "Prelude.^: negative exponent"
(^^)
    :: (Fractional a, Integral b) => a -> b -> a
    = if n >= 0 then x^n else recip (x^(-n))
fromIntegral :: (Integral a, Num b) => a -> b
fromIntegral = fromInteger . toInteger
realToFrac :: (Real a, Fractional b) => a -> b
realToFrac = fromRational . toRational
-- Monadic classes
class Functor f where
    fmap :: (a -> b) -> f a -> f b
class Monad m where
        (>>=) :: m a -> (a -> m b) -> m b
        (>>) :: m a -> m b -> m b
        return :: a -> m a
        fail :: String -> m a
            -- Minimal complete definition:
            -- (>>=), return
        m >> k = m >>= \__ -> k
        fail s = error s
sequence :: Monad m => [m a] -> m [a]
sequence = foldr mcons (return [])
                                where mcons p q = p >>= \x -> q >>= \y -> return (x:y)
sequence_ :: Monad m => [m a] -> m ()
sequence_ = foldr (>>) (return ())
-- The xxxM functions take list arguments, but lift the function or
-- list element to a monad type
mapM :: Monad m => (a -> m b) -> [a] -> m [b]
mapM f as = sequence (map f as)
mapM_ :: Monad m => (a -> m b) -> [a] -> m ()
mapM_f as = sequence_(map f as)
(=<<) :: Monad m => (a -> m b) -> m a -> m b
f =<< x = x >>= f
```

```
-- Trivial type
data () = () deriving (Eq, Ord, Enum, Bounded)
        -- Not legal Haskell; for illustration only
-- Function type
-- identity function
id :: a -> a
id x = x
-- constant function
const :: a -> b -> a
const x _ = x
-- function composition
(.) :: (b -> c) -> (a -> b) -> a -> c
f . g = \ x -> f (g x)
-- flip f takes its (first) two arguments in the reverse order of f.
flip :: (a -> b -> c) -> b -> a -> c
flip f x y = f y x
seq :: a -> b -> b
seq = ... -- Primitive
-- right-associating infix application operators
-- (useful in continuation-passing style)
($), ($!) :: (a -> b) -> a -> b
f $ x = f x
f $! x = x `seq` f x
-- Boolean type
data Bool = False | True deriving (Eq, Ord, Enum, Read, Show, Bounded)
-- Boolean functions
(&&), (||) :: Bool -> Bool -> Bool
True && x = x
False && _ = False
True || - = True
False || x = x
not :: Bool -> Bool
not True = False
not False = True
otherwise :: Bool
otherwise = True
-- Character type
data Char = ... 'a' | 'b' ... -- Unicode values
instance Eq Char where
    c == c' = fromEnum c == fromEnum c'
```

```
instance Ord Char where
    c <= c' = fromEnum c <= fromEnum c'
instance Enum Char where
    toEnum = primIntToChar
    fromEnum = primCharToInt
    enumFrom c = map toEnum [fromEnum c .. fromEnum (maxBound::Char)]
    enumFromThen c c' = map toEnum [fromEnum c, fromEnum c' .. fromEnum lastChar]
        where lastChar :: Char
        lastChar }{\begin{array}{l}{\mp@subsup{C}{}{\prime}<c}\\{\mathrm{ otherwise = minBound}}\\{= maxBound}
instance Bounded Char where
    minBound = '\0'
    maxBound = primUnicodeMaxChar
type String = [Char]
-- Maybe type
data Maybe a = Nothing | Just a deriving (Eq, Ord, Read, Show)
maybe :: b -> (a -> b) -> Maybe a -> b
maybe n f Nothing = n
maybe n f (Just x) = f x
instance Functor Maybe where
    fmap f Nothing = Nothing
    fmap f (Just x) = Just (f x)
instance Monad Maybe where
    (Just x) >>= k = k x
    Nothing >>= k = Nothing
    return = Just
    fail s = Nothing
-- Either type
data Either a b = Left a | Right b deriving (Eq, Ord, Read, Show)
either :: (a -> c) -> (b -> c) -> Either a b -> c
either f g (Left x) = f x
either f g (Right y) = g y
-- IO type
data IO a = ... -- abstract
instance Functor IO where
    fmap f x = x >>= (return . f)
instance Monad IO where
    (>>=) = ...
    return = ...
    fail s = ioError (userError s)
```

```
-- Ordering type
data Ordering = LT | EQ | GT
    deriving (Eq, Ord, Enum, Read, Show, Bounded)
-- Standard numeric types. The data declarations for these types cannot
-- be expressed directly in Haskell since the constructor lists would be
-- far too large.
data Int = minBound ... -1 | 0 | 1 ... maxBound
instance Eq Int where ...
instance Ord Int where ...
instance Num Int where ...
instance Real Int where ...
instance Integral Int where ...
instance Enum Int where ...
instance Bounded Int where ...
data Integer = ... -1 | 0 | 1 ...
instance Eq Integer where ...
instance Ord Integer where ...
instance Num Integer where ...
instance Real Integer where ...
instance Integral Integer where ...
instance Enum Integer where ...
data Float
instance Eq Float where ...
instance Ord Float where ...
instance Num Float where ...
instance Real Float where ...
instance Fractional Float where ...
instance Floating Float where ...
instance RealFrac Float where ...
instance RealFloat Float where ...
data Double
instance Eq Double where ...
instance Ord Double where ...
instance Num Double where ...
instance Real Double where ...
instance Fractional Double where ...
instance Floating Double where...
instance RealFrac Double where ...
instance RealFloat Double where ...
```

```
-- The Enum instances for Floats and Doubles are slightly unusual.
-- The 'toEnum' function truncates numbers to Int. The definitions
-- of enumFrom and enumFromThen allow floats to be used in arithmetic
-- series: [0,0.1 .. 0.95]. However, roundoff errors make these somewhat
-- dubious. This example may have either 10 or }11\mathrm{ elements, depending on
-- how 0.1 is represented.
instance Enum Float where
    succ x = x+1
    pred x = x-1
    toEnum = fromIntegral
    fromEnum = fromInteger . truncate -- may overflow
    enumFrom = numericEnumFrom
    enumFromThen = numericEnumFromThen
    enumFromTo = numericEnumFromTo
    enumFromThenTo = numericEnumFromThenTo
instance Enum Double where
    succ x = x+1
    pred x = x-1
    toEnum = fromIntegral
    fromEnum = fromInteger . truncate -- may overflow
    enumFrom = numericEnumFrom
    enumFromThen = numericEnumFromThen
    enumFromTo = numericEnumFromTo
    enumFromThenTo = numericEnumFromThenTo
numericEnumFrom :: (Fractional a) => a -> [a]
numericEnumFromThen :: (Fractional a) => a -> a -> [a]
numericEnumFromTo :: (Fractional a, Ord a) => a -> a -> [a]
numericEnumFromThenTo :: (Fractional a, Ord a) => a -> a -> a -> [a]
numericEnumFrom = iterate (+1)
numericEnumFromThen n m = iterate (+(m-n)) n
numericEnumFromTo n m = takeWhile (<= m+1/2) (numericEnumFrom n)
numericEnumFromThenTo n n' m = takeWhile p (numericEnumFromThen n n'
                                    where
                                    p { 的' >= n = (<= m + (n' - n)/2) 
-- Lists
data [a] = [] a : [a] deriving (Eq, Ord)
            -- Not legal Haskell; for illustration only
instance Functor [] where
    fmap = map
instance Monad [] where
    m >>= k = concat (map k m)
    return x = [x]
    fail s = []
```

-- Tuples

```
data (a,b) = (a,b) deriving (Eq, Ord, Bounded)
data (a,b,c) = (a,b,c) deriving (Eq, Ord, Bounded)
    -- Not legal Haskell; for illustration only
-- component projections for pairs:
-- (NB: not provided for triples, quadruples, etc.)
fst :: (a,b) -> a
fst (x,y) = x
snd :: (a,b) -> b
snd (x,y) = y
-- curry converts an uncurried function to a curried function;
-- uncurry converts a curried function to a function on pairs.
curry :: ((a, b) -> c) -> a -> b -> c
curry f x y = f (x, y)
uncurry :: (a -> b -> c) -> ((a, b) -> c)
uncurry f p = f (fst p) (snd p)
-- Misc functions
-- until p f yields the result of applying f until p holds.
until :: (a -> Bool) -> (a -> a) -> a -> a
until p f x
    px = x
    otherwise = until p f (f x)
-- asTypeOf is a type-restricted version of const. It is usually used
-- as an infix operator, and its typing forces its first argument
-- (which is usually overloaded) to have the same type as the second.
asTypeOf :: a -> a -> a
asTypeOf = const
-- error stops execution and displays an error message
error :: String -> a
error = primError
-- It is expected that compilers will recognize this and insert error
-- messages that are more appropriate to the context in which undefined
-- appears.
undefined :: a
undefined = error "Prelude.undefined"
```


### 8.1 Prelude PreludeList

```
-- Standard list functions
module PreludeList (
    map, (++), filter, concat, concatMap,
    head, last, tail, init, null, length, (!!),
    foldl, foldl1, scanl, scanl1, foldr, foldr1, scanr, scanr1,
    iterate, repeat, replicate, cycle,
    take, drop, splitAt, takeWhile, dropWhile, span, break,
    lines, words, unlines, unwords, reverse, and, or,
    any, all, elem, notElem, lookup,
    sum, product, maximum, minimum,
    zip, zip3, zipWith, zipWith3, unzip, unzip3)
    where
import qualified Char(isSpace)
infixl 9 !!
infixr 5 ++
infix 4 `elem`, `notElem`
-- Map and append
map :: (a -> b) -> [a] -> [b]
map f [] = []
map f (x:xs) = f x : map f xs
(++) :: [a] -> [a] -> [a]
[] ++ ys = ys
(x:xs) ++ ys = x : (xs ++ ys)
filter :: (a -> Bool) -> [a] -> [a]
filter p [] = []
filter p (x:xs) | p x = x : filter p xs
otherwise = filter p xs
concat :: [[a]] -> [a]
concat xss = foldr (++) [] xss
concatMap :: (a -> [b]) -> [a] -> [b]
concatMap f = concat . map f
-- head and tail extract the first element and remaining elements,
-- respectively, of a list, which must be non-empty. last and init
-- are the dual functions working from the end of a finite list,
-- rather than the beginning.
head :: [a] -> a
head (x:_) = x
head [] = error "Prelude.head: empty list"
tail :: [a] -> [a]
tail (_:xs) = xs
tail [] = error "Prelude.tail: empty list"
```

```
last :: [a] -> a
last [x] = x
last (_:xs) = last xs
last [] = error "Prelude.last: empty list"
init :: [a] -> [a]
init [x] = []
init (x:xs) = x : init xs
init [] = error "Prelude.init: empty list"
null :: [a] -> Bool
null [] = True
null (_:_) = False
-- length returns the length of a finite list as an Int.
```

```
length :: [a] -> Int
```

length :: [a] -> Int
length [] = 0
length [] = 0
length (_:l) = 1 + length l
length (_:l) = 1 + length l
-- List index (subscript) operator, O-origin
-- List index (subscript) operator, O-origin
(!!) :: [a] -> Int -> a
(!!) :: [a] -> Int -> a
xs !! n | n < 0 = error "Prelude.!!: negative index"
xs !! n | n < 0 = error "Prelude.!!: negative index"
[] !! _ = error "Prelude.!!: index too large"
[] !! _ = error "Prelude.!!: index too large"
(x:_) !! 0 = x
(x:_) !! 0 = x
(_:xs) !! n = xs !! (n-1)
(_:xs) !! n = xs !! (n-1)
-- foldl, applied to a binary operator, a starting value (typically the
-- left-identity of the operator), and a list, reduces the list using
-- the binary operator, from left to right:
-- foldl f z [x1, x2, ..., xn] == (...((z `f` x1) `f` x2) `f`...) `f` xn
-- foldl1 is a variant that has no starting value argument, and thus must
-- be applied to non-empty lists. scanl is similar to foldl, but returns
-- a list of successive reduced values from the left:
-- scanl f z [x1, x2, ...] == [z, z `f` x1, (z `f` x1) `f` x2, ...]
-- Note that last (scanl f z xs) == foldl f z xs.
-- scanll is similar, again without the starting element:
-- scanl1 f [x1, x2, ...] == [x1, x1 `f` x2, ...]
foldl :: (a -> b -> a) -> a -> [b] -> a
foldl f z [] = z
foldl f z (x:xs) = foldl f (f z x) xs
foldl1 :: (a -> a -> a) -> [a] -> a
foldl1 f (x:xs) = foldl f x xs
foldl1 _ [] = error "Prelude.foldl1: empty list"
scanl :: (a -> b -> a) -> a -> [b] -> [a]
scanl f q xs = q : (case xs of
[] -> []
x:xs -> scanl f (f q x) xs)
scanl1 :: (a -> a -> a) -> [a] -> [a]
scanl1 f (x:xs) = scanl f x xs
scanl1 - [] = []

```
```

-- foldr, foldr1, scanr, and scanrl are the right-to-left duals of the
-- above functions.
foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f z [] = z
foldr f z (x:xs) = f x (foldr f z xs)
foldr1 :: (a -> a -> a) -> [a] -> a
foldr1 f [x] = x
foldr1 f (x:xs) = f x (foldr1 f xs)
foldr1 _ [] = error "Prelude.foldr1: empty list"
scanr :: (a -> b -> b) -> b -> [a] -> [b]
scanr f q0 [] = [q0]
scanr f q0 (x:xs) = f x q : qs
where qs@(q:_) = scanr f q0 xs
scanr1 :: (a -> a -> a) -> [a] -> [a]
scanr1 f [] = []
scanr1 f [x] = [x]
scanr1 f (x:xs) = f x q : qs
where qs@(q:_) = scanr1 f xs
-- iterate f x returns an infinite list of repeated applications of f to x:
-- iterate f x == [x, f x, f (f x), ...]
iterate :: (a -> a) -> a -> [a]
iterate f x = x : iterate f (f x)
-- repeat x is an infinite list, with x the value of every element.
repeat :: a -> [a]
repeat x = xs where xs = x:xs
-- replicate n x is a list of length n with x the value of every element
replicate :: Int -> a -> [a]
replicate n x = take n (repeat x)
-- cycle ties a finite list into a circular one, or equivalently,
-- the infinite repetition of the original list. It is the identity
-- on infinite lists.
cycle :: [a] -> [a]
cycle [] = error "Prelude.cycle: empty list"
cycle xs = xs' where xs' = xs ++ xs'
-- take n, applied to a list xs, returns the prefix of xs of length n,
-- or xs itself if n > length xs. drop n xs returns the suffix of xs
-- after the first n elements, or [] if n > length xs. splitAt n xs
-- is equivalent to (take n xs, drop n xs).
take :: Int -> [a] -> [a]
take n _ | n <= 0 = []
take _ [] = []
take n (x:xs) = x : take (n-1) xs

```
```

drop :: Int -> [a] -> [a]
drop n xs | n <= 0 = xs
drop _ [] = []
drop n (_:xs) = drop (n-1) xs
splitAt :: Int -> [a] -> ([a],[a])
splitAt n xs = (take n xs, drop n xs)
-- takeWhile, applied to a predicate p and a list xs, returns the longest
-- prefix (possibly empty) of xs of elements that satisfy p. dropWhile p xs
-- returns the remaining suffix. span p xs is equivalent to
-- (takeWhile p xs, dropWhile p xs), while break p uses the negation of p.
takeWhile :: (a -> Bool) -> [a] -> [a]
takeWhile p [] = []
takeWhile p (x:xs)
p x llo x : takeWhile p xs
dropWhile :: (a -> Bool) -> [a] -> [a]
dropWhile p [] = []
dropWhile p xs@(x:xs')
p x = dropWhile p xs'
otherwise = xs
span, break :: (a -> Bool) -> [a] -> ([a],[a])
span p [] = ([],[])
span p xs@(x:xs')
px = (x:ys,zs)
| otherwise = ([],xs)
where (ys,zs) = span p xs'
break p = span (not . p)
-- lines breaks a string up into a list of strings at newline characters.
-- The resulting strings do not contain newlines. Similary, words
-- breaks a string up into a list of words, which were delimited by
-- white space. unlines and unwords are the inverse operations.
-- unlines joins lines with terminating newlines, and unwords joins
-- words with separating spaces.
lines "" (: String -> [String]
words :: String -> [String]
words s = case dropWhile Char.isSpace s of
"" -> []
s' -> w : words s'r
where (w, s'r) = break Char.isSpace s'

```
```

unlines :: [String] -> String
unlines = concatMap (++ "\n")
unwords :: [String] -> String
unwords [] = ""
unwords ws = foldr1 (\w s -> w ++ ' ':s) ws
-- reverse xs returns the elements of xs in reverse order. xs must be finite.
reverse :: [a] -> [a]
reverse = foldl (flip (:)) []
-- and returns the conjunction of a Boolean list. For the result to be
-- True, the list must be finite; False, however, results from a False
-- value at a finite index of a finite or infinite list. or is the
-- disjunctive dual of and.
and, or :: [Bool] -> Bool
and = foldr (\&\&) True
or = foldr (||) False
-- Applied to a predicate and a list, any determines if any element
-- of the list satisfies the predicate. Similarly, for all.
any, all :: (a -> Bool) -> [a] -> Bool
any p = or . map p
all p = and . map p
-- elem is the list membership predicate, usually written in infix form,
-- e.g., x `elem` xs. notElem is the negation.
elem, notElem :: (Eq a) => a -> [a] -> Bool
elem x = any (== x)
notElem x = all (/= x)
-- lookup key assocs looks up a key in an association list.
lookup :: (Eq a) => a -> [(a,b)] -> Maybe b
lookup key [] = Nothing
lookup key ((x,y):xys)
key == x = Just y
otherwise = lookup key xys
-- sum and product compute the sum or product of a finite list of numbers.
sum, product :: (Num a) => [a] -> a
sum = foldl (+) 0
product = foldl (*) 1
-- maximum and minimum return the maximum or minimum value from a list,
-- which must be non-empty, finite, and of an ordered type.
maximum, minimum :: (Ord a) => [a] -> a
maximum [] = error "Prelude.maximum: empty list"
maximum xs = foldl1 max xs
minimum [] = error "Prelude.minimum: empty list"
minimum xs = foldl1 min xs

```
-- zip takes two lists and returns a list of corresponding pairs. If one -- input list is short, excess elements of the longer list are discarded. -- zip3 takes three lists and returns a list of triples. Zips for larger -- tuples are in the List library
```

zip :: [a] -> [b] -> [(a,b)]
zip = zipWith (,)
zip3 :: [a] -> [b] -> [c] -> [(a,b,c)]
zip3 = zipWith3 (,,)

```
-- The zipWith family generalises the zip family by zipping with the
-- function given as the first argument, instead of a tupling function.
-- For example, zipWith (+) is applied to two lists to produce the list
-- of corresponding sums.
```

zipWith :: (a->b->c) -> [a]->[b]->[c]
zipWith z (a:as) (b:bs)
= z a b : zipWith z as bs
zipWith _ _ _ = []
zipWith3 :: (a->b->c->d) -> [a]->[b]->[c]->[d]
zipWith3 z (a:as) (b:bs) (c:cs)
= z a b c : zipWith3 z as bs cs
zipWith3 _ _ _ _ = []
-- unzip transforms a list of pairs into a pair of lists.
unzip :: [(a,b)] -> ([a],[b])
unzip = foldr (\(a,b) ~ (as,bs) -> (a:as,b:bs)) ([],[])
unzip3 :: [(a,b,c)] -> ([a],[b],[c])
unzip3 = foldr (\(a,b,c) ~ (as,bs,cs) -> (a:as,b:bs,c:cs))
([],[],[])

```

\subsection*{8.2 Prelude PreludeText}
```

module PreludeText (
ReadS, ShowS,
Read(readsPrec, readList),
Show(showsPrec, show, showList),
reads, shows, read, lex,
showChar, showString, readParen, showParen ) where
-- The instances of Read and Show for
-- Bool, Maybe, Either, Ordering
-- are done via "deriving" clauses in Prelude.hs
import Char(isSpace, isAlpha, isDigit, isAlphaNum,
showLitChar, readLitChar, lexLitChar)
import Numeric(showSigned, showInt, readSigned, readDec, showFloat,
readFloat, lexDigits)
type ReadS a = String -> [(a,String)]
type ShowS = String -> String
class Read a where
readsPrec :: Int -> ReadS a
readList :: ReadS [a]
-- Minimal complete definition:
-- readsPrec
readList = readParen False (\r -> [pr | ("[",s) <- lex r,
pr <- readl s])
where readl s = [([],t) ("]",t) <- lex s] ++
[(x:xs,u) | (x,t) <- reads s,
(xs,u) <- readl' t]
readl' s = [([],t) | ("]",t) <- lex s] ++
[(x:xs,v) | (",",t) <- lex s,
(x,u) <- reads t,
(xs,v) <- readl' u]
class Show a where
showsPrec :: Int -> a -> ShowS
show :: a -> String
showList :: [a] -> ShowS
-- Mimimal complete definition:
-- show or showsPrec
showsPrec _ x s = show x ++ s
show x = showsPrec 0 x ""
showList [] = showString "[]"
showList (x:xs) = showChar '[' . shows x . showl xs
where showl [] = showChar ']'
showl (x:xs) = showChar ',' . shows x .
showl xs

```
```

reads :: (Read a) => ReadS a
reads = readsPrec 0
shows :: (Show a) => a -> ShowS
shows = showsPrec 0
read :: (Read a) => String -> a
read s = case [x | (x,t) <- reads s, ("","") <- lex t] of
[x] -> x
[] -> error "Prelude.read: no parse"
_ -> error "Prelude.read: ambiguous parse"
showChar :: Char -> ShowS
showChar = (:)
showString :: String -> ShowS
showString = (++)
showParen :: Bool -> ShowS -> ShowS
showParen b p = if b then showChar '(' . p . showChar ')' else p
readParen :: Bool -> ReadS a -> ReadS a
readParen b g = if b then mandatory else optional
where optional r = g r ++ mandatory r
mandatory r = [(x,u) | ("(",s) <- lex r,
(x,t) <- optional s,
(")",u) <- lex t ]
-- This lexer is not completely faithful to the Haskell lexical syntax.
-- Current limitations:
-- Qualified names are not handled properly
-- Octal and hexidecimal numerics are not recognized as a single token
-- Comments are not treated properly
lex :: ReadS String
lex "" = [("","")]
lex (c:s)
| isSpace c = lex (dropWhile isSpace s)
lex ('\'':s) = [('\''':ch++"'", t) | (ch,'\'':t) <- lexLitChar s,
ch /= "'" ]
lex ('"':s) = [('"':str, t) | (str,t)<- lexString s]
where
lexString ('"':s) = [("\"",s)]
lexString s = [(ch++str, u)
| (ch,t) <- lexStrItem s,
(str,u) <- lexString t ]
lexStrItem ('<br>':'\&':s) = [("<br>\&",s)]
lexStrItem ('<br>':c:s) | isSpace c
= [("<br>\&",t) |
\prime<br>':t <-
[dropWhile isSpace s]]
lexStrItem s = lexLitChar s

```
```

lex (c:s) | isSingle c = [([c],s)]
isSym c = [(c:sym,t) | (sym,t) <- [span isSym s]]
isAlpha c = [(c:nam,t) | (nam,t) <- [span isIdChar s]]
isDigit c = [(c:ds++fe,t) | (ds,s) <- [span isDigit s],
(fe,t) <- lexFracExp s ]
| otherwise = [] -- bad character
where
isSingle c = c `elem` ",;()[]{}_`"                 isSym c = c `elem`"!@#$%&*+./<=>?\\^|:-~"                 isIdChar c = isAlphaNum c || c`elem`"_'"                 lexFracExp ('.':c:cs) | isDigit c                         = [('.':ds++e,u) | (ds,t) <- lexDigits (c:cs),                                     (e,u) <- lexExp t]             lexFracExp s = lexExp s             lexExp (e:s) | e`elem`"eE"                 = [(e:c:ds,u) | (c:t) <- [s], c`elem` "+-",
(ds,u) <- lexDigits t] ++
[(e:ds,t) | (ds,t) <- lexDigits s]
lexExp s = [("",s)]
instance Show Int where
showsPrec n = showsPrec n . toInteger
-- Converting to Integer avoids
-- possible difficulty with minInt
instance Read Int where
readsPrec p r = [(fromInteger i, t) | (i,t) <- readsPrec p r]
-- Reading at the Integer type avoids
-- possible difficulty with minInt
instance Show Integer where
showsPrec = showSigned showInt
instance Read Integer where
readsPrec p = readSigned readDec
instance Show Float where
showsPrec p = showFloat
instance Read Float where
readsPrec p = readSigned readFloat
instance Show Double where
showsPrec p = showFloat
instance Read Double where
readsPrec p = readSigned readFloat
instance Show () where
showsPrec p () = showString "()"

```
```

instance Read () where
readsPrec p = readParen False
(\r -> [((),t) | ("(",s) <- lex r,
(")",t) <- lex s ] )
instance Show Char where
showsPrec p '\'' = showString "'<br>''"
showsPrec p c = showChar '\'' . showLitChar c . showChar '\''
showList cs = showChar '"' . showl cs
where showl "" = showChar '"'
showl ('"':cs) = showString "<br>\"" . showl cs
showl (c:cs) = showLitChar c . showl cs
instance Read Char where
readsPrec p = readParen False
(\r -> [(c,t) | ('\'':s,t)<- lex r,
(c,"\'") <- readLitChar s])
readList = readParen False (\r -> [(l,t) | ('"':s, t) <- lex r,
(l,_) <- readl s ])
where readl ('"':s) = [("",s)]
readl ('<br>':'\&':s) = readl s
readl s = [(c:cs,u) | (c ,t) <- readLitChar s,
(cs,u) <- readl t ]
instance (Show a) => Show [a] where
showsPrec p = showList
instance (Read a) => Read [a] where
readsPrec p = readList
-- Tuples
instance (Show a, Show b) => Show (a,b) where
showsPrec p (x,y) = showChar '(' . shows x . showChar ',' .
shows y . showChar ')'
instance (Read a, Read b) => Read (a,b) where
readsPrec p = readParen False
(\r -> [((x,y), w) | ("(",s) <- lex r,
(x,t) <- reads s,
(",",u) <- lex t,
(y,v) <- reads u,
(")",w) <- lex v ] )

```
-- Other tuples have similar Read and Show instances

\subsection*{8.3 Prelude PreludeIO}
```

module PreludeIO (
FilePath, IOError, ioError, userError, catch,
putChar, putStr, putStrLn, print,
getChar, getLine, getContents, interact,
readFile, writeFile, appendFile, readIO, readLn
) where
import PreludeBuiltin
type FilePath = String
data IOError -- The internals of this type are system dependent
instance Show IOError where ...
instance Eq IOError where...
ioError :: IOError -> IO a
ioError = primIOError
userError :: String -> IOError
userError = primUserError
catch :: IO a -> (IOError -> IO a) -> IO a
catch = primCatch
putChar :: Char -> IO ()
putChar = primPutChar
putStr :: String -> IO ()
putStr s = mapM_ putChar s
putStrLn :: String -> IO ()
putStrLn s = do putStr s
putStr "\n"
print :: Show a => a -> IO ()
print x = putStrLn (show x)
getChar :: IO Char
getChar = primGetChar
getLine :: IO String
getLine = do c <- getChar
if c == '\n' then return "" else
do s <- getLine
return (c:s)
getContents :: IO String
getContents = primGetContents

```
```

interact :: (String -> String) -> IO ()
-- The hSetBuffering ensures the expected interactive behaviour
interact f = do hSetBuffering stdin NoBuffering
hSetBuffering stdout NoBuffering
s <- getContents
putStr (f s)
readFile :: FilePath -> IO String
readFile = primReadFile
writeFile :: FilePath -> String -> IO ()
writeFile = primWriteFile
appendFile :: FilePath -> String -> IO ()
appendFile = primAppendFile
-- raises an exception instead of an error
readIO :: Read a => String -> IO a
readIO s = case [x | (x,t) <- reads s, ("","") <- lex t] of
[x] -> return x
[] -> ioError (userError "Prelude.readIO: no parse")
_ -> ioError (userError "Prelude.readIO: ambiguous parse")
readLn :: Read a => IO a
readLn = do l <- getLine
r <- readIO l
return r

```

\section*{Chapter 9}

\section*{Syntax Reference}

\subsection*{9.1 Notational Conventions}

These notational conventions are used for presenting syntax:
\[
\begin{array}{cl}
{[\text { pattern }]} & \text { optional } \\
\{\text { pattern }\} & \text { zero or more repetitions } \\
(\text { pattern }) & \text { grouping } \\
\text { pat }_{1} \mid \text { pat }_{2} & \text { choice } \\
\text { pat }_{\left\langle\text {pat }{ }^{\prime}\right\rangle} & \text { difference-elements generated by pat } \\
& \text { except those generated by } \text { pat }^{\prime} \\
\text { fibonacci } & \text { terminal syntax in typewriter font }
\end{array}
\]

BNF-like syntax is used throughout, with productions having the form:
nonterm \(\quad \rightarrow \quad\) alt \(_{1} \mid\) alt \(_{2}|\ldots|\) alt \(_{n}\)
There are some families of nonterminals indexed by precedence levels (written as a superscript). Similarly, the nonterminals op, varop, and conop may have a double index: a letter \(l\), \(r\), or \(n\) for left-, right- or nonassociativity and a precedence level. A precedence-level variable \(i\) ranges from 0 to 9 ; an associativity variable \(a\) varies over \(\{l, r, n\}\). Thus, for example
\(\operatorname{aexp} \quad \rightarrow \quad\left(\exp ^{i+1} q o p^{(a, i)}\right)\)
actually stands for 30 productions, with 10 substitutions for \(i\) and 3 for \(a\).
In both the lexical and the context-free syntax, there are some ambiguities that are to be resolved by making grammatical phrases as long as possible, proceeding from left to right (in shift-reduce parsing, resolving shift/reduce conflicts by shifting). In the lexical syntax, this is the "maximal munch" rule. In the context-free syntax, this means that conditionals, let-expressions, and lambda abstractions extend to the right as far as possible.

\subsection*{9.2 Lexical Syntax}
```

program }->\mathrm{ {lexeme | whitespace }
lexeme }->\mathrm{ qvarid |qconid |qvarsym |qconsym
| literal | special| reservedop| reservedid
literal }->\mathrm{ integer |float | char | string
special }->\mathrm{ (|)|,|;|[|]|`|{|}
whitespace }->\mathrm{ whitestuff {whitestuff }
whitestuff }->\mathrm{ whitechar | comment | ncomment
whitechar }->\mathrm{ newline |vertab | space |tab|uniWhite
newline }->\mathrm{ return linefeed | return | linefeed |formfeed
return }->\mathrm{ a carriage return
linefeed }->\mathrm{ a line feed
vertab }->\mathrm{ a vertical tab
formfeed }->\mathrm{ a form feed
space }\quad->\quad\mathrm{ a space
tab }->\mathrm{ a horizontal tab
uniWhite }->\mathrm{ any Unicode character defined as whitespace

```

```

dashes }->\quad--{-
opencom }->\quad{
closecom }->\mathrm{ -}
ncomment }->\mathrm{ opencom ANYseq {ncomment ANYseq} closecom

```

```

ANY }->\mathrm{ graphic|whitechar
any }->\mathrm{ graphic | space |tab
graphic }->\mathrm{ small |large | symbol | digit | special |:| "|'
small }\quad->\quad\mathrm{ ascSmall |uniSmall |_
ascSmall }->\mathrm{ a|b|...|z
uniSmall }->\mathrm{ any Unicode lowercase letter
large }->\mathrm{ ascLarge |niLarge
ascLarge }->\mathrm{ A | B |...| Z
uniLarge }->\mathrm{ any uppercase or titlecase Unicode letter
symbol }\quad->\quad\mathrm{ ascSymbol | uniSymbol}\mp@subsup{l}{\mathrm{ special |_ | | | |'>}}{\mathrm{ ' }
ascSymbol -> ! |\#|\$|%|\&|*|+|.|/|<|=|>|?|@
| \|^|||-|
uniSymbol }->\mathrm{ any Unicode symbol or punctuation
digit }->\mathrm{ ascDigit |uniDigit

```
```

ascDigit }->\mathrm{ 0|1|_..|9
uniDigit }->\mathrm{ any Unicode decimal digit
octit }->00|1|···|
hexit }\quad->\quad\operatorname{digit | A | .. | F |a|...|f

```
```

varid $\quad \rightarrow \quad\left(\text { small }\left\{\text { small } \mid \text { large } \mid \text { digit }\left.\right|^{\prime}\right\}\right)_{\langle\text {reservedid }\rangle}$
conid $\rightarrow$ large $\left\{\right.$ small $\mid$ large $\mid$ digit $\left.\left.\right|^{\prime}\right\}$
reservedid $\rightarrow$ case|class|data|default|deriving|do|else
| if|import|in|infix|infixl|infixr|instance
| let|module|newtype|of|then|type|where|_
varsym $\quad \rightarrow \quad(\text { symbol }\{\text { symbol } \mid:\})_{\langle\text {reservedop }} \mid$ dashes $\rangle$
consym $\rightarrow(:\{\text { symbol } \mid:\})_{\langle\text {reservedop }\rangle}$
reservedop $\rightarrow \ldots|:|::|=|\backslash|||<-|->|@| \sim|=>$

```
\(\begin{array}{ll}\text { varid } \\ \text { conid } & \text { (variables) } \\ \text { (constructo }\end{array}\)
tyvar \(\quad \rightarrow \quad\) varid
tycon \(\quad \rightarrow\) conid
tycls \(\quad \rightarrow\) conid
modid \(\rightarrow\) conid
(modules)
```

qvarid }->\quad[modid.] varid
qconid }->\mathrm{ [modid .] conid
qtycon }->\quad[modid .] tyco
qtycls }->\quad[\mathrm{ modid .] tycls
qvarsym }->\quad[modid.] varsy
qconsym }->\mathrm{ [modid .] consym
decimal }->\mathrm{ digit{digit}
octal }->\mathrm{ octit{octit}
hexadecimal-> hexit{hexit}
integer }->\mathrm{ decimal
| 00 octal | 00 octal
| 0x hexadecimal | 0x hexadecimal
float }->\mathrm{ decimal . decimal [exponent]
| decimal exponent
exponent }->\mathrm{ (e|E)[+|-] decimal

```

```

string }->\quad"{graphic<" |<br>rangle | space| escape |gap} "
escape }->\mathrm{ \ (charesc |ascii | decimal | ○ octal | x hexadecimal)

```
```

charesc $\rightarrow \mathrm{a}|\mathrm{b}| \mathrm{f}|\mathrm{n}| \mathrm{r}|\mathrm{t}| \mathrm{v}|\backslash| \mathrm{l\mid}|\boldsymbol{\prime}| \&$
ascii $\quad \rightarrow \quad$ ^cntrl $\mid$ NUL $\mid$ SOH $\mid$ STX $|E T X| E O T|E N Q| A C K$
| BEL|BS|HT|LF|VT|FF|CR|SO|SI|DLE
| DC1 | DC2|DC3|DC4|NAK|SYN|ETB|CAN
| EM|SUB|ESC|FS|GS|RS|US|SP|DEL
cntrl $\rightarrow$ ascLarge $|@|[|\backslash|]|\wedge|-$
gap $\quad \rightarrow \quad \backslash$ whitechar $\{$ whitechar $\} \backslash$

```

\subsection*{9.3 Layout}

Section 2.7 gives an informal discussion of the layout rule. This section defines it more precisely.
The meaning of a Haskell program may depend on its layout. The effect of layout on its meaning can be completely described by adding braces and semicolons in places determined by the layout. The meaning of this augmented program is now layout insensitive.

The effect of layout is specified in this section by describing how to add braces and semicolons to a laid-out program. The specification takes the form of a function \(L\) that performs the translation. The input to \(L\) is:
- A stream of lexemes as specified by the lexical syntax in the Haskell report, with the following additional tokens:
- If a let, where, do, or of keyword is not followed by the lexeme \(\{\), the token \(\{n\}\) is inserted after the keyword, where \(n\) is the indentation of the next lexeme if there is one, or 0 if the end of file has been reached.
- If the first lexeme of a module is not \(\{\) or module, then it is preceded by \(\{n\}\) where \(n\) is the indentation of the lexeme.
- Where the start of a lexeme is preceded only by white space on the same line, this lexeme is preceded by \(\langle n\rangle\) where \(n\) is the indentation of the lexeme, provided that it is not, as a consequence of the first two rules, preceded by \(\{n\}\). (NB: a string literal may span multiple lines - Section 2.6. So in the fragment
\[
\begin{aligned}
& f=(\text { (Hello \} } \\
{\quad \backslash \text { Bill", "Jake") }}
\end{array}
\end{aligned}
\]

There is no \(\langle n\rangle\) inserted before the \(\backslash \mathrm{Bill}\), because it is not the beginning of a complete lexeme; nor before the , , because it is not preceded only by white space.)
- A stack of "layout contexts", in which each element is either:
- Zero, indicating that the enclosing context is explicit (i.e. the programmer supplied the opening brace. If the innermost context is 0 , then no layout tokens will be inserted until either the enclosing context ends or a new context is pushed.
- A positive integer, which is the indentation column of the enclosing layout context.

The "indentation" of a lexeme is the column number of the first character of that lexeme; the indentation of a line is the indentation of its leftmost lexeme. To determine the column number, assume a fixed-width font with the following conventions:
- The characters newline, return, linefeed, and formfeed, all start a new line.
- The first column is designated column 1 , not 0 .
- Tab stops are 8 characters apart.
- A tab character causes the insertion of enough spaces to align the current position with the next tab stop.

For the purposes of the layout rule, Unicode characters in a source program are considered to be of the same, fixed, width as an ASCII character. However, to avoid visual confusion, programmers should avoid writing programs in which the meaning of implicit layout depends on the width of non-space characters.

The application

\section*{Ltokens []}
delivers a layout-insensitive translation of tokens, where tokens is the result of lexically analysing a module and adding column-number indicators to it as described above. The definition of \(L\) is as follows, where we use ":" as a stream construction operator, and "[]" for the empty stream.
\[
\left.\begin{array}{lll}
L(<n>: t s)(m: m s) & =;:(L t s(m: m s)) & \\
& =\left\{\begin{array}{l}
\text { if } m=n \\
L(<n>: t s) m s
\end{array}\right. & =L \text { ts } m s \\
& & \\
\text { if } n<m
\end{array}\right)
\]

Note 1. A nested context must be further indented than the enclosing context ( \(n>m\) ). If not, \(L\) fails, and the compiler should indicate a layout error. An example is:
\[
\begin{aligned}
f \mathrm{x}= & \operatorname{let} \\
\mathrm{pz} \mathrm{z}= & \mathrm{z}=\text { let } \\
& \text { in } \mathrm{h}
\end{aligned}
\]

Here, the definition of \(p\) is indented less than the indentation of the enclosing context, which is set in this case by the definition of \(h\).

Note 2. If the first token after a where (say) is not indented more than the enclosing layout context, then the block must be empty, so empty braces are inserted. The \(\{n\}\) token is replaced by \(\langle n\rangle\), to mimic the situation if the empty braces had been explicit.

Note 3. By matching against 0 for the current layout context, we ensure that an explicit close brace can only match an explicit open brace. A parse error results if an explicit close brace matches an implicit open brace.

Note 4. This clause means that all brace pairs are treated as explicit layout contexts, including labelled construction and update (Section 3.15). This is a difference between this formulation and Haskell 1.4.

Note 5. The side condition parse-error \((t)\) is to be interpreted as follows: if the tokens generated so far by \(L\) together with the next token \(t\) represent an invalid prefix of the Haskell grammar, and the tokens generated so far by \(L\) followed by the token " \(\}\) " represent a valid prefix of the Haskell grammar, then parse-error \((t)\) is true.
The test \(m /=0\) checks that an implicitly-added closing brace would match an implicit open brace.

Note 6. At the end of the input, any pending close-braces are inserted. It is an error at this point to be within a non-layout context (i.e. \(m=0\) ).

If none of the rules given above matches, then the algorithm fails. It can fail for instance when the end of the input is reached, and a non-layout context is active, since the close brace is missing. Some error conditions are not detected by the algorithm, although they could be: for example let \(\}\).

Note 1 implements the feature that layout processing can be stopped prematurely by a parse error. For example
\[
\text { let } x=e ; y=x \text { in } e^{\prime}
\]
is valid, because it translates to
\[
\text { let }\{x=e ; y=x\} \text { in } e^{\prime}
\]

The close brace is inserted due to the parse error rule above. The parse-error rule is hard to implement in its full generality, because doing so involves fixities. For example, the expression
do \(\mathrm{a}==\mathrm{b}=\mathrm{c}\)
has a single unambiguous (albeit probably type-incorrect) parse, namely
(do \(\{\mathrm{a}=\mathrm{b}\) b \(\}=\mathrm{c}\)
because \((==)\) is non-associative. Programmers are therefore advised to avoid writing code that requires the parser to insert a closing brace in such situations.

\subsection*{9.4 Literate comments}

The "literate comment" convention, first developed by Richard Bird and Philip Wadler for Orwell, and inspired in turn by Donald Knuth's "literate programming", is an alternative style for encoding Haskell source code. The literate style encourages comments by making them the default. A line in which " \(>\) " is the first character is treated as part of the program; all other lines are comment.

The program text is recovered by taking only those lines beginning with " \(>\) ", and replacing the leading " \(>\) " with a space. Layout and comments apply exactly as described in Chapter 9 in the resulting text.

To capture some cases where one omits an " \(>\) " by mistake, it is an error for a program line to appear adjacent to a non-blank comment line, where a line is taken as blank if it consists only of whitespace.

By convention, the style of comment is indicated by the file extension, with ". hs" indicating a usual Haskell file and ". lhs" indicating a literate Haskell file. Using this style, a simple factorial program would be:
```

    This literate program prompts the user for a number
    and prints the factorial of that number:
    > main :: IO ()
> main = do putStr "Enter a number: "
> l <- readLine
> putStr "n!= "
> print (fact (read l))
This is the factorial function.
> fact :: Integer -> Integer
f fact 0 = 1
> fact n = n * fact (n-1)

```

An alternative style of literate programming is particularly suitable for use with the LaTeX text processing system. In this convention, only those parts of the literate program that are entirely enclosed between \(\backslash\) begin \(\{\) code \(\} .. . \backslash\) end \(\{\) code \(\}\) delimiters are treated as program text; all other lines are comment. More precisely:
- Program code begins on the first line following a line that begins \(\backslash\) begin \(\{\) code \(\}\).
- Program code ends just before a subsequent line that begins \end \{code\} (ignoring string } literals, of course).

It is not necessary to insert additional blank lines before or after these delimiters, though it may be stylistically desirable. For example,
```

\documentstyle{article}
\begin{document }

## 1. Introduction

This is a trivial program that prints the first 20 factorials.
$$
\begin{code}
main :: IO ()
main = print [ (n, product [1..n]) | n <- [1..20]]
\end{code}
$$
\end{document}

```

This style uses the same file extension. It is not advisable to mix these two styles in the same file.

\subsection*{9.5 Context-Free Syntax}
\begin{tabular}{|c|c|c|}
\hline module & & ```
module modid [exports] where body
body
``` \\
\hline body & & ```
{ impdecls; topdecls }
{ impdecls }
{ topdecls }
``` \\
\hline impdecls & \(\rightarrow\) & impdecl \(_{1} ; \ldots\); impdecl \(_{n} \quad(n \geq 1)\) \\
\hline exports & \(\rightarrow\) & \(\left(\right.\) export \(\left._{1}, \ldots, \operatorname{export}_{n}[],\right) \quad(n \geq 0)\) \\
\hline export & \[
\overrightarrow{\mid}
\] & \begin{tabular}{l}
quar \\
qtycon \(\left[(\ldots) \mid\left(\right.\right.\) cname \(_{1}, \ldots\), cname \(\left.\left._{n}\right)\right] \quad(n \geq 0)\) \\
qtycls \(\left[(\ldots) \mid\left(q u a r_{1}, \ldots, q v a r_{n}\right)\right] \quad(n \geq 0)\) \\
module modid
\end{tabular} \\
\hline impdecl & & import [qualified] modid [as modid] [impspec] (empty declaration) \\
\hline impspec & & \begin{tabular}{ll} 
( import,\(\ldots\), import \(\left._{n}[],\right)\) & \((n \geq 0)\) \\
hiding \(\left(\right.\) import \(_{1}, \ldots\), import \(\left._{n}[],\right)\) & \((n \geq 0)\)
\end{tabular} \\
\hline import & \[
\overrightarrow{\mid}
\] & \begin{tabular}{l}
var \\
tycon \(\left[(.) \mid.\left(\right.\right.\) cname \(_{1}, \ldots\), cname \(\left.\left._{n}\right)\right] \quad(n \geq 0)\) \\
tycls \(\left[(\ldots) \mid\left(\operatorname{var}_{1}, \ldots, \operatorname{var}_{n}\right)\right] \quad(n \geq 0)\)
\end{tabular} \\
\hline cname & \(\rightarrow\) & \(v a r \mid \operatorname{con}\) \\
\hline topdecls topdecl & \[
\begin{aligned}
& \vec{~} \\
& \overrightarrow{\mid} \\
& \mid \\
& \mid \\
& \mid \\
& \mid
\end{aligned}
\] & ```
topdecl ; ; ...; topdecln
(n\geq0)
type simpletype = type
data [context =>] simpletype = constrs [deriving]
newtype [context =>] simpletype = newconstr [deriving]
class [scontext =>] tycls tyvar [where cdecls]
instance [scontext =>] qtycls inst [where idecls]
default (type }\mp@subsup{|}{1}{\prime}...,\mp@subsup{type}{n}{\prime}
( }n\geq0
decl
``` \\
\hline \begin{tabular}{l}
decls \\
decl
\end{tabular} & &  \\
\hline
\end{tabular}

```

constrs }->\mp@subsup{\mathrm{ constr }}{1}{}|···|\mp@subsup{\mathrm{ constr }}{n}{
constr }->\mathrm{ con[!] atype }\mp@subsup{A}{1}{···}\mathrm{ ..[!] atype }\mp@subsup{k}{}{\prime
| (btype | !atype) conop (btype | ! atype)
(infix conop)
| con {\mp@subsup{fielddecl l , ..., fielddecl n } ( }{n}{\prime}=0)
newconstr }->\mathrm{ con atype
| con {var :: type }
fielddecl }->\mathrm{ vars::(type|!atype)
deriving }->\mathrm{ deriving(dclass |(dclass }\mp@subsup{|}{1}{\prime}...,\mp@subsup{\mathrm{ dclass }}{n}{}))(n\geq0
dclass }->\mathrm{ qtycls

```

\begin{tabular}{|c|c|c|c|}
\hline funlhs & \[
\overrightarrow{\mid}
\] & \begin{tabular}{l}
var apat \(\{\) apat \(\}\) \\
pat \({ }^{i+1}\) varop \(^{(a, i)}\) pat \(^{i+1}\) \\
lpat \({ }^{i}\) varop \({ }^{(1, i)}\) pat \(^{i+1}\) \\
pat \({ }^{i+1}\) varop \(^{(r, i)}\) rpat \(^{i}\) \\
(funlhs) apat \{ apat \}
\end{tabular} & \\
\hline rhs & & \begin{tabular}{l}
\(=\exp\) [where decls] \\
gdrhs [where decls]
\end{tabular} & \\
\hline gdrhs & \(\rightarrow\) & \(g d=\exp [g d r h s]\) & \\
\hline \(g d\) & \(\rightarrow\) & \(\mid e x p{ }^{0}\) & \\
\hline \(\exp\) & & \[
\exp ^{0}::[\text { context }=>] \text { type }
\] & (expression type signature) \\
\hline \(e x p{ }^{i}\) & \[
\rightarrow
\] & \[
\begin{aligned}
& \exp ^{i+1}\left[q o p^{(\mathrm{n}, i)} \exp ^{i+1}\right] \\
& \operatorname{lexp}^{i} \\
& \exp ^{i}
\end{aligned}
\] & \\
\hline \[
\begin{aligned}
& l^{e x p}{ }^{i} \\
& \text { lexp }^{6}
\end{aligned}
\] & \(\xrightarrow{\rightarrow}\) & \[
\begin{aligned}
& \left(\operatorname{lexp}^{i} \mid \exp ^{i+1}\right) q o p^{(1, i)} \exp ^{i+1} \\
& -\exp ^{\gamma}
\end{aligned}
\] & \\
\hline rexp \({ }^{\text {i }}\) & \(\rightarrow\) & \(\exp ^{i+1} q_{o p}{ }^{(r, i)}\left(\operatorname{rexp}^{i} \mid \exp ^{i+1}\right)\) & \\
\hline exp \({ }^{10}\) & \(\rightarrow\) & \[
\begin{aligned}
& \text { \apat } t_{1} \ldots \text { apat } t_{n}->\text { exp } \\
& \text { let decls in exp } \\
& \text { if exp then exp else exp } \\
& \text { case exp of }\{\text { alts }\}
\end{aligned}
\] & \begin{tabular}{l}
(lambda abstraction, \(n \geq 1\) ) \\
(let expression) \\
(conditional) \\
(case expression)
\end{tabular} \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|}
\hline fexp & \[
\rightarrow
\] & \begin{tabular}{l}
do \(\{\) stmts \(\}\) \\
fexp \\
[fexp] aexp
\end{tabular} & (do expression)
(function application) \\
\hline \multirow[t]{13}{*}{aexp} & \(\rightarrow\) & quar & (variable) \\
\hline & & gcon & (general constructor) \\
\hline & & literal & \\
\hline & & ( exp ) & (parenthesized expression) \\
\hline & & \(\left(\exp _{1}, \ldots, e x p_{k}\right)\) & (tuple, \(k \geq 2\) ) \\
\hline & & \(\left[\exp _{1}, \ldots, \exp _{k}\right]\) & (list, \(k \geq 1\) ) \\
\hline & & [ \(\exp _{1}\left[, \exp _{2}\right] \ldots\left[\exp _{3}\right]\) ] & (arithmetic sequence) \\
\hline & & \[
\left[\text { exp } \mid \text { qual }_{1}, \ldots, \text { qual }_{n}\right]
\] & (list comprehension, \(n \geq 1\) ) \\
\hline & & \[
\left(e x p^{i+1} q o p^{(a, i)}\right)
\] & (left section)
(left section) \\
\hline & & \[
\left(\text { lexp }^{i} q_{q p^{(l, i)}}\right)
\] & (left section) \\
\hline & | &  & (right section) \\
\hline & | & \(\left(\right.\) qop \(_{\langle-\rangle}^{(r, i)} \mathrm{rexp}^{\text {i }}\) ) & (right section) \\
\hline & & qcon \(\left\{\right.\) fbind \(_{1}, \ldots\), fbind \(\left._{n}\right\}\) \(\operatorname{aexp}_{\{q \text { con }\rangle}\left\{\right.\) fbind \(_{1}, \ldots\), fbind \(\left._{n}\right\}\) & \begin{tabular}{l}
(labeled construction, \(n \geq 0\) ) \\
(labeled update, \(n \geq 1\) )
\end{tabular} \\
\hline \multirow[t]{3}{*}{qual} & \(\rightarrow\) & pat <- exp & (generator) \\
\hline & | & let decls & (local declaration) \\
\hline & | & exp & (guard) \\
\hline \multirow[t]{3}{*}{alts alt} & \(\rightarrow\) & alt \(_{1} ; \ldots\); \({ }^{\text {alt }}\) & \((n \geq 1)\) \\
\hline & \(\rightarrow\) & \begin{tabular}{l}
pat -> exp [where decls] \\
pat gdpat [where decls]
\end{tabular} & \\
\hline & | & & (empty alternative) \\
\hline gdpat & \(\rightarrow\) & gd \(\rightarrow\) exp [ gdpat ] & \\
\hline \multirow[t]{4}{*}{stmts
stmt} & \(\rightarrow\) & \(s t m t_{1} \ldots . . s t m t_{n} \exp [\%]\) & \((n \geq 0)\) \\
\hline & \(\rightarrow\) & exp; & \\
\hline & & pat <-exp ; & \\
\hline & & \begin{tabular}{l}
let decls; \\
;
\end{tabular} & (empty statement) \\
\hline fbind & \(\rightarrow\) & quar \(=\) exp & \\
\hline pat & \(\rightarrow\) & var + integer & (successor pattern) \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|c|}
\hline \(p a t^{i}\) & \(\rightarrow\) & \[
\begin{aligned}
& \text { pat }^{0} \\
& \text { pat }^{i+1}\left[\text { qconop }^{(\mathrm{n}, i)} \text { pat }^{i+1}\right] \\
& \text { lpat }^{i} \\
& \text { rpat }^{i}
\end{aligned}
\] & \\
\hline \(l p a t^{i}\) & \(\rightarrow\) & \(\left(l p a t^{i} \mid p a t^{i+1}\right)\) qconop \(^{(1, i)}\) pat \(^{i+1}\) & \\
\hline lpat \({ }^{6}\) & \(\rightarrow\) & - (integer \(\mid\) float \()\) & (negative literal) \\
\hline rpat \({ }^{\text {i }}\) & \(\rightarrow\) & pat \({ }^{i+1}\) qconop \(^{(\mathrm{r}, i)}\left(\right.\) rpat \(^{i} \mid\) pat \(\left.^{i+1}\right)\) & \\
\hline \(p a t^{10}\) & \[
\overrightarrow{\mid}
\] & \begin{tabular}{l}
apat \\
gcon apat \({ }_{1} \ldots\) apat \(_{k}\)
\end{tabular} & \((\) arity \(g c o n=k, k \geq 1)\) \\
\hline \multirow[t]{9}{*}{apat} & \multirow[t]{9}{*}{\(\rightarrow\)} & var [ @ apat] & (as pattern) \\
\hline & & gcon & (arity gcon \(=0\) ) \\
\hline & & qcon \(\left\{\right.\) fpat \(_{1}, \ldots\), fpat \(\left._{k}\right\}\) & (labeled pattern, \(k \geq 0\) ) \\
\hline & & literal & \\
\hline & & - & (wildcard) \\
\hline & & ( pat) & (parenthesized pattern) \\
\hline & & \(\left(p^{\prime} t_{1}, \ldots, p^{\prime}, \ldots\right)\) & (tuple pattern, \(k \geq\) 2) \\
\hline & & [ \(\left.\mathrm{pat}_{1}, \ldots, \mathrm{pat}_{k}\right]\) & (list pattern, \(k \geq 1\) ) \\
\hline & & ~ apat & (irrefutable pattern) \\
\hline fpat & \(\rightarrow\) & \(q v a r=p a t\) & \\
\hline \multirow[t]{4}{*}{gcon} & \multirow[t]{4}{*}{\(\rightarrow\)} & () & \\
\hline & & [] & \\
\hline & & (,, 1,\(\}\) ) & \\
\hline & & qcon & \\
\hline var & \(\rightarrow\) & varid \(\mid\) ( varsym) & (variable) \\
\hline quar & \(\rightarrow\) & qvarid | ( qvarsym ) & (qualified variable) \\
\hline con & \(\rightarrow\) & conid | ( consym) & (constructor) \\
\hline qcon & \(\rightarrow\) & qconid | ( gconsym) & (qualified constructor) \\
\hline varop & \(\rightarrow\) & varsym |`varid & (variable operator) \\
\hline quarop & \(\rightarrow\) & qvarsym |` quarid` & (qualified variable operator) \\
\hline conop & \(\rightarrow\) & consym | ` conid` & (constructor operator) \\
\hline qconop & \(\rightarrow\) & gconsym | ¢ qconid` & (qualified constructor operator) \\
\hline op & \(\rightarrow\) & varop | conop & (operator) \\
\hline qop & \(\rightarrow\) & quarop | qconop & (qualified operator) \\
\hline gconsym & \(\rightarrow\) & : | qconsym & \\
\hline
\end{tabular}

\section*{Chapter 10}

\section*{Specification of Derived Instances}

A derived instance is an instance declaration that is generated automatically in conjunction with a data or newtype declaration. The body of a derived instance declaration is derived syntactically from the definition of the associated type. Derived instances are possible only for classes known to the compiler: those defined in either the Prelude or a standard library. In this chapter, we describe the derivation of classes defined by the Prelude.

If \(T\) is an algebraic datatype declared by:
\[
\begin{aligned}
\text { data } c x=>T u_{1} \ldots u_{k}= & K_{1} t_{11} \ldots t_{1 k_{1}}|\ldots| K_{n} t_{n 1} \ldots t_{n k_{n}} \\
& \text { deriving }\left(C_{1}, \ldots, C_{m}\right)
\end{aligned}
\]
(where \(m \geq 0\) and the parentheses may be omitted if \(m=1\) ) then a derived instance declaration is possible for a class \(C\) if these conditions hold:
1. \(C\) is one of Eq, Ord, Enum, Bounded, Show, or Read.
2. There is a context \(c x^{\prime}\) such that \(c x^{\prime} \Rightarrow C t_{i j}\) holds for each of the constituent types \(t_{i j}\).
3. If \(C\) is Bounded, the type must be either an enumeration (all constructors must be nullary) or have only one constructor.
4. If \(C\) is Enum, the type must be an enumeration.
5. There must be no explicit instance declaration elsewhere in the program that makes \(T u_{1} \ldots u_{k}\) an instance of \(C\).

For the purposes of derived instances, a newt ype declaration is treated as a dat a declaration with a single constructor.

If the deriving form is present, an instance declaration is automatically generated for \(T u_{1} \ldots u_{k}\) over each class \(C_{i}\). If the derived instance declaration is impossible for any of the
\(C_{i}\) then a static error results. If no derived instances are required, the deriving form may be omitted or the form deriving () may be used.

Each derived instance declaration will have the form:
\[
\text { instance }\left(c x, c x^{\prime}\right) \Rightarrow C_{i}\left(T u_{1} \ldots u_{k}\right) \text { where }\{d\}
\]
where \(d\) is derived automatically depending on \(C_{i}\) and the data type declaration for \(T\) (as will be described in the remainder of this section).

The context \(c x^{\prime}\) is the smallest context satisfying point (2) above. For mutually recusive data types, the compiler may need to perform a fixpoint calculation to compute it.

The remaining details of the derived instances for each of the derivable Prelude classes are now given. Free variables and constructors used in these translations always refer to entities defined by the Prelude.

\subsection*{10.1 Derived instances of Eq and Ord}

The class methods automatically introduced by derived instances of Eq and Ord are (==), (/=), compare, \((<),(<=),(>),(>=)\), max, and min. The latter seven operators are defined so as to compare their arguments lexicographically with respect to the constructor set given, with earlier constructors in the datatype declaration counting as smaller than later ones. For example, for the Bool datatype, we have that (True > False) \(==\) True.

Derived comparisons always traverse constructors from left to right. These examples illustrate this property:
```

(1,undefined) == (2,undefined) => False
(undefined,1) == (undefined,2) =>

```

All derived operations of class Eq and Ord are strict in both arguments. For example, False < \(\perp\) is \(\perp\), even though False is the first constructor of the Bool type.

\subsection*{10.2 Derived instances of Enum}

Derived instance declarations for the class Enum are only possible for enumerations (data types with only nullary constructors).

The nullary constructors are assumed to be numbered left-to-right with the indices 0 through \(n-1\). The succ and pred operators give the successor and predecessor respectively of a value, under this numbering scheme. It is an error to apply succ to the maximum element, or pred to the minimum element.

The toEnum and fromEnum operators map enumerated values to and from the Int type; toEnum raises a runtime error if the Int argument is not the index of one of the constructors.

The definitions of the remaining methods are
```

enumFrom x = enumFromTo x lastCon
enumFromThen x y = enumFromThenTo x y bound
where
bound | fromEnum y >= fromEnum x = lastCon
| otherwise = firstCon
enumFromTo x y = map toEnum [fromEnum x .. fromEnum y]
enumFromThenTo x y z = map toEnum [fromEnum x, fromEnum y .. fromEnum z]

```
where firstCon and lastCon are respectively the first and last constructors listed in the data declaration. For example, given the datatype:
```

data Color = Red | Orange | Yellow | Green deriving (Enum)

```
we would have:
```

[Orange ..] == [Orange, Yellow, Green]
fromEnum Yellow == 2

```

\subsection*{10.3 Derived instances of Bounded}

The Bounded class introduces the class methods minBound and maxBound, which define the minimal and maximal elements of the type. For an enumeration, the first and last constructors listed in the data declaration are the bounds. For a type with a single constructor, the constructor is applied to the bounds for the constituent types. For example, the following datatype:
```

data Pair a b = Pair a b deriving Bounded

```
would generate the following Bounded instance:
```

instance (Bounded a,Bounded b) => Bounded (Pair a b) where
minBound = Pair minBound minBound
maxBound = Pair maxBound maxBound

```

\subsection*{10.4 Derived instances of Read and Show}

The class methods automatically introduced by derived instances of Read and Show are showsPrec, readsPrec, showList, and readList. They are used to coerce values into strings and parse strings into values.

The function showsPrec \(d \times r\) accepts a precedence level \(d\) (a number from 0 to 11), a value \(x\), and a string \(r\). It returns a string representing \(x\) concatenated to \(r\). showsPrec satisfies the law:
```

showsPrec d x r ++ s == showsPrec d x (r ++ s)

```

The representation will be enclosed in parentheses if the precedence of the top-level constructor in \(x\) is less than \(d\). Thus, if \(d\) is 0 then the result is never surrounded in parentheses; if \(d\) is 11 it is always surrounded in parentheses, unless it is an atomic expression (recall that function application has precedence 10). The extra parameter \(r\) is essential if tree-like structures are to be printed in linear time rather than time quadratic in the size of the tree.

The function readsPrec \(d s\) accepts a precedence level \(d\) (a number from 0 to 10) and a string s, and attempts to parse a value from the front of the string, returning a list of (parsed value, remaining string) pairs. If there is no successful parse, the returned list is empty. Parsing of an unparenthesised infix operator application succeeds only if the precedence of the operator is greater than or equal to d .

It should be the case that
\[
(x, " \|) \text { is an element of (readsPrec } d \text { (showsPrec } d x \text { } " \|) \text { ) }
\]

That is, readsPrec should be able to parse the string produced by showsPrec, and should deliver the value that showsPrec started with.
showList and readList allow lists of objects to be represented using non-standard denotations. This is especially useful for strings (lists of Char).
readsPrec will parse any valid representation of the standard types apart from strings, for which only quoted strings are accepted, and other lists, for which only the bracketed form [...] is accepted. See Chapter 8 for full details.

The result of show is a syntactically correct Haskell expression containing only constants, given the fixity declarations in force at the point where the type is declared. It contains only the constructor names defined in the data type, parentheses, and spaces. When labelled constructor fields are used, braces, commas, field names, and equal signs are also used. Parentheses are only added where needed, ignoring associativity. No line breaks are added. The result of show is readable by read if all component types are readable. (This is true for all instances defined in the Prelude but may not be true for user-defined instances.)

Derived instances of Read make the following assumptions, which derived instances of Show obey:
- If the constructor is defined to be an infix operator, then the derived Read instance will parse only infix applications of the constructor (not the prefix form).
- Associativity is not used to reduce the occurrence of parentheses, although precedence may be. For example, given
```

infixr 4 :\$
data T = Int :\$ T NT

```
then:
- show (1 : \$ \(2: \$ N T\) ) produces the string "1 :
- read "1 : \(\$(2: \$ N T)\) " succeeds, with the obvious result.
- read "1 : \$ 2 : \(\$ \mathrm{NT}\) "fails.
- If the constructor is defined using record syntax, the derived Read will parse only the recordsyntax form, and furthermore, the fields must be given in the same order as the original declaration.
- The derived Read instance allows arbitrary Haskell whitespace between tokens of the input string. Extra parentheses are also allowed.

The derived Read and Show instances may be unsuitable for some uses. Some problems include:
- Circular structures cannot be printed or read by these instances.
- The printer loses shared substructure; the printed representation of an object may be much larger than necessary.
- The parsing techniques used by the reader are very inefficient; reading a large structure may be quite slow.
- There is no user control over the printing of types defined in the Prelude. For example, there is no way to change the formatting of floating point numbers.

\subsection*{10.5 An Example}

As a complete example, consider a tree datatype:
```

data Tree a = Leaf a | Tree a :^: Tree a
deriving (Eq, Ord, Read, Show)

```

Automatic derivation of instance declarations for Bounded and Enum are not possible, as Tree is not an enumeration or single-constructor datatype. The complete instance declarations for Tree are shown in Figure 10.1, Note the implicit use of default class method definitions-for example, only \(<=\) is defined for Ord, with the other class methods ( \(<,>,>=\), max, and min) being defined by the defaults given in the class declaration shown in Figure 6.1 (page 83).
```

infixr 5 :^:
data Tree a = Leaf a | Tree a :^: Tree a
instance (Eq a) => Eq (Tree a) where
Leaf m == Leaf n = m==n
u:^:v == x:^:y = u==x \&\& v==y
_== _ = False
instance (Ord a) => Ord (Tree a) where
Leaf m <= Leaf n = m<=n
Leaf m <= x:^:y = True
u:^:v <= Leaf n = False
u:^:v <= x:^:y = u<x || u==x \&\& v<=y
instance (Show a) => Show (Tree a) where
showsPrec d (Leaf m) = showParen (d > app_prec) showStr
where
showStr = showString "Leaf " . showsPrec (app_prec+1) m
showsPrec d (u :`: v) = showParen (d > up_prec) showStr
where
showStr = showsPrec (up_prec+1) u .
showString " :^: " .
showsPrec (up_prec+1) v
-- Note: right-associativity of :^: ignored
instance (Read a) => Read (Tree a) where
readsPrec d r = readParen (d > up_prec)
(\r -> [(u:^:v,w) |
(u,s) <- readsPrec (up_prec+1) r,
(":^:",t) <- lex s,
(v,w) <- readsPrec (up_prec+1) t]) r
++ readParen (d > app_prec)
(\r -> [(Leaf m,t) |
("Leaf",s) <- lex r,
(m,t) <- readsPrec (app_prec+1) s]) r
up_prec = 5 -- Precedence of :^:
app_prec = 10 -- Application has precedence one more than
-- the most tightly-binding operator

```

Figure 10.1: Example of Derived Instances

\section*{Chapter 11}

\section*{Compiler Pragmas}

Some compiler implementations support compiler pragmas, which are used to give additional instructions or hints to the compiler, but which do not form part of the Haskell language proper and do not change a program's semantics. This chapter summarizes this existing practice. An implementation is not required to respect any pragma, but the pragma should be ignored if an implementation is not prepared to handle it. Lexically, pragmas appear as comments, except that the enclosing syntax is \(\{-\# \#-\}\).

\subsection*{11.1 Inlining}
```

decl }\quad->\quad{-\# INLINE qvars \#-
decl }->\quad{-\# NOINLINE qvars \#-

```

The INLINE pragma instructs the compiler to inline the specified variables at their use sites. Compilers will often automatically inline simple expressions. This may be prevented by the NOINLINE pragma.

\subsection*{11.2 Specialization}
```

decl }\quad->\quad{-\# SPECIALIZE spec ci, .., specck \#-} (k\geq1
spec }->\mathrm{ vars :: type

```

Specialization is used to avoid inefficiencies involved in dispatching overloaded functions. For example, in
```

factorial :: Num a => a -> a
factorial 0 = 0
factorial n = n * factorial (n-1)
{-\# SPECIALIZE factorial :: Int -> Int,
factorial :: Integer -> Integer \#-}

```
calls to factorial in which the compiler can detect that the parameter is either Int or Integer will use specialized versions of factorial which do not involve overloaded numeric operations.

\section*{Part II}

\section*{The Haskell 98 Libraries}

\section*{Chapter 12}

\section*{Rational Numbers}
```

module Ratio (
Ratio, Rational, (%), numerator, denominator, approxRational ) where
infixl 7 %
data (Integral a) => Ratio a = ...
type Rational = Ratio Integer
(%) :: (Integral a) => a -> a -> Ratio a
numerator, denominator :: (Integral a) => Ratio a -> a
approxRational :: (RealFrac a) => a -> a -> Rational
instance (Integral a) => Eq (Ratio a) where ...
instance (Integral a) => Ord (Ratio a) where ...
instance (Integral a) => Num (Ratio a) where ...
instance (Integral a) => Real (Ratio a) where ...
instance (Integral a) => Fractional (Ratio a) where ...
instance (Integral a) => RealFrac (Ratio a) where ...
instance (Integral a) => Enum (Ratio a) where ...
instance (Read a,Integral a) => Read (Ratio a) where ...
instance (Integral a) => Show (Ratio a) where ...

```

For each Integral type \(t\), there is a type Ratio \(t\) of rational pairs with components of type \(t\). The type name Rational is a synonym for Ratio Integer.

Ratio is an instance of classes Eq, Ord, Num, Real, Fractional, RealFrac, Enum, Read, and Show. In each case, the instance for Rat io \(t\) simply "lifts" the corresponding operations over \(t\). If \(t\) is a bounded type, the results may be unpredictable; for example Ratio Int may give rise to integer overflow even for rational numbers of small absolute size.

The operator ( \(\%\) ) forms the ratio of two integral numbers, reducing the fraction to terms with no common factor and such that the denominator is positive. The functions numerator and denominator extract the components of a ratio; these are in reduced form with a positive denominator. Ratio is an abstract type. For example, \(12 \% 8\) is reduced to \(3 / 2\) and \(12 \%(-8)\) is reduced to \((-3) / 2\).

The approxRational function, applied to two real fractional numbers \(x\) and epsilon, returns the simplest rational number within the open interval ( \(\mathrm{x}-\mathrm{epsilon}, \mathrm{x}+\mathrm{epsilon}\) ). A rational number \(n / d\) in reduced form is said to be simpler than another \(n^{\prime} / d^{\prime}\) if \(|n| \leq\left|n^{\prime}\right|\) and \(d \leq d^{\prime}\). Note that it can be proved that any real interval contains a unique simplest rational.

\subsection*{12.1 Library Ratio}
```

-- Standard functions on rational numbers
module Ratio (
Ratio, Rational, (%), numerator, denominator, approxRational ) where
infixl 7 %
ratPrec = 7 :: Int
data (Integral a) => Ratio a = !a :% !a deriving (Eq)
type Rational = Ratio Integer
(%)
:: (Integral a) => a -> a -> Ratio a
numerator, denominator :: (Integral a) => Ratio a -> a
approxRational :: (RealFrac a) => a -> a -> Rational
-- "reduce" is a subsidiary function used only in this module.
-- It normalises a ratio by dividing both numerator
-- and denominator by their greatest common divisor.
--
-- E.g., 12 `reduce' 8 == 3 :% 2 -- 12 'reduce' (-8) == 3:% (-2) reduce _ 0 = error "Ratio.% : zero denominator" reduce x y = (x `quot`d) :% (y`quot` d)
where d = gcd x y
x % y = reduce (x * signum y) (abs y)
numerator (x :% _) = x
denominator (_ :% y) = y
instance (Integral a) => Ord (Ratio a) where
(x:%y) <= (x':%y') = x * y' <= x' * y
(x:%y) < (x':%y') = x * y' < x' * y
instance (Integral a) => Num (Ratio a) where
(x:%y) + (x':%y') = reduce (x* y' + x'*y) ( }\mp@subsup{y}{}{*}\mp@subsup{y}{}{\prime}
(x:%y) * (x':%y') = reduce (x * x') (y * y')
negate (x:%y) = (-x) :% y
abs (x:%y) = abs x :% y
signum (x:%y) = signum x :% 1
fromInteger x = fromInteger x :% 1
instance (Integral a) => Real (Ratio a) where
toRational (x:%y) = toInteger x :% toInteger y
instance (Integral a) => Fractional (Ratio a) where
(x:%y) / (x':% y') = (x* y') % ( }\mp@subsup{y}{}{*}\mp@subsup{\textrm{X}}{}{\prime}
recip (x:%y) = y % x
fromRational (x:%y) = fromInteger x :% fromInteger y

```
```

instance (Integral a) => RealFrac (Ratio a) where
properFraction (x:%y) = (fromIntegral q, r:%y)
where (q,r) = quotRem x y
instance (Integral a) => Enum (Ratio a) where
succ x = x+1
pred x = x-1
toEnum = fromIntegral
fromEnum = fromInteger . truncate -- May overflow
enumFrom = numericEnumFrom -- These numericEnumXXX functions
enumFromThen = numericEnumFromThen -- are as defined in Prelude.hs
enumFromTo = numericEnumFromTo -- but not exported from it!
enumFromThenTo = numericEnumFromThenTo
instance (Read a, Integral a) => Read (Ratio a) where
readsPrec p = readParen (p > ratPrec)
(\r -> [(x%y,u) | (x,s) <- readsPrec (ratPrec+1) r,
("%",t) <- lex s,
(y,u) <- readsPrec (ratPrec+1) t ])
instance (Integral a) => Show (Ratio a) where
showsPrec p (x:%y) = showParen (p > ratPrec)
(showsPrec (ratPrec+1) x .
showString " % " .
showsPrec (ratPrec+1) y)
approxRational x eps = simplest (x-eps) (x+eps)
where simplest x y | y < x = simplest y x
x == y = xr
x > 0 = simplest' n d n' d'
y < 0 - - simplest' (-n') d' (-n) d
otherwise = 0 :% 1
where xr@(n:%d) = toRational x
(n':%d') = toRational y
simplest' n d n' d' -- assumes 0 < n%d < n'%d'
r == 0 = q :% 1
q/= q' = (q+1) :% 1
otherwise = (q* n''+d'') :% n''
where (q,r) = quotRem n d
(q', r') = quotRem n' d'
(n'\prime:%d'\prime) = simplest' d' r' d r

```

\section*{Chapter 13}

\section*{Complex Numbers}
```

module Complex (
Complex((:+)), realPart, imagPart, conjugate,
mkPolar, cis, polar, magnitude, phase ) where
infix 6 :+
data (RealFloat a) => Complex a = !a :+ !a
realPart, imagPart :: (RealFloat a) => Complex a -> a
conjugate :: (RealFloat a) => Complex a -> Complex a
mkPolar :: (RealFloat a) => a -> a -> Complex a
cis :: (RealFloat a) => a -> Complex a
polar :: (RealFloat a) => Complex a -> (a,a)
magnitude, phase :: (RealFloat a) => Complex a -> a
instance (RealFloat a) => Eq (Complex a) where ...
instance (RealFloat a) => Read (Complex a) where ...
instance (RealFloat a) => Show (Complex a) where ...
instance (RealFloat a) => Num (Complex a) where ...
instance (RealFloat a) => Fractional (Complex a) where ...
instance (RealFloat a) => Floating (Complex a) where ...

```

Complex numbers are an algebraic type. The constructor (:+) forms a complex number from its real and imaginary rectangular components. This constructor is strict: if either the real part or the imaginary part of the number is \(\perp\), the entire number is \(\perp\). A complex number may also be formed from polar components of magnitude and phase by the function mkPolar. The function cis produces a complex number from an angle \(t\). Put another way, cis \(t\) is a complex value with magnitude 1 and phase \(t\) (modulo \(2 \pi\) ).

The function polar takes a complex number and returns a (magnitude, phase) pair in canonical form: The magnitude is nonnegative, and the phase, in the range \((-\pi, \pi]\); if the magnitude is zero, then so is the phase.

The functions realPart and imagPart extract the rectangular components of a complex number and the functions magnitude and phase extract the polar components of a complex number. The function conjugate computes the conjugate of a complex number in the usual way.

The magnitude and sign of a complex number are defined as follows:
```

abs z = magnitude z :+ 0
signum 0 = 0
signum z@(x:+y) = x/r :+ y/r where r = magnitude z

```

That is, abs \(z\) is a number with the magnitude of \(z\), but oriented in the positive real direction, whereas signum \(z\) has the phase of \(z\), but unit magnitude.

\subsection*{13.1 Library Complex}
```

module Complex(Complex((:+)), realPart, imagPart, conjugate, mkPolar,
cis, polar, magnitude, phase) where
infix 6 :+
data (RealFloat a) => Complex a = !a :+ !a deriving (Eq,Read,Show)
realPart, imagPart :: (RealFloat a) => Complex a -> a
realPart (x:+y) = x
imagPart (x:+y) = y
conjugate :: (RealFloat a) => Complex a -> Complex a
conjugate (x:+y) = x :+ (-y)
mkPolar :: (RealFloat a) => a -> a -> Complex a
mkPolar r theta = r * cos theta :+ r * sin theta
cis :: (RealFloat a) => a -> Complex a
cis theta = cos theta :+ sin theta
polar :: (RealFloat a) => Complex a -> (a,a)
polar z = (magnitude z, phase z)
magnitude :: (RealFloat a) => Complex a -> a
magnitude (x:+y) = scaleFloat k
(sqrt ((scaleFloat mk x)^2 + (scaleFloat mk y)^2))
where k = max (exponent x) (exponent y)
mk = - k
phase :: (RealFloat a) => Complex a -> a
phase (0 :+ 0) = 0
phase (x :+ y) = atan2 y x

```
```

instance (RealFloat a) => Num (Complex a) where
(x:+y) + (x':+y') = (x+\mp@subsup{x}{}{\prime}) :+ (y+y')
(x:+y) - (x':+y') = (x-x') :+ (y-y')
(x:+y) * ( }\mp@subsup{x}{}{\prime}:+\mp@subsup{y}{}{\prime})=(\mp@subsup{x}{}{*}\mp@subsup{x}{}{\prime}-\mp@subsup{y}{}{*}\mp@subsup{y}{}{\prime}) :+ (x* y'+y*\mp@subsup{x}{}{\prime}
negate (x:+y) = negate x :+ negate y
abs z = magnitude z :+ 0
signum 0 = 0
signum z@(x:+y) = x/r :+ y/r where r = magnitude z
fromInteger n = fromInteger n :+ 0
instance (RealFloat a) => Fractional (Complex a) where

```

```

    where x'' = scaleFloat k x'
        y'r = scaleFloat k y'
        k = - max (exponent x') (exponent y')
        d = x'**'' + y'* '''
    fromRational a = fromRational a :+ 0
    ```
```

instance (RealFloat a) => Floating (Complex a) where
pi = pi :+ 0
exp (x:+y) = expx * cos y :+ expx * sin y
where expx = exp x
log z = log (magnitude z) :+ phase z
sqrt 0 = 0
sqrt z@(x:+y) = u :+ (if y < 0 then -v else v)
where (u,v) = if x < O then (v',u') else (u', v')
v' = abs y / (u'*2)
u' = sqrt ((magnitude z + abs x) / 2)
sin (x:+y) = sin x * cosh y :+ cos x * sinh y
cos (x:+y) = cos x * cosh y :+ (- sin x * sinh y)
tan (x:+y) = (sinx*coshy:+cosx*sinhy)/(cosx*coshy:+(-sinx*sinhy))
where sinx = sin x
cosx = cos x
sinhy = sinh y
coshy = cosh y
sinh (x:+y) = cos y * sinh x :+ sin y * cosh x
cosh (x:+y) = cos y * cosh x :+ sin y * sinh x
tanh (x:+y) = (cosy*sinhx:+siny*coshx)/(cosy*coshx:+siny*sinhx)
where siny = sin y
cosy = cos y
sinhx = sinh x
coshx = cosh x
asin z@(x:+y) = y':+(-x')
where (x':+y') = log (((-y):+x) + sqrt (1 - z*z))
acos z@(x:+y) = y'':+(-\mp@subsup{x}{}{\prime\prime})
where (x'':+y'') = log (z + ((-y'):+\mp@subsup{x}{}{\prime}))
(x':+y') = sqrt (1 - z*z)
atan z@(x:+y) = y':+(-x')
where (x':+y') = log (((1-y):+x) / sqrt (1+z*z))
asinh z = log (z + sqrt (1+z*z))
acosh z = log (z + (z+1) * sqrt ((z-1)/(z+1)))
atanh z = log ((1+z) / sqrt (1-z*z))

```

\section*{Chapter 14}

\section*{Numeric}
```

module Numeric(fromRat,
showSigned, showIntAtBase,
showInt, showOct, showHex,
readSigned, readInt,
readDec, readOct, readHex,
floatToDigits,
showEFloat, showFFloat, showGFloat, showFloat,
readFloat, lexDigits) where
fromRat :: (RealFloat a) => Rational -> a
showSigned :: (Real a) => (a -> ShowS) -> Int -> a -> ShowS
showIntAtBase :: Integral a => a -> (Int -> Char) -> a -> ShowS
showInt :: Integral a => a -> ShowS
showOct :: Integral a => a -> ShowS
showHex :: Integral a => a -> ShowS
readSigned :: (Real a) => ReadS a -> ReadS a
readInt :: (Integral a) =>
a -> (Char -> Bool) -> (Char -> Int) -> ReadS a
readDec :: (Integral a) => ReadS a
readOct :: (Integral a) => ReadS a
readHex :: (Integral a) => ReadS a
showEFloat :: (RealFloat a) => Maybe Int -> a -> ShowS
showFFloat :: (RealFloat a) => Maybe Int -> a -> ShowS
showGFloat :: (RealFloat a) => Maybe Int -> a -> ShowS
showFloat :: (RealFloat a) => a -> ShowS
floatToDigits :: (RealFloat a) => Integer -> a -> ([Int], Int)
readFloat :: (RealFrac a) => ReadS a
lexDigits :: ReadS String

```

This library contains assorted numeric functions, many of which are used in the standard Prelude.
In what follows, recall the following type definitions from the Prelude:
```

type ShowS = String -> String
type ReadS = String -> [(a,String)]

```

\subsection*{14.1 Showing functions}
- showSigned : : (Real a) => (a -> ShowS) -> Int -> a -> ShowS converts a possibly-negative Real value of type a to a string. In the call (showSigned show prec val), val is the value to show, prec is the precedence of the enclosing context, and show is a function that can show unsigned values.
- showIntAtBase :: Integral a => a -> (Int -> Char) -> a -> ShowS shows a non-negative Integral number using the base specified by the first argument, and the character representation specified by the second.
- showInt, showOct, showHex :: Integral a => a -> ShowS show non-negative Integral numbers in base 10,8 , and 16 respectively.
- showFFloat, showEFloat, showGFloat
: : (RealFloat a) => Maybe Int -> a -> ShowS
These three functions all show signed Realfloat values:
- showFFloat uses standard decimal notation (e.g. 245000,0.0015).
- showEFloat uses scientific (exponential) notation (e.g. 2.45e2, 1.5e-3).
- showGFloat uses standard decimal notation for arguments whose absolute value lies between 0.1 and \(9,999,999\), and scientific notation otherwise.

In the call (showEFloat digs val), if digs is Nothing, the value is shown to full precision; if digs is Just \(d\), then at most \(d\) digits after the decimal point are shown. Exactly the same applies to the digs argument of the other two functions.
- floatToDigits : (Realfloat a) => Integer -> a \(->\) ([Int], Int) converts a base and a value to the representation of the value in digits, plus an exponent. More specifically, if
\[
\text { floatToDigits } b r=\left(\left[d_{1}, d_{2}, \ldots d_{n}\right], e\right)
\]
then the following properties hold:
- \(r=0 . d_{1} d_{2} \ldots, d_{n} * b^{e}\)
\(-n \geq 0\)
- \(d_{1} \neq 0\) (when \(\left.n>0\right)\)
- \(0 \leq d_{i} \leq b-1\)

\subsection*{14.2 Reading functions}
- readSigned : : (Real a) \(=>\) ReadS a \(->\) ReadS a reads a signed Real value, given a reader for an unsigned value.
- readInt : : (Integral a) \(=>\) a \(->\) (Char->Bool) \(->\) (Char->Int) -> ReadS a reads an unsigned Integral value in an arbitrary base. In the call (readInt base \(i s d i g\) d2i), base is the base, \(i s d i g\) is a predicate distinguishing valid digits in this base, and \(d 2 i\) converts a valid digit character to an Int.
- readFloat : : (RealFrac a) => ReadS a reads an unsigned RealFrac value, expressed in decimal scientific notation.
- readDec, readOct, readHex : (Integral a) => ReadS a each read an unsigned number, in decimal, octal, and hexadecimal notation respectively. In the hexadecimal case, both upper or lower case letters are allowed.
- lexDigits : : ReadS String reads a non-empty string of decimal digits.
(NB: readInt is the "dual" of showIntAtBase, and readDec is the "dual" of showInt. The inconsistent naming is a historical accident.)

\subsection*{14.3 Miscellaneous}
- fromRat : : (RealFloat a) => Rational \(->\) a converts a Rational value into any type in class Realfloat.

\subsection*{14.4 Library Numeric}
```

module Numeric(fromRat,
showSigned, showIntAtBase,
showInt, showOct, showHex,
readSigned, readInt,
readDec, readOct, readHex,
floatToDigits,
showEFloat, showFFloat, showGFloat, showFloat,
readFloat, lexDigits) where
import Char ( isDigit, isOctDigit, isHexDigit
, digitToInt, intToDigit )
import Ratio ( (%), numerator, denominator )
import Array ( (!), Array, array )

```
-- This converts a rational to a floating. This should be used in the -- Fractional instances of Float and Double.
fromRat :: (RealFloat a) => Rational -> a
```

fromRat x =

```
    if \(x=0\) then encodeFloat \(0 \quad 0 \quad\)-- Handle exceptional cases
    else if \(x<0\) then - fromRat' (-x) -- first.
    else fromRat' \(x\)
-- Conversion process:
-- Scale the rational number by the RealFloat base until
-- it lies in the range of the mantissa (as used by decodeFloat/encodeFloat).
-- Then round the rational to an Integer and encode it with the exponent
-- that we got from the scaling.
-- To speed up the scaling process we compute the \(\log 2\) of the number to get
-- a first guess of the exponent.
fromRat' :: (RealFloat a) => Rational -> a
fromRat' \(\mathrm{x}=\mathrm{r}\)
    where \(b=\) floatRadix r
        \(p=\) floatDigits \(r\)
        (minExp0, _) = floatRange r
        minExp \(=\operatorname{minExp} 0-p \quad\)-- the real minimum exponent
        xMin = toRational (expt b (p-1))
        xMax = toRational (expt b p)
        p0 = (integerLogBase b (numerator \(x\) ) -
            integerLogBase b (denominator x) - p) `max' minExp
        \(\mathrm{f}=\) if \(\mathrm{p} 0<0\) then \(1 \%\) expt \(b(-\mathrm{p} 0)\) else expt \(b \mathrm{p} 0 \% 1\)
        \(\left(x^{\prime}, p^{\prime}\right)=\) scaleRat (toRational b) minExp xMin xMax p0 (x / f)
        \(r=\) encodeFloat (round \(x^{\prime}\) ) \(p^{\prime}\)
-- Scale \(x\) until xMin \(<=x\) < xMax, or \(p\) (the exponent) <= minExp.
scaleRat :: Rational -> Int -> Rational -> Rational ->
                Int -> Rational -> (Rational, Int)
scaleRat \(b\) minExp xMin \(x M a x ~ p ~ x=\)
    if \(p<=\) minExp then
            ( \(\mathrm{x}, \mathrm{p}\) )
    else if \(x>=x M a x\) then
        scaleRat \(b\) minExp \(x M i n \operatorname{xMax}(p+1)(x / b)\)
    else if \(x<x M i n\) then
            scaleRat b minExp xMin xMax (p-1) ( \(\mathrm{x} * \mathrm{~b}\) )
        else
            ( \(\mathrm{x}, \mathrm{p}\) )
-- Exponentiation with a cache for the most common numbers.
minExpt \(=0::\) Int
maxExpt = 1100: :Int
expt :: Integer -> Int -> Integer
expt base \(\mathrm{n}=\)
    if base \(==2 \& \& \mathrm{n}>=\operatorname{minExpt} \& \& \mathrm{n}<=\) maxExpt then
        expts!n
    else
        base^n
```

expts :: Array Int Integer
expts = array (minExpt,maxExpt) [(n,2^n) | n <- [minExpt .. maxExpt]]
-- Compute the (floor of the) log of i in base b.
-- Simplest way would be just divide i by b until it's smaller then b,
-- but that would be very slow! We are just slightly more clever.
integerLogBase :: Integer -> Integer -> Int
integerLogBase b i =
if i < b then
0
else
-- Try squaring the base first to cut down the number of divisions.
let l = 2 * integerLogBase (b*b) i
doDiv :: Integer -> Int -> Int
doDiv i l = if i < b then l else doDiv (i `div` b) (l+1)
in doDiv (i `div` (b^l)) l
-- Misc utilities to show integers and floats
showSigned :: Real a => (a -> ShowS) -> Int -> a -> ShowS
showSigned showPos p x
x < 0 = showParen (p > 6) (showChar '_' . showPos (-x))
otherwise = showPos x
-- showInt, showOct, showHex are used for positive numbers only
showInt, showOct, showHex :: Integral a => a -> ShowS
showOct = showIntAtBase 8 intToDigit
showInt = showIntAtBase 10 intToDigit
showHex = showIntAtBase 16 intToDigit
showIntAtBase :: Integral a
=> a -- base
-> (Int -> Char) -- digit to char
-> a -- number to show
-> ShowS
showIntAtBase base intToDig n rest
n < 0 = error "Numeric.showIntAtBase: can't show negative numbers"
n' == 0 = rest'
otherwise = showIntAtBase base intToDig n' rest'
where
(n',d) = quotRem n base
rest' = intToDig (fromIntegral d) : rest
readSigned :: (Real a) => ReadS a -> ReadS a
readSigned readPos = readParen False read'
where read' r = read'' r ++
[(-x,t) | ("-",s) <- lex r,
(x,t) <- read'r s]
read''r r = [(n,s) | (str,s) <- lex r,
(n,"") <- readPos str]

```
```

-- readInt reads a string of digits using an arbitrary base.
-- Leading minus signs must be handled elsewhere.
readInt :: (Integral a) => a -> (Char -> Bool) -> (Char -> Int) -> ReadS a
readInt radix isDig digToInt s =
[(foldl1 (\n d -> n * radix + d) (map (fromIntegral . digToInt) ds), r)
| (ds,r) <- nonnull isDig s ]
-- Unsigned readers for various bases
readDec, readOct, readHex :: (Integral a) => ReadS a
readDec = readInt 10 isDigit digitToInt
readOct = readInt 8 isOctDigit digitToInt
readHex = readInt 16 isHexDigit digitToInt
showEFloat :: (RealFloat a) => Maybe Int -> a -> ShowS
showFFloat :: (RealFloat a) => Maybe Int -> a -> ShowS
showGFloat :: (RealFloat a) => Maybe Int -> a -> ShowS
showFloat :: (RealFloat a) => a -> ShowS
showEFloat d x = showString (formatRealFloat FFExponent d x)
showFFloat d x = showString (formatRealFloat FFFixed d x)
showGFloat d x = showString (formatRealFloat FFGeneric d x)
showFloat = showGFloat Nothing
-- These are the format types. This type is not exported.
data FFFormat = FFExponent | FFFixed | FFGeneric

```
```

formatRealFloat :: (RealFloat a) => FFFormat -> Maybe Int -> a -> String
formatRealFloat fmt decs x
= s
where
base = 10
s = if isNaN x then
"NaN"
else if isInfinite x then
if x < O then "-Infinity" else "Infinity"
else if x < 0 || isNegativeZero x then
'-' : doFmt fmt (floatToDigits (toInteger base) (-x))
else
doFmt fmt (floatToDigits (toInteger base) x)
doFmt fmt (is, e)
= let
ds = map intToDigit is
in
case fmt of
FFGeneric ->
doFmt (if e < 0 || e > 7 then FFExponent else FFFixed)
(is, e)
FFExponent ->
case decs of
Nothing ->
case ds of
[] -> "0.0e0"
[d] -> d : ".Oe" ++ show (e-1)
d:ds -> d : '.' : ds ++ 'e':show (e-1)
Just dec ->
let dec' = max dec 1 in
case is of
[] -> '0':'.':take dec' (repeat '0') ++ "e0"
_ ->
let (ei, is') = roundTo base (dec'+1) is
d:ds = map intToDigit
(if ei > O then init is' else is')
in d:'.':ds ++ "e" ++ show (e-1+ei)
FFFixed ->
case decs of
Nothing -- Always prints a decimal point
| e > 0 -> take e (ds ++ repeat '0')

```
```

            ++ '.' : mk0 (drop e ds)
            | otherwise -> "0." ++ mk0 (replicate (-e) '0' ++ ds)
            Just dec -> -- Print decimal point iff dec > 0
                let dec' = max dec 0 in
                if e >= O then
                let (ei, is') = roundTo base (dec' + e) is
                    (ls, rs) = splitAt (e+ei)
                                    (map intToDigit is')
                in mkO ls ++ mkdot0 rs
        else
                let (ei, is') = roundTo base dec'
                    (replicate (-e) 0 ++ is)
                        d : ds = map intToDigit
                        (if ei > O then is' else 0:is')
                in d : mkdot0 ds
            where
        mk0 "" = "0" -- Print 0.34, not . 34
        mk0 s = s
        mkdot0 "" = "" -- Print 34, not 34.
        mkdot0 s = '.' : s -- when the format specifies no
            -- digits after the decimal point
    roundTo :: Int -> Int -> [Int] -> (Int, [Int])
roundTo base d is = case f d is of
(0, is) -> (0, is)
(1, is) -> (1, 1 : is)
where b2 = base `div` 2
f n [] = (0, replicate n 0)
f 0 (i:_) = (if i >= b2 then 1 else 0, [])
f d (i:is) =
let (c, ds) = f (d-1) is
i' = c + i
in if i' == base then (1, 0:ds) else (0, i':ds)
-- Based on "Printing Floating-Point Numbers Quickly and Accurately"
-- by R.G. Burger and R. K. Dybvig, in PLDI 96.
-- The version here uses a much slower logarithm estimator.
-- It should be improved.
-- This function returns a non-empty list of digits (Ints in [0..base-1])
-- and an exponent. In general, if
-- floatToDigits r = ([a, b, ... z], e)
-- then
-- r = 0.ab..z * base^e
--
floatToDigits :: (RealFloat a) => Integer -> a -> ([Int], Int)

```
```

floatToDigits _ 0 = ([], 0)
floatToDigits base x =
let (f0, e0) = decodeFloat x
(minExp0, _) = floatRange x
p = floatDigits x
b = floatRadix x
minExp = minExp0 - p -- the real minimum exponent
-- Haskell requires that f be adjusted so denormalized numbers
-- will have an impossibly low exponent. Adjust for this.
f :: Integer
e :: Int
(f, e) = let n = minExp - e0
in if n > 0 then (f0 `div` (b^n), e0+n) else (f0, e0)
(r, s,mUp, mDn) =
if e >= 0 then
let be = b^e in
if f == b^(p-1) then
(f*be*b*2, 2*b, be*b, b)
else
(f*be*2, 2, be, be)
else
if e > minExp \&\& f == b^(p-1) then
(f*b*2, b^(-e+1)*2, b, 1)
else
(f*2, b^(-e)*2, 1, 1)
k =
let k0 =
if b==2 \&\& base==10 then
-- logBase 10 2 is slightly bigger than 3/10 so
-- the following will err on the low side. Ignoring
-- the fraction will make it err even more.
-- Haskell promises that p-1 <= logBase b f < p.
(p - 1 + e0) * 3 `div` 10
else
ceiling ((log (fromInteger (f+1)) +
fromIntegral e * log (fromInteger b)) /
log (fromInteger base))
fixup n =
if n >= 0 then
if r + mUp <= expt base n * s then n else fixup (n+1)
else
if expt base (-n) * (r + mUp) <= s then n

```
```

                                    else fixup (n+1)
            in fixup k0
        gen ds rn sN mUpN mDnN =
        let (dn, rn') = (rn * base) `divMod` sN
            mUpN' = mUpN * base
            mDnN' = mDnN * base
        in case (rn' < mDnN', rn' + mUpN' > sN) of
            (True, False) -> dn : ds
            (False, True) -> dn+1 : ds
            (True, True) -> if rn' * 2 < sN then dn : ds else dn+1 : ds
            (False, False) -> gen (dn:ds) rn' sN mUpN' mDnN'
        rds =
        if k >= 0 then
            gen [] r (s * expt base k) mUp mDn
        else
            let bk = expt base (-k)
            in gen [] (r * bk) s (mUp * bk) (mDn * bk)
    in (map fromIntegral (reverse rds), k)
    -- This floating point reader uses a less restrictive syntax for floating
-- point than the Haskell lexer. The '.' is optional.
readFloat :: (RealFrac a) => ReadS a
readFloat r = [(fromRational ((n%1)*10^^(k-d)),t) | (n,d,s) <- readFix r,
(k,t) <- readExp s] ++
[ (0/0, t) | ("NaN",t) <- lex r] ++
[ (1/0, t) | ("Infinity",t) <- lex r]
where
readFix r = [(read (ds++ds'), length ds', t)
| (ds,d) <- lexDigits r,
(ds',t) <- lexFrac d ]
lexFrac ('.':ds) = lexDigits ds
lexFrac s = [("",s)]
readExp (e:s) | e `elem' "eE" = readExp' s
readExp s = [(0,s)]
readExp' ('_':s) = [(-k,t) | (k,t) <- readDec s]
readExp' ('+':s) = readDec s
readExp' s = readDec s
lexDigits :: ReadS String
lexDigits = nonnull isDigit
nonnull :: (Char -> Bool) -> ReadS String
nonnull p s = [(cs,t) | (cs@(_:_),t) <- [span p s]]

```

\section*{Chapter 15}

\section*{Indexing Operations}
```

module Ix ( Ix(range, index, inRange, rangeSize) ) where
class Ord a => Ix a where
range :: (a,a) -> [a]
index :: (a,a) -> a -> Int
inRange :: (a,a) -> a -> Bool
rangeSize :: (a,a) -> Int
instance Ix Char where ...
instance Ix Int where ...
instance Ix Integer where ...
instance (Ix a, Ix b) => Ix (a,b) where ...
-- et cetera
instance Ix Bool where ...
instance Ix Ordering where ...

```

The Ix class is used to map a contiguous subrange of values in a type onto integers. It is used primarily for array indexing (see Chapter 16). The Ix class contains the methods range, index, and inRange. The index operation maps a bounding pair, which defines the lower and upper bounds of the range, and a subscript, to an integer. The range operation enumerates all subscripts; the inRange operation tells whether a particular subscript lies in the range defined by a bounding pair.

An implementation is entitled to assume the following laws about these operations:
```

range (l,u) !! index (l,u) i == i -- when i is in range
inRange (l,u) i == i `elem` range (l,u)
map index (range (l,u)) == [0..rangeSize (l,u)]

```

\subsection*{15.1 Deriving Instances of Ix}

It is possible to derive an instance of Ix automatically, using a deriving clause on a data declaration (Section 4.3.3). Such derived instance declarations for the class Ix are only possible for enumerations (i.e. datatypes having only nullary constructors) and single-constructor datatypes, whose constituent types are instances of Ix. A Haskell implementation must provide Ix instances for tuples up to at least size 15 .
- For an enumeration, the nullary constructors are assumed to be numbered left-to-right with the indices being 0 to \(n-1\) inclusive. This is the same numbering defined by the Enum class. For example, given the datatype:
\[
\text { data Colour }=\text { Red } \mid \text { Orange } \mid \text { Yellow } \mid \text { Green } \mid \text { Blue } \mid \text { Indigo } \mid \text { Violet }
\]
we would have:
```

range (Yellow,Blue) == [Yellow,Green,Blue]
index (Yellow,Blue) Green == 1
inRange (Yellow,Blue) Red == False

```
- For single-constructor datatypes, the derived instance declarations are as shown for tuples in Figure 15.1.
```

instance (Ix a, Ix b) => Ix (a,b) where
range ((l, l'),(u, u'))
= [(i,i') | i <- range (l,u), i' <- range (l',u')]
index ((l, l'),(u,u')) (i,i')
= index (l,u) i * rangeSize (l',u') + index ( ( ', ,u') i
inRange ((l, l'),(u,u')) (i,i')
= inRange (l,u) i \&\& inRange (l',u') i'
-- Instances for other tuples are obtained from this scheme:
--
instance (Ix a1, Ix a2, ... , Ix ak) => Ix (a1,a2,...,ak) where
range ((l1,l2,···.,lk),(u1,u2,···,uk)) =
[(i1,i2,...,ik) | i1 <- range (l1,u1),
i2 <- range (l2,u2),
...
ik <- range (lk,uk)]
index ((l1,l2,...,lk),(u1,u2,...,uk)) (i1,i2,...,ik) =
index (lk,uk) ik + rangeSize (lk,uk) * (
index (lk-1,uk-1) ik-1 + rangeSize (lk-1,uk-1) * (
...
index (l1,u1)))
inRange ((l1,l2,...lk),(u1,u2,...,uk)) (i1,i2,...,ik) =
inRange (l1,u1) i1 \&\& inRange (l2,u2) i2 \&\&
... \&\& inRange (lk,uk) ik

```

Figure 15.1: Derivation of Ix instances

\subsection*{15.2 Library Ix}
```

module Ix ( Ix(range, index, inRange, rangeSize) ) where
class Ord a => Ix a where
range :: (a,a) -> [a]
index :: (a,a) -> a -> Int
inRange :: (a,a) -> a -> Bool
rangeSize :: (a,a) -> Int
rangeSize b@(l,h) | null (range b) = 0
otherwise = index b h + 1
-- NB: replacing "null (range b)" by "not (l <= h)"
-- fails if the bounds are tuples. For example,
-- (1,2) <= (2,1)
-- but the range is nevertheless empty
-- range ((1,2),(2,1)) = []
instance Ix Char where
range (m,n) = [m..n]
index b@(c,c') ci
| inRange b ci = fromEnum ci - fromEnum c
otherwise = error "Ix.index: Index out of range."
inRange (c,C') i = c <= i \&\& i <= c'
instance Ix Int where
range (m,n) = [m..n]
index b@(m,n) i
inRange b i = i - m
| otherwise = error "Ix.index: Index out of range."
inRange (m,n) i = m <= i \&\& i <= n
instance Ix Integer where
range (m,n) = [m..n]
index b@(m,n) i
inRange b i = fromInteger (i - m)
| otherwise = error "Ix.index: Index out of range."
inRange (m,n) i = m <= i \&\& i <= n
instance (Ix a,Ix b) => Ix (a, b) -- as derived, for all tuples
instance Ix Bool -- as derived
instance Ix Ordering -- as derived
instance Ix () -- as derived

```

\section*{Chapter 16}

\section*{Arrays}
```

module Array (
module Ix, -- export all of Ix for convenience
Array, array, listArray, (!), bounds, indices, elems, assocs,
accumArray, (//), accum, ixmap ) where
import Ix
infixl 9 !, //
data (Ix a) => Array a b = ... -- Abstract
array :: (Ix a) => (a,a) -> [(a,b)] -> Array a b
listArray :: (Ix a) => (a,a) -> [b] -> Array a b
(!) :: (Ix a) => Array a b -> a -> b
bounds :: (Ix a) => Array a b -> (a,a)
indices :: (Ix a) => Array a b -> [a]
elems :: (Ix a) => Array a b -> [b]
assocs :: (Ix a) => Array a b -> [(a,b)]
accumArray :: (Ix a) => (b -> c -> b) -> b -> (a,a) -> [(a,c)]
-> Array a b
(//) :: (Ix a) => Array a b -> [(a,b)] -> Array a b
accum :: (Ix a) => (b -> c -> b) -> Array a b -> [(a,c)]
-> Array a b
ixmap :: (Ix a, Ix b) => (a,a) -> (a -> b) -> Array b c
-> Array a c
instance Functor (Array a) where ...
instance (Ix a, Eq b) => Eq (Array a b) where ...
instance (Ix a, Ord b) => Ord (Array a b) where ...
instance (Ix a, Show a, Show b) => Show (Array a b) where ...
instance (Ix a, Read a, Read b) => Read (Array a b) where ...

```

Haskell provides indexable arrays, which may be thought of as functions whose domains are isomorphic to contiguous subsets of the integers. Functions restricted in this way can be implemented efficiently; in particular, a programmer may reasonably expect rapid access to the components. To ensure the possibility of such an implementation, arrays are treated as data, not as general functions.

Since most array functions involve the class Ix, this module is exported from Array so that modules need not import both Array and Ix.

\subsection*{16.1 Array Construction}

If \(a\) is an index type and \(b\) is any type, the type of arrays with indices in a and elements in \(b\) is written Array a b. An array may be created by the function array. The first argument of array is a pair of bounds, each of the index type of the array. These bounds are the lowest and highest indices in the array, in that order. For example, a one-origin vector of length 10 has bounds \((1,10)\), and a one-origin 10 by 10 matrix has bounds \(((1,1),(10,10)\) ).

The second argument of array is a list of associations of the form (index, value). Typically, this list will be expressed as a comprehension. An association (i, x) defines the value of the array at index \(i\) to be \(x\). The array is undefined (i.e. \(\perp\) ) if any index in the list is out of bounds. If any two associations in the list have the same index, the value at that index is undefined (i.e. \(\perp\) ). Because the indices must be checked for these errors, array is strict in the bounds argument and in the indices of the association list, but nonstrict in the values. Thus, recurrences such as the following are possible:
\[
a=\operatorname{array}(1,100)((1,1):[(i, i \not * a!(i-1)) \mid i<-[2 . .100]])
\]

Not every index within the bounds of the array need appear in the association list, but the values associated with indices that do not appear will be undefined (i.e. \(\perp\) ). Figure 16.1 shows some examples that use the array constructor.

The (!) operator denotes array subscripting. The bounds function applied to an array returns its bounds. The functions indices, elems, and assocs, when applied to an array, return lists of the indices, elements, or associations, respectively, in index order. An array may be constructed from a pair of bounds and a list of values in index order using the function listArray.

If, in any dimension, the lower bound is greater than the upper bound, then the array is legal, but empty. Indexing an empty array always gives an array-bounds error, but bounds still yields the bounds with which the array was constructed.

\subsection*{16.1.1 Accumulated Arrays}

Another array creation function, accumArray, relaxes the restriction that a given index may appear at most once in the association list, using an accumulating function which combines the values of associations with the same index. The first argument of accumArray is the accumulating
```

-- Scaling an array of numbers by a given number:
scale :: (Num a, Ix b) => a -> Array b a -> Array b a
scale x a = array b [(i, a!i * x) | i <- range b]
where b = bounds a
-- Inverting an array that holds a permutation of its indices
invPerm :: (Ix a) => Array a a -> Array a a
invPerm a = array b [(a!i, i) | i <- range b]
where b = bounds a
-- The inner product of two vectors
inner :: (Ix a, Num b) => Array a b -> Array a b -> b
inner v w = if b == bounds w
then sum [v!i * w!i | i <- range b]
else error "inconformable arrays for inner product"
where b = bounds v

```

Figure 16.1: Array examples
function; the second is an initial value; the remaining two arguments are a bounds pair and an association list, as for the array function. For example, given a list of values of some index type, hist produces a histogram of the number of occurrences of each index within a specified range:
```

hist :: (Ix a, Num b) => (a,a) -> [a] -> Array a b
hist bnds is = accumArray (+) 0 bnds [(i, 1) | i<-is, inRange bnds i]

```

If the accumulating function is strict, then accumArray is strict in the values, as well as the indices, in the association list. Thus, unlike ordinary arrays, accumulated arrays should not in general be recursive.

\subsection*{16.2 Incremental Array Updates}

The operator (//) takes an array and a list of pairs and returns an array identical to the left argument except that it has been updated by the associations in the right argument. (As with the array function, the indices in the association list must be unique for the updated elements to be defined.) For example, if \(m\) is a 1 -origin, \(n\) by \(n\) matrix, then \(m / /[((i, i), 0) \mid i<-[1 \ldots n]]\) is the same matrix, except with the diagonal zeroed.
accum \(f\) takes an array and an association list and accumulates pairs from the list into the array with the accumulating function \(f\). Thus accumArray can be defined using accum:
```

accumArray f z b = accum f (array b [(i, z) | i <- range b])

```
```

-- A rectangular subarray
subArray :: (Ix a) => (a,a) -> Array a b -> Array a b
subArray bnds = ixmap bnds (\i->i)
-- A row of a matrix
row :: (Ix a, Ix b) => a }->>\mathrm{ Array (a,b) c -> Array b c
row i x = ixmap ( l',u') (\j-> (i,j)) x where ((_, l'),(_,u')) = bounds x
-- Diagonal of a matrix (assumed to be square)
diag :: (Ix a) => Array (a,a) b -> Array a b
diag x = ixmap (l,u) (\i->(i,i)) x
where
((l,_),(u,_)) = bounds x
-- Projection of first components of an array of pairs
firstArray :: (Ix a) => Array a (b,c) -> Array a b
firstArray = fmap (\(x,y)->x)

```

Figure 16.2: Derived array examples

\subsection*{16.3 Derived Arrays}

The two functions fmap and ixmap derive new arrays from existing ones; they may be thought of as providing function composition on the left and right, respectively, with the mapping that the original array embodies. The fmap function transforms the array values while ixmap allows for transformations on array indices. Figure 16.2 shows some examples.

\subsection*{16.4 Library Array}
```

module Array (
module Ix, -- export all of Ix
Array, array, listArray, (!), bounds, indices, elems, assocs,
accumArray, (//), accum, ixmap ) where
import Ix
import List( (<br>) )
infixl 9 !, //
data (Ix a) => Array a b = MkArray (a,a) (a -> b) deriving ()

```
```

array :: (Ix a) => (a,a) -> [(a,b)] -> Array a b
array b ivs =
if and [inRange b i | (i,_) <- ivs]
then MkArray b
(\j -> case [v | (i,v) <- ivs, i == j] of
[v] -> v
[] -> error "Array.!: \
\undefined array element"
_ -> error "Array.!: \
\multiply defined array element")
else error "Array.array: out-of-range array association"
listArray :: (Ix a) => (a,a) -> [b] -> Array a b
listArray b vs = array b (zipWith (\ a b -> (a,b)) (range b) vs)
(!)
(!) (MkArray - f) = f
bounds :: (Ix a) => Array a b -> (a,a)
bounds (MkArray b _) = b
indices :: (Ix a) => Array a b -> [a]
indices = range . bounds
elems :: (Ix a) => Array a b -> [b]
elems a = [a!i | i <- indices a]
assocs :: (Ix a) => Array a b -> [(a,b)]
assocs a = [(i, a!i) | <- indices a]
(//) :: (Ix a) => Array a b -> [(a,b)] -> Array a b
a // new_ivs = array (bounds a) (old_ivs ++ new_ivs)
where
old_ivs = [(i,a!i) | i <- indices a,
i `notElem` new_is]
new_is = [i | (i,_) <- new_ivs]
accum :: (Ix a) => (b -> c -> b) -> Array a b -> [(a,c)]
-> Array a b
accum f = foldl (\a (i,v) -> a // [(i,f (a!i) v)])
accumArray :: (Ix a) => (b -> c -> b) -> b -> (a,a) -> [(a,c)]
-> Array a b
accumArray f z b = accum f (array b [(i,z) | i <- range b])
ixmap :: (Ix a, Ix b) => (a,a) -> (a -> b) -> Array b c
-> Array a c
ixmap b f a = array b [(i, a ! f i) | i <- range b]
instance (Ix a) => Functor (Array a) where
fmap fn (MkArray b f) = MkArray b (fn . f)
instance (Ix a, Eq b) => Eq (Array a b) where
a == a' = assocs a == assocs a'

```
```

instance (Ix a, Ord b) => Ord (Array a b) where
a <= a' = assocs a <= assocs a'
instance (Ix a, Show a, Show b) => Show (Array a b) where
showsPrec p a = showParen (p > arrPrec) (
showString "array " .
showsPrec (arrPrec+1) (bounds a) . showChar ' ' .
showsPrec (arrPrec+1) (assocs a)
instance (Ix a, Read a, Read b) => Read (Array a b) where
readsPrec p = readParen (p > arrPrec)
(\r -> [ (array b as, u)
| ("array",s) <- lex r,
(b,t) <- readsPrec (arrPrec+1) s,
(as,u) <- readsPrec (arrPrec+1) t ])
-- Precedence of the 'array' function is that of application itself
arrPrec = 10

```

\section*{Chapter 17}

\section*{List Utilities}
```

module List (
elemIndex, elemIndices,
find, findIndex, findIndices,
nub, nubBy, delete, deleteBy, (<br>), deleteFirstsBy,
union, unionBy, intersect, intersectBy,
intersperse, transpose, partition, group, groupBy,
inits, tails, isPrefixOf, isSuffixOf,
mapAccumL, mapAccumR,
sort, sortBy, insert, insertBy, maximumBy, minimumBy,
genericLength, genericTake, genericDrop,
genericSplitAt, genericIndex, genericReplicate,
zip4, zip5, zip6, zip7,
zipWith4, zipWith5, zipWith6, zipWith7,
unzip4, unzip5, unzip6, unzip7, unfoldr,
-- ...and what the Prelude exports
-- []((:), []), -- This is built-in syntax
map, (++), concat, filter,
head, last, tail, init, null, length, (!!),
foldl, foldl1, scanl, scanl1, foldr, foldr1, scanr, scanr1,
iterate, repeat, replicate, cycle,
take, drop, splitAt, takeWhile, dropWhile, span, break,
lines, words, unlines, unwords, reverse, and, or,
any, all, elem, notElem, lookup,
sum, product, maximum, minimum, concatMap,
zip, zip3, zipWith, zipWith3, unzip, unzip3
) where
infix 5 <br>
elemIndex :: Eq a => a -> [a] -> Maybe Int
elemIndices :: Eq a => a -> [a] -> [Int]
find :: (a -> Bool) -> [a] -> Maybe a
findIndex :: (a -> Bool) -> [a] -> Maybe Int
findIndices :: (a -> Bool) -> [a] -> [Int]
nub
:: Eq a => [a] -> [a]
nubBy :: (a -> a -> Bool) -> [a] -> [a]
delete :: Eq a => a -> [a] -> [a]
deleteBy :: (a -> a -> Bool) -> a -> [a] -> [a]
(<br>) :: Eq a => [a] -> [a] -> [a]
deleteFirstsBy :: (a -> a -> Bool) -> [a] -> [a] -> [a]
union :: Eq a => [a] -> [a] -> [a]
unionBy :: (a -> a -> Bool) -> [a] -> [a] -> [a]

```
\begin{tabular}{|c|c|}
\hline intersect & : : Eq a => [a] -> [a] -> [a] \\
\hline intersectBy & :: (a -> a -> Bool) -> [a] -> [a] -> [a] \\
\hline intersperse & :: a -> [a] -> [a] \\
\hline transpose & : : [ [a]] -> [[a]] \\
\hline partition & :: (a -> Bool) -> [a] -> ([a], [a]) \\
\hline group & :: Eq a => [a] -> [[a]] \\
\hline groupBy & :: (a -> a -> Bool) -> [a] -> [[a]] \\
\hline inits & :: [a] -> [ [a]] \\
\hline tails & : : [a] -> [ [a]] \\
\hline isPrefixOf & :: Eq a => [a] -> [a] -> Bool \\
\hline isSuffixOf & :: Eq a => [a] -> [a] -> Bool \\
\hline mapAccumL & :: (a -> b -> (a, c) ) -> a -> [b] -> (a, [c]) \\
\hline mapAccumR & :: (a -> b -> ( \(\mathrm{a}, \mathrm{c}\) ) ) -> a -> [b] -> (a, [c]) \\
\hline unfoldr & :: (b -> Maybe (a,b)) -> b -> [a] \\
\hline sort & :: Ord a => [a] -> [a] \\
\hline sortBy & :: (a -> a -> Ordering) -> [a] -> [a] \\
\hline insert & :: Ord a => a -> [a] -> [a] \\
\hline insertBy & :: (a -> a -> Ordering) -> a -> [a] -> [a] \\
\hline maximumby & :: (a -> a -> Ordering) -> [a] -> a \\
\hline minimumby & :: (a -> a -> Ordering) -> [a] -> a \\
\hline genericLength & : : Integral a => [b] -> a \\
\hline generictake & :: Integral a => a -> [b] -> [b] \\
\hline genericDrop & :: Integral a \(=>\) a \(->\) [b] -> [b] \\
\hline genericSplitAt & :: Integral a => a -> [b] -> ([b], [b]) \\
\hline genericIndex & :: Integral a => [b] -> a -> b \\
\hline genericReplicate & :: Integral a => a -> b -> [b] \\
\hline zip4 & :: [a] -> [b] -> [c] -> [d] -> [ \((a, b, c, d)\) ] \\
\hline zip5 & :: [a] -> [b] -> [c] -> [d] -> [e] -> [(a,b, c, d, e)] \\
\hline zip6 & :: [a] -> [b] -> [c] -> [d] -> [e] -> [f] \\
\hline & -> [ \((a, b, c, d, e, f)]\) \\
\hline zip7 & :: [a] -> [b] -> [c] -> [d] -> [e] -> [f] -> [g] \\
\hline & \(\rightarrow\) [ \((a, b, c, d, e, f, g)]\) \\
\hline zipWith4 & :: (a->b->c->d->e) -> [a]-> [b]-> [c]-> [d]-> [e] \\
\hline zipWith5 & : : (a->b->c->d->e->f) -> \\
\hline & [a]-> [b]-> [c]-> [d]-> [e]-> [f] \\
\hline zipWith6 & \(::(a->b->c->d->e->f->g)->\) \\
\hline & [a]-> [b]-> [c]-> [d]-> [e]-> [f]-> [g] \\
\hline zipWith7 & : : (a->b->c->d->e->f->g->h) -> \\
\hline & [a]-> [b]-> [c]-> [d]-> [e]-> [f]-> [g]-> [h] \\
\hline unzip4 & : : [ \(\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\) ) ] \(\rightarrow\) ( \([\mathrm{a}],[\mathrm{b}],[\mathrm{c}],[\mathrm{d}])\) \\
\hline unzip5 & :: [ (a,b,c,d,e)] -> ([a], [b], [c], [d], [e]) \\
\hline unzip6 & : : [ \(\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}, \mathrm{f})]\)-> ([a],[b],[c],[d],[e],[f]) \\
\hline unzip7 & : : [ (a,b,c,d,e,f,g)] -> ([a], [b], [c], [d], [e], [f], [g]) \\
\hline
\end{tabular}

This library defines some lesser-used operations over lists.

\subsection*{17.1 Indexing lists}
- elemIndex val list returns the index of the first occurrence, if any, of val in list as Just index. Nothing is returned if not (val `elem` list).
- elemIndices val list returns an in-order list of indices, giving the occurrences of val in list.
- find returns the first element of a list that satisfies a predicate, or Nothing, if there is no such element. findIndex returns the corresponding index. findIndices returns a list of all such indices.

\section*{17.2 "Set" operations}

There are a number of "set" operations defined over the List type. nub (meaning "essence") removes duplicates elements from a list. delete, ( \(\backslash \backslash\) ), union and intersect (and their By variants) preserve the invariant that their result does not contain duplicates, provided that their first argument contains no duplicates.
- nub removes duplicate elements from a list. For example:
\[
\text { nu.b }[1,3,1,4,3,3]=[1,3,4]
\]
- delete x removes the first occurrence of x from its list argument, e.g.,
```

delete 'a' "banana" == "bnana"

```
- ( \(\backslash \backslash\) ) is list difference (non-associative). In the result of \(\mathrm{xs} \backslash \backslash \mathrm{ys}\), the first occurrence of each element of ys in turn (if any) has been removed from xs. Thus, (xs ++ ys) \(\backslash \backslash x s==y s\).
- union is list union, e.g.,
"dog" `union` "cow" == "dogcw"
- intersect is list intersection, e.g.,
\[
[1,2,3,4] \text { 'intersect` }[2,4,6,8]==[2,4]
\]

\subsection*{17.3 List transformations}
- intersperse sep inserts sep between the elements of its list argument, e.g.,
intersperse ',' "abcde" == "a,b,c,d,e"
- transpose transposes the rows and columns of its argument, e.g.,
```

transpose [[1,2,3],[4,5,6]] == [[1,4],[2,5],[3,6]]

```
- partition takes a predicate and a list and returns a pair of lists: those elements of the argument list that do and do not satisfy the predicate, respectively; i.e.,
```

partition p xs == (filter p xs, filter (not . p) xs)

```
- sort implement a stable sorting algorithm, here specified in terms of the insertBy function, which inserts objects into a list according to the specified ordering relation.
- insert inserts a new element into an ordered list (arranged in increasing order).
- group splits its list argument into a list of lists of equal, adjacent elements. For example
group "Mississippi" == ["M","i","ss","i","ss","i","pp","i"]
- inits returns the list of initial segments of its argument list, shortest first.
```

inits "abc" == ["","a","ab","abc"]

```
- tails returns the list of all final segments of its argument list, longest first.
tails "abc" == ["abc", "bc", "c",""]
- mapAccumL f \(s\) lapplies \(f\) to an accumulating "state" parameter \(s\) and to each element of 1 in turn.
- mapAccumR is similar to mapAccumL except that the list is processed from right-to-left rather than left-to-right.

\section*{17.4 unfoldr}

The unfoldr function is a "dual" to foldr: while foldr reduces a list to a summary value, unfoldr builds a list from a seed value. For example:
```

iterate f == unfoldr (\x -> Just (x, f x))

```

In some cases, unfoldr can undo a foldr operation:
```

unfoldr f' (foldr f z xs) == xs

```
if the following holds:
```

f' (f x y) = Just (x,y)
f'z = Nothing

```

\subsection*{17.5 Predicates}
isPrefixOf and isSuffixOf check whether the first argument is a prefix (resp. suffix) of the second argument.

\subsection*{17.6 The "By" operations}

By convention, overloaded functions have a non-overloaded counterpart whose name is suffixed with "By". For example, the function nub could be defined as follows:
```

nub :: (Eq a) => [a] -> [a]
nub [] = []
nub (x:xs) = x : nub (filter (\y }->>\operatorname{not (x == y)) xs)

```

However, the equality method may not be appropriate in all situations. The function:
```

nubBy :: (a -> a -> Bool) -> [a] -> [a]
nubBy eq [] = []
nubBy eq (x:xs) = x : nubBy eq (filter (\y -> not (eq x y)) xs)

```
allows the programmer to supply their own equality test. When the "By" function replaces an Eq context by a binary predicate, the predicate is assumed to define an equivalence; when the "By" function replaces an Ord context by a binary predicate, the predicate is assumed to define a total ordering.

The "By" variants are as follows: nubBy, deleteBy, deleteFirstsBy (the By variant of \\), unionBy, intersectBy, groupBy, sortBy, insertBy, maximumBy, minimumBy.

The library does not provide elemBy, because any (eq x) does the same job as elemBy eq x would. A handful of overloaded functions (elemIndex, elemIndices, isPrefixOf, isSuffix \(0 f\) ) were not considered important enough to have "By" variants.

\subsection*{17.7 The "generic" operations}

The prefix "generic" indicates an overloaded function that is a generalised version of a Prelude function. For example,
```

genericLength :: Integral a => [b] -> a

```
is a generalised version of length.
The "generic" operations are as follows: genericLength, genericTake, genericDrop, genericSplitAt, genericIndex (the generic version of !!), genericReplicate.

\subsection*{17.8 Further "zip" operations}

The Prelude provides zip, zip3, unzip, unzip3, zipWith, and zipWith3. The List library provides these same three operations for \(4,5,6\), and 7 arguments.

\subsection*{17.9 Library List}
```

module List (
elemIndex, elemIndices,
find, findIndex, findIndices,
nub, nubBy, delete, deleteBy, (<br>), deleteFirstsBy,
union, unionBy, intersect, intersectBy,
intersperse, transpose, partition, group, groupBy,
inits, tails, isPrefixOf, isSuffixOf,
mapAccumL, mapAccumR,
sort, sortBy, insert, insertBy, maximumBy, minimumBy,
genericLength, genericTake, genericDrop,
genericSplitAt, genericIndex, genericReplicate,
zip4, zip5, zip6, zip7,
zipWith4, zipWith5, zipWith6, zipWith7,
unzip4, unzip5, unzip6, unzip7, unfoldr,
-- ...and what the Prelude exports
-- []((:), []), -- This is built-in syntax
map, (++), concat, filter,
head, last, tail, init, null, length, (!!),
foldl, foldl1, scanl, scanl1, foldr, foldr1, scanr, scanr1,
iterate, repeat, replicate, cycle,
take, drop, splitAt, takeWhile, dropWhile, span, break,
lines, words, unlines, unwords, reverse, and, or,
any, all, elem, notElem, lookup,
sum, product, maximum, minimum, concatMap,
zip, zip3, zipWith, zipWith3, unzip, unzip3
) where
import Maybe( listToMaybe )
infix 5 <br>
elemIndex :: Eq a => a -> [a] -> Maybe Int
elemIndex x = findIndex (x ==)
elemIndices :: Eq a => a -> [a] -> [Int]
elemIndices x = findIndices (x ==)
find :: (a -> Bool) -> [a] -> Maybe a
find p = listToMaybe . filter p
findIndex :: (a -> Bool) -> [a] -> Maybe Int
findIndex p = listToMaybe . findIndices p
findIndices :: (a -> Bool) -> [a] -> [Int]
findIndices p xs = [ i | (x,i) <- zip xs [0..], p x ]
nub :: Eq a => [a] -> [a]
nub = nubBy (==)

```
```

nubBy
:: (a -> a -> Bool) -> [a] -> [a]
nubBy eq [] = []
nubBy eq (x:xs) = x : nubBy eq (filter (\y -> not (eq x y)) xs)
delete :: Eq a => a -> [a] -> [a]
delete = deleteBy (==)
deleteBy :: (a -> a -> Bool) -> a -> [a] -> [a]
deleteBy eq x [] = []
deleteBy eq x (y:ys) = if x `eq` y then ys else y : deleteBy eq x ys
(<br>) :: Eq a => [a] -> [a] -> [a]
(<br>) = foldl (flip delete)
deleteFirstsBy :: (a -> a -> Bool) -> [a] -> [a] -> [a]
deleteFirstsBy eq = foldl (flip (deleteBy eq))
union :: Eq a => [a] -> [a] -> [a]
union = unionBy (==)
unionBy :: (a -> a -> Bool) -> [a] -> [a] -> [a]
unionBy eq xs ys = xs ++ deleteFirstsBy eq (nubBy eq ys) xs
intersect :: Eq a => [a] -> [a] -> [a]
intersect = intersectBy (==)
intersectBy :: (a -> a -> Bool) -> [a] -> [a] -> [a]
intersectBy eq xs ys = [x | x <- xs, any (eq x) ys]
intersperse :: a -> [a] -> [a]
intersperse sep [] = []
intersperse sep [x] = [x]
intersperse sep (x:xs) = x : sep : intersperse sep xs
-- transpose is lazy in both rows and columns,
-- and works for non-rectangular 'matrices'
-- For example, transpose [[1,2],[3,4,5],[]] = [[1,3],[2,4],[5]]
-- Note that [h | (h:t) <- xss] is not the same as (map head xss)
-- because the former discards empty sublists inside xss
transpose :: [[a]] -> [[a]]
transpose [] = []
transpose ([] : xss) = transpose xss
transpose ((x:xs) : xss) = (x : [h | (h:t) <- xss]) :
transpose (xs : [t | (h:t) <- xss])
partition :: (a -> Bool) -> [a] -> ([a],[a])
partition p xs = (filter p xs, filter (not . p) xs)
-- group splits its list argument into a list of lists of equal, adjacent
-- elements. e.g.,
-- group "Mississippi" == ["M","i","ss","i","ss","i","pp","i"]
group :: Eq a => [a] -> [[a]]
group = groupBy (==)

```
```

groupBy :: (a -> a -> Bool) -> [a] -> [[a]]
groupBy eq [] = []
groupBy eq (x:xs) = (x:ys) : groupBy eq zs
where (ys,zs) = span (eq x) xs
-- inits xs returns the list of initial segments of xs, shortest first.
-- e.g., inits "abc" == ["","a","ab","abc"]
inits :: [a] -> [[a]]
inits [] = [[]]
inits (x:xs) = [[]] ++ map (x:) (inits xs)
-- tails xs returns the list of all final segments of xs, longest first.
-- e.g., tails "abc" == ["abc", "bc", "c",""]
tails :: [a] -> [[a]]
tails [] = [[]]
tails xxs@(_:xs) = xxs : tails xs
isPrefixOf :: Eq a => [a] -> [a] -> Bool
isPrefixOf [] - = True
isPrefixOf [] = False
isPrefixOf (x:xs) (y:ys) = x == y \&\& isPrefixOf xs ys
isSuffixOf :: Eq a => [a] -> [a] -> Bool
isSuffixOf x y = reverse x `isPrefixOf` reverse y
mapAccumL :: (a -> b -> (a, c)) -> a -> [b] -> (a, [c])
mapAccumL f s [] = (s, [])
mapAccumL f s (x:xs) = (s'r,y:ys)
where (s', y ) = f s x
(s'',ys) = mapAccumL f s' xs
mapAccumR :: (a -> b -> (a, c)) -> a -> [b] -> (a, [c])
mapAccumR f s [] = (s, [])
mapAccumR f s (x:xs) = (s'', y:ys)
where (s'r,y ) = f s' x
(s', ys) = mapAccumR f s xs
unfoldr :: (b -> Maybe (a,b)) -> b -> [a]
unfoldr f b = case f b of
Nothing -> []
Just (a,b) -> a : unfoldr f b
sort :: (Ord a) => [a] -> [a]
sort = sortBy compare
sortBy :: (a -> a -> Ordering) -> [a] -> [a]
sortBy cmp = foldr (insertBy cmp) []
insert :: (Ord a) => a -> [a] -> [a]
insert = insertBy compare

```
```

insertBy :: (a -> a -> Ordering) -> a -> [a] -> [a]
insertBy cmp x [] = [x]
insertBy cmp x ys@(y:ys')
= case cmp x y of
GT -> y : insertBy cmp x ys'
_ -> x : ys
maximumBy :: (a -> a -> Ordering) -> [a] -> a
maximumBy cmp [] = error "List.maximumBy: empty list"
maximumBy cmp xs = foldl1 max xs
where
max x y = case cmp x y of
GT -> x
_ -> y
minimumBy :: (a -> a -> Ordering) -> [a] -> a
minimumBy cmp [] = error "List.minimumBy: empty list"
minimumBy cmp xs = foldl1 min xs
where
min x y = case cmp x y of
GT -> y
_ -> x
genericLength :: (Integral a) => [b] -> a
genericLength [] = 0
genericLength (x:xs) = 1 + genericLength xs
genericTake :: (Integral a) => a -> [b] -> [b]
genericTake _ [] = []
genericTake 0 _
= []
genericTake n (x:xs)
| > 0 = x : genericTake (n-1) xs
| otherwise = error "List.genericTake: negative argument"
genericDrop :: (Integral a) => a -> [b] -> [b]
genericDrop 0 xs = xs
genericDrop _ [] = []
genericDrop n (_:xs)
| > 0 = genericDrop (n-1) xs
otherwise = error "List.genericDrop: negative argument"
genericSplitAt :: (Integral a) => a -> [b] -> ([b],[b])
genericSplitAt 0 xs = ([],xs)
genericSplitAt _ [] = ([],[])
genericSplitAt n (x:xs)
n > 0 = (x:xs', xs'')
otherwise = error "List.genericSplitAt: negative argument"
where (xs',xs'') = genericSplitAt (n-1) xs

```
```

genericIndex :: (Integral a) => [b] -> a -> b
genericIndex (x:_) 0 = x
genericIndex (_:xs) n
n > 0 = genericIndex xs (n-1)
| otherwise = error "List.genericIndex: negative argument"
genericIndex _ _ = error "List.genericIndex: index too large"
genericReplicate :: (Integral a) => a -> b -> [b]
genericReplicate n x = genericTake n (repeat x)
zip4 :: [a] -> [b] -> [c] -> [d] -> [(a,b,c,d)]
zip4 = zipWith4 (,,,)
zip5 :: [a] -> [b] -> [c] -> [d] -> [e] -> [(a,b,c,d,e)]
zip5 = zipWith5 (,,,,)
zip6 :: [a] -> [b] -> [c] -> [d] -> [e] -> [f] ->
[(a,b,c,d,e,f)]
zip6 = zipWith6 (,,',,)
zip7 :: [a] -> [b] -> [c] -> [d] -> [e] -> [f] ->
[g] -> [(a,b,c,d,e,f,g)]
zip7 = zipWith7 (r,r,r,)
zipWith4 :: (a->b->c->d->e) -> [a]->[b]->[c]->[d]->[e]
zipWith4 z (a:as) (b:bs) (c:cs) (d:ds)
= z a b c d : zipWith4 z as bs cs ds
zipWith4 - - - - - = []
zipWith5 :: (a->b->c->d->e->f) ->
[a]-> [b]-> [c]-> [d]-> [e]-> [f]
zipWith5 z (a:as) (b:bs) (c:cs) (d:ds) (e:es)
= z a b c d e : zipWith5 z as bs cs ds es
zipWith5 _ _ _ _ _ _ = []
zipWith6 :: (a->b->c->d->e->f->g) ->
[a]-> [b]-> [c]-> [d]-> [e]-> [f]-> [g]
zipWith6 z (a:as) (b:bs) (c:cs) (d:ds) (e:es) (f:fs)
= z a b c d e f : zipWith6 z as bs cs ds es fs
zipWith6 _ _ _ _ _ _ _ = []
zipWith7 :: (a->b->c->d->e->f->g->h) ->
[a]-> [b]-> [c]-> [d]-> [e]-> [f]-> [g]-> [h]
zipWith7 z (a:as) (b:bs) (c:cs) (d:ds) (e:es) (f:fs) (g:gs)
= z a b c d e f g : zipWith7 z as bs cs ds es fs gs
zipWith7 _ _ _ _ _ _ _ _ = []
unzip4 :: [(a,b,c,d)] -> ([a],[b],[c],[d])
unzip4 = foldr (\(a,b,c,d) ~ (as,bs,cs,ds) ->
(a:as,b:bs,c:cs,d:ds))
([],[],[],[])

```
```

unzip5 :: [(a,b,c,d,e)] -> ([a],[b],[c],[d],[e])
unzip5 = foldr (\(a,b,c,d,e) ~ (as,bs,cs,ds,es) ->
(a:as,b:bs,c:cs,d:ds,e:es))
([],[],[],[],[])
unzip6 :: [(a,b,c,d,e,f)] -> ([a],[b],[c],[d],[e],[f])
unzip6 = foldr (\ (a,b,c,d,e,f) ~ (as,bs,cs,ds,es,fs) ->
(a:as,b:bs,c:cs,d:ds,e:es,f:fs))
([],[],[],[],[],[])
unzip7 :: [(a,b,c,d,e,f,g)] -> ([a],[b],[c],[d],[e],[f],[g])
unzip7 = foldr (\(a,b,c,d,e,f,g) ~ (as,bs,cs,ds,es,fs,gs) ->
(a:as,b:bs,c:cs,d:ds,e:es,f:fs,g:gs))
([],[],[],[],[],[],[])

```

\section*{Chapter 18}

\section*{Maybe Utilities}
```

module Maybe(
isJust, isNothing,
fromJust, fromMaybe, listToMaybe, maybeToList,
catMaybes, mapMaybe,
-- ...and what the Prelude exports
Maybe(Nothing, Just),
maybe
) where
isJust, isNothing :: Maybe a -> Bool
fromJust :: Maybe a -> a
fromMaybe :: a -> Maybe a -> a
listToMaybe :: [a] -> Maybe a
maybeToList :: Maybe a -> [a]
catMaybes :: [Maybe a] -> [a]
mapMaybe :: (a -> Maybe b) -> [a] -> [b]

```

The type constructor Maybe is defined in Prelude as
```

data Maybe a = Nothing | Just a

```

The purpose of the Maybe type is to provide a method of dealing with illegal or optional values without terminating the program, as would happen if error were used, and without using IOError from the IO monad, which would cause the expression to become monadic. A correct result is encapsulated by wrapping it in Just; an incorrect result is returned as Nothing.

Other operations on Maybe are provided as part of the monadic classes in the Prelude.

\subsection*{18.1 Library Maybe}
```

module Maybe(
isJust, isNothing,
fromJust, fromMaybe, listToMaybe, maybeToList,
catMaybes, mapMaybe,
-- ...and what the Prelude exports
Maybe(Nothing, Just),
maybe
) where
isJust :: Maybe a -> Bool
isJust (Just a) = True
isJust Nothing = False
isNothing :: Maybe a -> Bool
isNothing = not . isJust
fromJust :: Maybe a -> a
fromJust (Just a) = a
fromJust Nothing = error "Maybe.fromJust: Nothing"
fromMaybe :: a -> Maybe a -> a
fromMaybe d Nothing = d
fromMaybe d (Just a) = a
maybeToList :: Maybe a -> [a]
maybeToList Nothing = []
maybeToList (Just a) = [a]
listToMaybe :: [a] -> Maybe a
listToMaybe [] = Nothing
listToMaybe (a:_) = Just a
catMaybes :: [Maybe a] -> [a]
catMaybes ms = [ m | Just m <- ms ]
mapMaybe :: (a -> Maybe b) -> [a] -> [b]
mapMaybe f = catMaybes . map f

```

\section*{Chapter 19}

\section*{Character Utilities}
```

module Char (
isAscii, isLatin1, isControl, isPrint, isSpace, isUpper, isLower,
isAlpha, isDigit, isOctDigit, isHexDigit, isAlphaNum,
digitToInt, intToDigit,
toUpper, toLower,
ord, chr,
readLitChar, showLitChar, lexLitChar,
-- ...and what the Prelude exports
Char, String
) where
isAscii, isLatin1, isControl, isPrint, isSpace, isUpper, isLower,
isAlpha, isDigit, isOctDigit, isHexDigit, isAlphaNum :: Char -> Bool
toUpper, toLower :: Char -> Char
digitToInt :: Char -> Int
intToDigit :: Int -> Char
ord :: Char -> Int
chr :: Int -> Char
lexLitChar :: ReadS String
readLitChar :: ReadS Char
showLitChar :: Char -> ShowS

```

This library provides a limited set of operations on the Unicode character set. The first 128 entries of this character set are identical to the ASCII set; with the next 128 entries comes the remainder of the Latin- 1 character set. This module offers only a limited view of the full Unicode character set; the full set of Unicode character attributes is not accessible in this library.

Unicode characters may be divided into five general categories: non-printing, lower case alphabetic, other alphabetic, numeric digits, and other printable characters. For the purposes of Haskell, any
alphabetic character which is not lower case is treated as upper case (Unicode actually has three cases: upper, lower, and title). Numeric digits may be part of identifiers but digits outside the ASCII range are not used by the reader to represent numbers.

For each sort of Unicode character, here are the predicates which return True:
\begin{tabular}{|l|llll|}
\hline Character Type & Predicates & & \\
\hline Lower Case Alphabetic & isPrint & isAlphaNum & isAlpha & isLower \\
Other Alphabetic & isPrint & isAlphaNum & isAlpha & isUpper \\
Digits & isPrint & isAlphaNum & & \\
Other Printable & isPrint & & & \\
Non-printing & & & \\
\hline
\end{tabular}

The isDigit, isOctDigit, and isHexDigit functions select only ASCII characters. int ToDigit and digitToInt convert between a single digit Char and the corresponding Int. digitToInt operates fails unless its argument satisfies isHexDigit, but recognises both upper and lower-case hexadecimal digits (i.e. \({ }^{\prime} 0^{\prime} . .{ }^{\prime} 9^{\prime},{ }^{\prime} a^{\prime} . . \mathrm{I}^{\prime} \mathrm{f}^{\prime}, \mathrm{A}^{\prime} \mathrm{A}^{\prime} . \mathrm{.l}^{\prime} \mathrm{F}^{\prime}\) ). intToDigit fails unless its argument is in the range \(0 . .15\), and generates lower-case hexadecimal digits.

The isSpace function recognizes only white characters in the Latin-1 range.
The function showLitchar converts a character to a string using only printable characters, using Haskell source-language escape conventions. The function lexLitChar does the reverse, returning the sequence of characters that encode the character. The function readLitchar does the same, but in addition converts the to the character that it encodes. For example:
```

showLitChar '\n's = "<br>n" ++ s
lexLitChar "<br>nHello" = [("<br>n", "Hello")]
readLitChar "<br>nHello" = [('\n', "Hello")]

```

Function toUpper converts a letter to the corresponding upper-case letter, leaving any other character unchanged. Any Unicode letter which has an upper-case equivalent is transformed. Similarly, toLower converts a letter to the corresponding lower-case letter, leaving any other character unchanged.

The ord and chr functions are fromEnum and toEnum restricted to the type Char.

\subsection*{19.1 Library Char}
```

module Char (
isAscii, isLatin1, isControl, isPrint, isSpace, isUpper, isLower,
isAlpha, isDigit, isOctDigit, isHexDigit, isAlphaNum,
digitToInt, intToDigit,
toUpper, toLower,
ord, chr,
readLitChar, showLitChar, lexLitChar,
-- ...and what the Prelude exports
Char, String
) where
import Array -- Used for character name table.
import Numeric (readDec, readOct, lexDigits, readHex)
import UnicodePrims -- Source of primitive Unicode functions.
-- Character-testing operations
isAscii, isLatin1, isControl, isPrint, isSpace, isUpper, isLower,
isAlpha, isDigit, isOctDigit, isHexDigit, isAlphaNum :: Char -> Bool
isAscii c = c < '\x80'
isLatin1 c = c <= '\xff'
isControl c = C < ' ' || C >= '\DEL' \&\& C <= '\x9f'
isPrint = primUnicodeIsPrint
isSpace c = c `elem` " \t\n\r\f\v\xA0"
-- Only Latin-1 spaces recognized
isUpper = primUnicodeIsUpper -- 'A'..'Z'
isLower = primUnicodeIsLower -- 'a'..'z'
isAlpha c = isUpper c || isLower c
isDigit c = c >= '0' \&\& c <= '9'
isOctDigit c = c >= '0' \&\& c <= '7'
isHexDigit c = isDigit c || C >= 'A' \&\& c <= 'F' ||
c >= 'a' \&\& c <= 'f'
isAlphaNum = primUnicodeIsAlphaNum
-- Digit conversion operations
digitToInt :: Char -> Int
digitToInt c
isDigit c = fromEnum c - fromEnum '0'
c >= 'a' \&\& c <= 'f' = fromEnum c - fromEnum 'a' + 10
c >= 'A' \&\& c <= ' F' = fromEnum c - fromEnum 'A' + 10
otherwise = error "Char.digitToInt: not a digit"

```
```

intToDigit :: Int -> Char
intToDigit i
i >= 0 \&\& i <= 9 = toEnum (fromEnum '0' + i)
i >= 10 \&\& i <= 15 = toEnum (fromEnum 'a' + i - 10)
otherwise = error "Char.intToDigit: not a digit"
-- Case-changing operations
toUpper :: Char -> Char
toUpper = primUnicodeToUpper
toLower :: Char -> Char
toLower = primUnicodeToLower
-- Character code functions
ord :: Char -> Int
ord = fromEnum
chr :: Int -> Char
chr = toEnum
-- Text functions
readLitChar :: ReadS Char
readLitChar ('<br>':s) = readEsc s
readLitChar (c:s) = [(c,s)]
readEsc :: ReadS Char
readEsc ('a':s) = [('\a',s)]
readEsc ('b':s) = [('\b', s)]
readEsc ('f':s) = [('\f',s)]
readEsc ('n':s) = [('\n',s)]
readEsc ('r':s) = [('\r',s)]
readEsc ('t':s) = [('\t',s)]
readEsc ('v':s) = [('\v',s)]
readEsc ('<br>''s) = [('<br>',s)]
readEsc ('"':s) = [('"',s)]
readEsc ('\'':s) = [('\'',s)]
readEsc ('^'':c:s) | c >= '@' \&\& c <= '_'
= [(chr (ord c - ord '@'), s)]
readEsc s@(d:_) | isDigit d
= [(chr n, t) ( n,t) <- readDec s]
readEsc ('o':s) = [(chr n, t) | (n,t) <- readOct s]
readEsc ('x':s) = [(chr n, t) | ( n,t) <- readHex s]
readEsc s@(c:_) | isUpper c
= let table = ('\DEL', "DEL") : assocs asciiTab
in case [(c, s') | (c, mne) <- table,
([],s') <- [match mne s]]
of (pr:_) -> [pr]
[] -> []
readEsc _ = []
match :: (Eq a) => [a] -> [a] -> ([a],[a])
match (x:xs) (y:ys) | x == y = match xs ys
match xs ys = (xs,ys)

```
```

showLitChar :: Char -> ShowS
showLitChar c | c > '\DEL' = showChar '<br>'.
protectEsc isDigit (shows (ord c))
showLitChar '\DEL' = showString "<br>DEL"
showLitChar '<br>' = showString "<br><br>"
showLitChar c | c >= , , = showChar c
showLitChar '\a' = showString "<br>a"
showLitChar '\b' = showString "<br>\b"
showLitChar '\f' = showString "<br>£"
showLitChar '\n' = showString "<br>n"
showLitChar '\r' = showString "<br>r"
showLitChar '\t' = showString "<br>t"
showLitChar '\v' = showString "<br>v"
showLitChar '\SO' = protectEsc (== 'H') (showString "<br>SO")
showLitChar c = showString ('<br>' : asciiTab!c)
protectEsc p f = f . cont
where cont s@(c:_) | p c = "<br>\&" ++ s
cont s = s
asciiTab = listArray ('\NUL', ' ')
["NUL", "SOH", "STX", "ETX", "EOT", "ENQ", "ACK", "BEL",
"BS", "HT", "LF", "VT", "FF", "CR", "SO", "SI",
"DLE", "DC1", "DC2", "DC3", "DC4", "NAK", "SYN", "ETB",
"CAN", "EM", "SUB", "ESC", "FS", "GS", "RS", "US",
"SP"]
lexLitChar :: ReadS String
lexLitChar ('<br>':s) = map (prefix '<br>') (lexEsc s)
where
lexEsc (c:s) | c `elem` "abfnrtv<br>\"'" = [([c],s)]
lexEsc ('^'':c:s) | c >= '@' \&\& c <= '_' = [(['^',c],s)]
-- Numeric escapes
lexEsc ('o':s) = [prefix 'o' (span isOctDigit s)]
lexEsc ('x':s) = [prefix 'x' (span isHexDigit s)]
lexEsc s@(d:_) | isDigit d = [span isDigit s]
-- Very crude approximation to \XYZ.
lexEsc s@(c:_) | isUpper c = [span isCharName s]
lexEsc _ = []
isCharName c = isUpper c || isDigit c
prefix c (t,s) = (c:t, s)
lexLitChar (c:s) = [([c],s)]
lexLitChar "" = []

```

\section*{Chapter 20}

\section*{Monad Utilities}
```

module Monad (
MonadPlus(mzero, mplus),
join, guard, when, unless, ap,
msum,
filterM, mapAndUnzipM, zipWithM, zipWithM_, foldM,
liftM, liftM2, liftM3, liftM4, liftM5,
-- ...and what the Prelude exports
Monad((>>=), (>>), return, fail),
Functor(fmap),
mapM, mapM_, sequence, sequence_, (=<<),
) where
class Monad m => MonadPlus m where
mzero :: m a
mplus :: m a -> m a -> m a
join :: Monad m => m (m a) -> m a
guard :: MonadPlus m => Bool -> m ()
when :: Monad m => Bool -> m () -> m ()
unless :: Monad m => Bool -> m () -> m ()
ap :: Monad m m m (a -> b) >> ma mab
mapAndUnzipM :: Monad m => (a -> m (b, c)) -> [a] -> m ([b], [c])
zipWithM :: Monad m => (a -> b -> m c) -> [a] -> [b] -> m [c]
zipWithM_ : Monad m => (a -> b -> m c) -> [a] -> [b] -> m ()
foldM :: Monad m m (a -> b -> ma) -> a -> [b] -> m a
filterM :: Monad m => (a m m Bool) -> [a] -> m [a]
msum :: MonadPlus m => [m a] -> m a
liftM :: Monad m m (a -> b) -> (m a -> m b)
liftM2 :: Monad m => (a -> b -> c) -> (mac>m b m m c)
liftM3 :: Monad m => (a -> b -> c -> d) ->
(m a -> m b -> m c -> m d)
liftM4 :: Monad m => (a -> b -> c -> d -> e) ->
(m a -> m b -> m c -> m d -> m e)
liftM5 :: Monad m => (a -> b -> c -> d -> e -> f) ->
(ma m m b mm c mm d -> m e -> m f

```

The Monad library defines the MonadP lus class, and provides some useful operations on monads.

\subsection*{20.1 Naming conventions}

The functions in this library use the following naming conventions:
- A postfix "M" always stands for a function in the Kleisli category: \(m\) is added to function results (modulo currying) and nowhere else. So, for example,
```

filter :: (a -> Bool) -> [a] -> [a]
filterM :: Monad m => (a -> m Bool) -> [a] -> m [a]

```
- A postfix "_" changes the result type from (ma) to (m ()). Thus (in the Prelude):
```

sequence :: Monad m => [m a] -> m [a]
sequence_ :: Monad m => [m a] -> m ()

```
- A prefix " \(m\) " generalises an existing function to a monadic form. Thus, for example:
```

sum :: Num a => [a] -> a
msum :: MonadPlus m => [m a] -> m a

```

\subsection*{20.2 Class MonadPlus}

The MonadPlus class is defined as follows:
```

class Monad m => MonadPlus m where
mzero :: m a
mplus :: m a -> m a -> m a

```

The class methods mzero and mplus are the zero and plus of the monad.
Lists and the Maybe type are instances of MonadPlus, thus:
```

instance MonadPlus Maybe where
mzero = Nothing
Nothing 'mplus'ys = ys
xs 'mplus` ys = xs
instance MonadPlus [] where
mzero = []
mplus = (++)

```

\subsection*{20.3 Functions}

The join function is the conventional monad join operator. It is used to remove one level of monadic structure, projecting its bound argument into the outer level.

The mapAndUnzipm function maps its first argument over a list, returning the result as a pair of lists. This function is mainly used with complicated data structures or a state-transforming monad.

The zipWithm function generalises zipWith to arbitrary monads. For instance the following function displays a file, prefixing each line with its line number,
```

listFile :: String -> IO ()
listFile nm =
do cts <- readFile nm
zipWithM_ (\i line -> do putStr (show i); putStr ": "; putStrLn line)
[1..]
(lines cts)

```

The foldm function is analogous to foldl, except that its result is encapsulated in a monad. Note that foldm works from left-to-right over the list arguments. This could be an issue where (>>) and the "folded function" are not commutative.
```

        foldM f al [x1, x2, ..., xm ]
    ==
do
a2 <- f a1 x1
a3 <- f a2 x2
...
f am xm

```

If right-to-left evaluation is required, the input list should be reversed.
The when and unless functions provide conditional execution of monadic expressions. For example,
```

when debug (putStr "Debugging\n")

```
will output the string "Debugging \(\backslash \mathrm{n}\) " if the Boolean value debug is True, and otherwise do nothing.

The monadic lifting operators promote a function to a monad. The function arguments are scanned left to right. For example,
```

liftM2 (+) [0,1] [0,2] = [0,2,1,3]
liftM2 (+) (Just 1) Nothing = Nothing

```

In many situations, the liftM operations can be replaced by uses of ap, which promotes function application.
return f ‘ap' \(x 1\) `ap` ... ‘ap' \(x n\)
is equivalent to
\(\operatorname{liftMn} \mathrm{f} x 1 \mathrm{x} 2 \ldots \mathrm{xn}\)

\subsection*{20.4 Library Monad}
```

module Monad (
MonadPlus(mzero, mplus),
join, guard, when, unless, ap,
msum,
filterM, mapAndUnzipM, zipWithM, zipWithM_, foldM,
liftM, liftM2, liftM3, liftM4, liftM5,
-- ...and what the Prelude exports
Monad((>>=), (>>), return, fail),
Functor(fmap),
mapM, mapM_, sequence, sequence_, (=<<),
) where
-- The MonadPlus class definition
class (Monad m) => MonadPlus m where
mzero :: m a
mplus :: m a -> m a -> m a
-- Instances of MonadPlus
instance MonadPlus Maybe where
mzero = Nothing
Nothing `mplus` ys = ys
xs `mplus` ys = xs
instance MonadPlus [] where
mzero = []
mplus = (++)
-- Functions
msum :: MonadPlus m => [m a] -> m a
msum xs = foldr mplus mzero xs
join :: (Monad m) => m (m a) -> m a
join x = x >>= id
when :: (Monad m) => Bool -> m () -> m ()
when p s = if p then s else return ()
unless :: (Monad m) => Bool -> m () -> m ()
unless p s = when (not p) s
ap :: (Monad m) => m (a -> b) -> m a -> m b
ap = liftM2 (\$)
guard :: MonadPlus m => Bool -> m ()
guard p = if p then return () else mzero
mapAndUnzipM :: (Monad m) => (a -> m (b, c)) -> [a] -> m ([b], [c])
mapAndUnzipM f xs = sequence (map f xs) >>= return . unzip

```
```

zipWithM :: (Monad m) => (a -> b -> m c) -> [a] -> [b] -> m [c]
zipWithM f xs ys = sequence (zipWith f xs ys)
zipWithM_ :: (Monad m) => (a -> b -> m c) -> [a] -> [b] -> m ()
zipWithM_f xs ys = sequence_ (zipWith f xs ys)
foldM :: (Monad m) => (a -> b -> m a) -> a -> [b] -> m a
foldM f a [] = return a
foldM f a (x:xs) = f a x >>= \ y -> foldM f y xs
filterM :: Monad m => (a -> m Bool) -> [a] -> m [a]
filterM p [] = return []
filterM p (x:xs) = do { b <- p x;
ys <- filterM p xs;
return (if b then (x:ys) else ys)
}
liftM :: (Monad m) => (a -> b) -> (m a -> m b)
liftM f = \a -> do { a' <- a; return (f a') }
liftM2 :: (Monad m) => (a -> b -> c) -> (m a -> m b -> m c)
liftM2 f = \a b -> do { a' <- a; b' <- b; return (f a' b') }
liftM3 :: (Monad m) => (a -> b -> c -> d) ->
(m a -> m b -> m c -> m d)
liftM3 f = \a b c -> do { a' <- a; b' <- b; c' <- c;
return (f a' b' c') }
liftM4 :: (Monad m) => (a -> b -> c -> d -> e) ->
(m a -> m b -> m c -> m d -> m e)
liftM4 f = \a b c d -> do { a' <- a; b' <- b; c' <- c; d' <- d;
return (f a' b' c' d') }
liftM5 :: (Monad m) => (a -> b >> c -> d -> e -> f) ->
(m a -> m b -> m c -> m d -> m e -> m f)
liftM5 f = \a b c d e -> do { a' <- a; b' <- b; c' <- c; d' <- d;
e' <- e; return (f a' b' c' d' e') }

```

\section*{Chapter 21}

\section*{Input/Output}
```

module IO (
Handle, HandlePosn,
IOMode (ReadMode,WriteMode,AppendMode,ReadWriteMode),
BufferMode (NoBuffering,LineBuffering,BlockBuffering),
SeekMode (AbsoluteSeek,RelativeSeek,SeekFromEnd),
stdin, stdout, stderr,
openFile, hClose, hFileSize, hIsEOF, isEOF,
hSetBuffering, hGetBuffering, hFlush,
hGetPosn, hSetPosn, hSeek,
hWaitForInput, hReady, hGetChar, hGetLine, hLookAhead, hGetContents,
hPutChar, hPutStr, hPutStrLn, hPrint,
hIsOpen, hIsClosed, hIsReadable, hIsWritable, hIsSeekable,
isAlreadyExistsError, isDoesNotExistError, isAlreadyInUseError,
isFullError, isEOFError,
isIllegalOperation, isPermissionError, isUserError,
ioeGetErrorString, ioeGetHandle, ioeGetFileName,
try, bracket, bracket_,
-- ...and what the Prelude exports
IO, FilePath, IOError, ioError, userError, catch, interact,
putChar, putStr, putStrLn, print, getChar, getLine, getContents,
readFile, writeFile, appendFile, readIO, readLn
) where
import Ix(Ix)
data Handle = ... -- implementation-dependent
instance Eq Handle where ...
instance Show Handle where .. -- implementation-dependent
data HandlePosn = ... -- implementation-dependent
instance Eq HandlePosn where ...
instance Show HandlePosn where --- -- implementation-dependent

```

```

hFileSize :: Handle -> IO Integer
hIsEOF :: Handle -> IO Bool
isEOF :: IO Bool
isEOF = hIsEOF stdin
hSetBuffering :: Handle -> BufferMode -> IO ()
hGetBuffering :: Handle -> IO BufferMode
hFlush
hGetPosn
hSetPosn
hSeek
hWaitForInput :: Handle -> Int -> IO Bool
hReady
hReady h
hGetChar
hGetLine
hLookAhead
hGetContents
hPutChar
hPutStr
hPutStrLn
hPrint :: Show a => Handle -> a -> IO ()
hIsOpen :: Handle -> IO Bool
hIsClosed :: Handle -> IO Bool
hIsReadable :: Handle -> IO Bool
hIsWritable :: Handle -> IO Bool
hIsSeekable :: Handle -> IO Bool
isAlreadyExistsError :: IOError -> Bool
isDoesNotExistError :: IOError -> Bool
isAlreadyInUseError :: IOError -> Bool
isFullError :: IOError -> Bool
isEOFError :: IOError -> Bool
isIllegalOperation :: IOError -> Bool
isPermissionError :: IOError -> Bool
isUserError :: IOError -> Bool
ioeGetErrorString :: IOError -> String
ioeGetHandle :: IOError -> Maybe Handle
ioeGetFileName :: IOError -> Maybe FilePath
try :: IO a -> IO (Either IOError a)
bracket :: IO a -> (a -> IO b) -> (a -> IO c) -> IO c
bracket_ :: IO a -> (a -> IO b) -> IO c -> IO c

```

The monadic I/O system used in Haskell is described by the Haskell language report. Commonly used I/O functions such as print are part of the standard prelude and need not be explicitly imported. This library contain more advanced I/O features. Some related operations on file systems
are contained in the Directory library.

\subsection*{21.1 I/O Errors}

Errors of type IOError are used by the I/O monad. This is an abstract type; the library provides functions to interrogate and construct values in IOError:
- isAlreadyExistsError - the operation failed because one of its arguments already exists.
- isDoesNotExistError - the operation failed because one of its arguments does not exist.
- isAlreadyInUseError - the operation failed because one of its arguments is a singleuse resource, which is already being used (for example, opening the same file twice for writing might give this error).
- isFullError - the operation failed because the device is full.
- isEOFError - the operation failed because the end of file has been reached.
- isIllegaloperation - the operation is not possible.
- isPermissionError - the operation failed because the user does not have sufficient operating system privilege to perform that operation.
- isUserError - a programmer-defined error value has been raised using fail.

All these functions return a Bool, which is True if its argument is the corresponding kind of error, and False otherwise.

Any computation which returns an IO result may fail with isIllegalOperation. Additional errors which could be raised by an implementation are listed after the corresponding operation. In some cases, an implementation will not be able to distinguish between the possible error causes. In this case it should return isIllegalOperation.

Three additional functions are provided to obtain information about an error value. These are ioeGetHandle which returns Just \(h d l\) if the error value refers to handle \(h d l\) and Nothing otherwise; ioeGetFileName which returns Just name if the error value refers to file name, and Nothing otherwise; and ioeGetErrorString which returns a string. For "user" errors (those which are raised using fail), the string returned by ioeGetErrorString is the argument that was passed to fail; for all other errors, the string is implementation-dependent.

The try function returns an error in a computation explicitly using the Either type.
The bracket function captures a common allocate, compute, deallocate idiom in which the deallocation step must occur even in the case of an error during computation. This is similar to try-catch-finally in Java.

\subsection*{21.2 Files and Handles}

Haskell interfaces to the external world through an abstract file system. This file system is a collection of named file system objects, which may be organised in directories (see Directory). In some implementations, directories may themselves be file system objects and could be entries in other directories. For simplicity, any non-directory file system object is termed a file, although it could in fact be a communication channel, or any other object recognised by the operating system. Physical files are persistent, ordered files, and normally reside on disk.

File and directory names are values of type String, whose precise meaning is operating system dependent. Files can be opened, yielding a handle which can then be used to operate on the contents of that file.

Haskell defines operations to read and write characters from and to files, represented by values of type Handle. Each value of this type is a handle: a record used by the Haskell run-time system to manage I/O with file system objects. A handle has at least the following properties:
- whether it manages input or output or both;
- whether it is open, closed or semi-closed;
- whether the object is seekable;
- whether buffering is disabled, or enabled on a line or block basis;
- a buffer (whose length may be zero).

Most handles will also have a current I/O position indicating where the next input or output operation will occur. A handle is readable if it manages only input or both input and output; likewise, it is writable if it manages only output or both input and output. A handle is open when first allocated. Once it is closed it can no longer be used for either input or output, though an implementation cannot re-use its storage while references remain to it. Handles are in the Show and Eq classes. The string produced by showing a handle is system dependent; it should include enough information to identify the handle for debugging. A handle is equal according to \(==\) only to itself; no attempt is made to compare the internal state of different handles for equality.

\subsection*{21.2.1 Standard Handles}

Three handles are allocated during program initialisation. The first two (stdin and stdout) manage input or output from the Haskell program's standard input or output channel respectively. The third (stderr) manages output to the standard error channel. These handles are initially open.

\subsection*{21.2.2 Semi-Closed Handles}

The operation hGetContents \(h d l\) (Section 21.9.4) puts a handle \(h d l\) into an intermediate state, semi-closed. In this state, \(h d l\) is effectively closed, but items are read from \(h d l\) on demand and accumulated in a special list returned by hGetContents \(h d l\).

Any operation that fails because a handle is closed, also fails if a handle is semi-closed. The only exception is hClose. A semi-closed handle becomes closed:
- if hClose is applied to it;
- if an I/O error occurs when reading an item from the handle;
- or once the entire contents of the handle has been read.

Once a semi-closed handle becomes closed, the contents of the associated list becomes fixed. The contents of this final list is only partially specified: it will contain at least all the items of the stream that were evaluated prior to the handle becoming closed.

Any I/O errors encountered while a handle is semi-closed are simply discarded.

\subsection*{21.2.3 File locking}

Implementations should enforce as far as possible, at least locally to the Haskell process, multiplereader single-writer locking on files. That is, there may either be many handles on the same file which manage input, or just one handle on the file which manages output. If any open or semiclosed handle is managing a file for output, no new handle can be allocated for that file. If any open or semi-closed handle is managing a file for input, new handles can only be allocated if they do not manage output. Whether two files are the same is implementation-dependent, but they should normally be the same if they have the same absolute path name and neither has been renamed, for example.

Warning: the readFile operation (Section 7.1) holds a semi-closed handle on the file until the entire contents of the file have been consumed. It follows that an attempt to write to a file (using writeFile, for example) that was earlier opened by readFile will usually result in failure with isAlreadyInUseError.

\subsection*{21.3 Opening and Closing Files}

\subsection*{21.3.1 Opening Files}

Computation openFile file mode allocates and returns a new, open handle to manage the file file. It manages input if mode is ReadMode, output if mode is WriteMode or AppendMode, and both input and output if mode is ReadWriteMode.

If the file does not exist and it is opened for output, it should be created as a new file. If mode is WriteMode and the file already exists, then it should be truncated to zero length. Some operating systems delete empty files, so there is no guarantee that the file will exist following an openFile with mode WriteMode unless it is subsequently written to successfully. The handle is positioned at the end of the file if mode is AppendMode, and otherwise at the beginning (in which case its internal I/O position is 0 ). The initial buffer mode is implementation-dependent.

If openFile fails on a file opened for output, the file may still have been created if it did not already exist.

Error reporting: the openFile computation may fail with isAlready InUseError if the file is already open and cannot be reopened; isDoesNotExistError if the file does not exist; or isPermissionError if the user does not have permission to open the file.

\subsection*{21.3.2 Closing Files}

Computation hClose \(h d l\) makes handle \(h d l\) closed. Before the computation finishes, if \(h d l\) is writable its buffer is flushed as for hFlush . Performing hClose on a handle that has already been closed has no effect; doing so not an error. All other operations on a closed handle will fail. If hClose fails for any reason, any further operations (apart from hClose) on the handle will still fail as if \(h d l\) had been successfully closed.

\subsection*{21.4 Determining the Size of a File}

For a handle \(h d l\) which is attached to a physical file, hFileSize \(h d l\) returns the size of that file in 8-bit bytes \((\geq 0)\).

\subsection*{21.5 Detecting the End of Input}

For a readable handle \(h d l\), computation hIsEOF \(h d l\) returns True if no further input can be taken from \(h d l\); for a handle attached to a physical file this means that the current I/O position is equal to the length of the file. Otherwise, it returns False. The computation isEOF is identical, except that it works only on stdin.

\subsection*{21.6 Buffering Operations}

Three kinds of buffering are supported: line-buffering, block-buffering or no-buffering. These modes have the following effects. For output, items are written out, or flushed, from the internal buffer according to the buffer mode:
- line-buffering: the entire buffer is flushed whenever a newline is output, the buffer overflows, a hFlush is issued, or the handle is closed.
- block-buffering: the entire buffer is written out whenever it overflows, a hFlush is issued, or the handle is closed.
- no-buffering: output is written immediately, and never stored in the buffer.

An implementation is free to flush the buffer more frequently, but not less frequently, than specified above. The buffer is emptied as soon as it has been written out.

Similarly, input occurs according to the buffer mode for handle hdl.
- line-buffering: when the buffer for \(h d l\) is not empty, the next item is obtained from the buffer; otherwise, when the buffer is empty, characters are read into the buffer until the next newline character is encountered or the buffer is full. No characters are available until the newline character is available or the buffer is full.
- block-buffering: when the buffer for \(h d l\) becomes empty, the next block of data is read into the buffer.
- no-buffering: the next input item is read and returned. The hLookAhead operation (Section 21.9.3) implies that even a no-buffered handle may require a one-character buffer.

For most implementations, physical files will normally be block-buffered and terminals will normally be line-buffered.

Computation hSetBuffering \(h d l\) mode sets the mode of buffering for handle \(h d l\) on subsequent reads and writes.
- If mode is LineBuffering, line-buffering is enabled if possible.
- If mode is BlockBuffering size, then block-buffering is enabled if possible. The size of the buffer is \(n\) items if size is Just \(n\) and is otherwise implementation-dependent.
- If mode is NoBuffering, then buffering is disabled if possible.

If the buffer mode is changed from BlockBuffering or LineBuffering to NoBuffering, then
- if \(h d l\) is writable, the buffer is flushed as for \(h F l u s h ;\)
- if \(h d l\) is not writable, the contents of the buffer is discarded.

Error reporting: the hSetBuffering computation may fail with isPermissionError if the handle has already been used for reading or writing and the implementation does not allow the buffering mode to be changed.

Computation hGetBuffering \(h d l\) returns the current buffering mode for \(h d l\).
The default buffering mode when a handle is opened is implementation-dependent and may depend on the file system object which is attached to that handle.

\subsection*{21.6.1 Flushing Buffers}

Computation hF lush \(h d l\) causes any items buffered for output in handle \(h d l\) to be sent immediately to the operating system.

Error reporting: the hFlush computation may fail with: isFullError if the device is full; isPermissionError if a system resource limit would be exceeded. It is unspecified whether the characters in the buffer are discarded or retained under these circumstances.

\subsection*{21.7 Repositioning Handles}

\subsection*{21.7.1 Revisiting an I/O Position}

Computation hGetPosn \(h d l\) returns the current I/O position of \(h d l\) as a value of the abstract type HandlePosn. If a call to hGetPosn \(h\) returns a position \(p\), then computation hSetPosn \(p\) sets the position of \(h\) to the position it held at the time of the call to hGetPosn.

Error reporting: the hSetPosn computation may fail with: isPermissionError if a system resource limit would be exceeded.

\subsection*{21.7.2 Seeking to a new Position}

Computation hSeek \(h d l\) mode \(i\) sets the position of handle \(h d l\) depending on mode. If mode is:
- AbsoluteSeek: the position of \(h d l\) is set to \(i\).
- RelativeSeek: the position of \(h d l\) is set to offset \(i\) from the current position.
- SeekFromEnd: the position of \(h d l\) is set to offset \(i\) from the end of the file.

The offset is given in terms of 8-bit bytes.
If \(h d l\) is block- or line-buffered, then seeking to a position which is not in the current buffer will first cause any items in the output buffer to be written to the device, and then cause the input buffer
to be discarded. Some handles may not be seekable (see hIsSeekable), or only support a subset of the possible positioning operations (for instance, it may only be possible to seek to the end of a tape, or to a positive offset from the beginning or current position). It is not possible to set a negative I/O position, or for a physical file, an I/O position beyond the current end-of-file.

Error reporting: the hSeek computation may fail with: isPermissionError if a system resource limit would be exceeded.

\subsection*{21.8 Handle Properties}

The functions hIsOpen, hIsClosed, hIsReadable, hIsWritable and hIsSeekable return information about the properties of a handle. Each of these returns True if the handle has the specified property, and False otherwise.

\subsection*{21.9 Text Input and Output}

Here we define a standard set of input operations for reading characters and strings from text files, using handles. Many of these functions are generalizations of Prelude functions. I/O in the Prelude generally uses stdin and stdout; here, handles are explicitly specified by the I/O operation.

\subsection*{21.9.1 Checking for Input}

Computation hWaitFor Input \(h d l t\) waits until input is available on handle \(h d l\). It returns True as soon as input is available on \(h d l\), or False if no input is available within \(t\) milliseconds.

Computation hReady \(h d l\) indicates whether at least one item is available for input from handle \(h d l\).

Error reporting: the hWaitForInput and hReady computations fail with isEOFError if the end of file has been reached.

\subsection*{21.9.2 Reading Input}

Computation hGet Char \(h d l\) reads a character from the file or channel managed by \(h d l\).
Computation hGetLine \(h d l\) reads a line from the file or channel managed by \(h d l\). The Prelude's getLine is a shorthand for hGetLine stdin.

Error reporting: the hGetChar computation fails with isEOFError if the end of file has been reached. The hGetLine computation fails with isEOFError if the end of file is encountered when reading the first character of the line. If hGetLine encounters end-of-file at any other point while reading in a line, it is treated as a line terminator and the (partial) line is returned.

\subsection*{21.9.3 Reading Ahead}

Computation hLookAhead \(h d l\) returns the next character from handle \(h d l\) without removing it from the input buffer, blocking until a character is available.

Error reporting: the hLookAhead computation may fail with: isEOFError if the end of file has been reached.

\subsection*{21.9.4 Reading The Entire Input}

Computation hGet Contents \(h d l\) returns the list of characters corresponding to the unread portion of the channel or file managed by \(h d l\), which is made semi-closed.

Error reporting: the hGetContents computation may fail with: isEOFError if the end of file has been reached.

\subsection*{21.9.5 Text Output}

Computation hPutChar \(h d l c\) writes the character \(c\) to the file or channel managed by \(h d l\). Characters may be buffered if buffering is enabled for \(h d l\).

Computation hPutStr \(h d l s\) writes the string \(s\) to the file or channel managed by \(h d l\).
Computation hPrint \(h d l t\) writes the string representation of \(t\) given by the shows function to the file or channel managed by \(h d l\) and appends a newline.

Error reporting: the hPutChar, hPutStr and hPrint computations may fail with: isFullError if the device is full; or isPermissionError if another system resource limit would be exceeded.

\subsection*{21.10 Examples}

Here are some simple examples to illustrate Haskell I/O.

\subsection*{21.10.1 Summing Two Numbers}

This program reads and sums two Integers.
```

import IO
main = do
hSetBuffering stdout NoBuffering
putStr "Enter an integer: "
x1 <- readNum
putStr "Enter another integer: "
x2 <- readNum
putStr ("Their sum is " ++ show (x1+x2) ++ "\n")
where readNum :: IO Integer
-- Providing a type signature avoids reliance on
-- the defaulting rule to fix the type of x1,x2
readNum = readLn

```

\subsection*{21.10.2 Copying Files}

A simple program to create a copy of a file, with all lower-case characters translated to upper-case. This program will not allow a file to be copied to itself. This version uses character-level I/O. Note that exactly two arguments must be supplied to the program.
```

import IO
import System
import Char( toUpper )
main = do
[f1,f2] <- getArgs
h1 <- openFile f1 ReadMode
h2 <- openFile f2 WriteMode
copyFile h1 h2
hclose h1
hClose h2
copyFile h1 h2 = do
eof <- hIsEOF h1
if eof then return () else
do
c <- hGetChar h1
hPutChar h2 (toUpper c)
copyFile h1 h2

```

An equivalent but much shorter version, using string I/O is:
```

import System
import Char( toUpper )
main = do
[f1,f2] <- getArgs
s <- readFile f1
writeFile f2 (map toUpper s)

```

\subsection*{21.11 Library IO}
```

module IO {- export list omitted -} where
-- Just provide an implementation of the system-independent
-- actions that IO exports.
try :: IO a -> IO (Either IOError a)
try f = catch (do r <- f
return (Right r))
(return . Left)
bracket :: IO a -> (a -> IO b) -> (a -> IO c) -> IO c
bracket before after m = do
x <- before
rs <- try (m x)
after x
case rs of
Right r -> return r
Left e -> ioError e
-- variant of the above where middle computation doesn't want x
bracket_ :: IO a -> (a -> IO b) -> IO c -> IO c
bracket_ before after m = do
x <- before
rs <- try m
after x
case rs of
Right r -> return r
Left e -> ioError e

```

\section*{Chapter 22}

\section*{Directory Functions}
```

module Directory (
Permissions( Permissions, readable, writable, executable, searchable ),
createDirectory, removeDirectory, removeFile,
renameDirectory, renameFile, getDirectoryContents,
getCurrentDirectory, setCurrentDirectory,
doesFileExist, doesDirectoryExist,
getPermissions, setPermissions,
getModificationTime ) where
import Time ( ClockTime )
data Permissions = Permissions {
readable, writable,
executable, searchable :: Bool
}
instance Eq Permissions where ...
instance Ord Permissions where ...
instance Read Permissions where ...
instance Show Permissions where ...
createDirectory :: FilePath -> IO ()
removeDirectory :: FilePath -> IO ()
removeFile :: FilePath -> IO ()
renameDirectory :: FilePath -> FilePath -> IO ()
renameFile :: FilePath -> FilePath -> IO ()
getDirectoryContents :: FilePath -> IO [FilePath]
getCurrentDirectory :: IO FilePath
setCurrentDirectory :: FilePath -> IO ()
doesFileExist :: FilePath -> IO Bool
doesDirectoryExist :: FilePath -> IO Bool
getPermissions :: FilePath -> IO Permissions
setPermissions :: FilePath -> Permissions -> IO ()
getModificationTime :: FilePath -> IO ClockTime

```

These functions operate on directories in the file system.
Any Directory operation could raise an isIllegalOperation, as described in Section 21.1; all other permissible errors are described below. Note that, in particular, if an implementation does not support an operation it should raise an isIllegalOperation. A directory contains a series of entries, each of which is a named reference to a file system object (file, directory etc.). Some entries may be hidden, inaccessible, or have some administrative function (for instance, "." or ".." under POSIX), but all such entries are considered to form part of the directory contents. Entries in sub-directories are not, however, considered to form part of the directory contents. Although there may be file system objects other than files and directories, this library does not distinguish between physical files and other non-directory objects. All such objects should therefore be treated as if they are files.

Each file system object is referenced by a path. There is normally at least one absolute path to each file system object. In some operating systems, it may also be possible to have paths which are relative to the current directory.

Computation createDirectory dir creates a new directory dir which is initially empty, or as near to empty as the operating system allows.

Error reporting. The createDirectory computation may fail with: isPermissionError if the user is not permitted to create the directory; isAlreadyExistsError if the directory already exists; or isDoesNotExistError if the new directory's parent does not exist.

Computation removeDirectory dir removes an existing directory dir. The implementation may specify additional constraints which must be satisfied before a directory can be removed (for instance, the directory has to be empty, or may not be in use by other processes). It is not legal for an implementation to partially remove a directory unless the entire directory is removed. A conformant implementation need not support directory removal in all situations (for instance, removal of the root directory).

Computation removeFile file removes the directory entry for an existing file file, where file is not itself a directory. The implementation may specify additional constraints which must be satisfied before a file can be removed (for instance, the file may not be in use by other processes).

Error reporting. The removeDirectory and removeFile computations may fail with: isPermissionError if the user is not permitted to remove the file/directory; or isDoesNotExistError if the file/directory does not exist.

Computation renameDirectory old new changes the name of an existing directory from old to new. If the new directory already exists, it is atomically replaced by the old directory. If the new directory is neither the old directory nor an alias of the old directory, it is removed as if by removeDirectory. A conformant implementation need not support renaming directories in all situations (for instance, renaming to an existing directory, or across different physical devices), but the constraints must be documented.

Computation renameFile old new changes the name of an existing file system object from old to new. If the new object already exists, it is atomically replaced by the old object. Neither path
may refer to an existing directory. A conformant implementation need not support renaming files in all situations (for instance, renaming across different physical devices), but the constraints must be documented.

Error reporting. The renameDirectory and renameFile computations may fail with: isPermissionError if the user is not permitted to rename the file/directory, or if either argument to renameFile is a directory; or isDoesNotExistError if the file/directory does not exist.

Computation getDirectoryContents dir returns a list of all entries in dir. Each entry in the returned list is named relative to the directory \(d i r\), not as an absolute path.

If the operating system has a notion of current directories, getCurrentDirectory returns an absolute path to the current directory of the calling process.

Error reporting. The getDirectoryContents and getCurrentDirectory computations may fail with: isPermissionError if the user is not permitted to access the directory; or isDoesNotExistError if the directory does not exist.

If the operating system has a notion of current directories, setCurrentDirectory dir changes the current directory of the calling process to dir.

Error reporting. setCurrentDirectory may fail with: isPermissionError if the user is not permitted to change directory to that specified; or isDoesNotExistError if the directory does not exist.

The Permissions type is used to record whether certain operations are permissible on a file/directory. getPermissions and setPermissions get and set these permissions, respectively. Permissions apply both to files and directories. For directories, the executable field will be False, and for files the searchable field will be False. Note that directories may be searchable without being readable, if permission has been given to use them as part of a path, but not to examine the directory contents.

Note that to change some, but not all permissions, a construct on the following lines must be used.
```

makeReadable f = do
p <- getPermissions f
setPermissions f (p {readable = True})

```

The operation doesDirectoryExist returns True if the argument file exists and is a directory, and False otherwise. The operation doesFileExist returns True if the argument file exists and is not a directory, and False otherwise.

The getModificationTime operation returns the clock time at which the file/directory was last modified.

Error reporting. get(set)Permissions, doesFile(Directory)Exist, and getModificationTime may fail with: isPermissionError if the user is not permitted to access the appropriate information; or isDoesNotExistError if the file/directory
does not exist. The setPermissions computation may also fail with: isPermissionError if the user is not permitted to change the permission for the specified file or directory; or isDoesNotExistError if the file/directory does not exist.

\section*{Chapter 23}

\section*{System Functions}
```

module System (
ExitCode(ExitSuccess,ExitFailure),
getArgs, getProgName, getEnv, system, exitWith, exitFailure
) where
data ExitCode = ExitSuccess | ExitFailure Int
deriving (Eq, Ord, Read, Show)
getArgs :: IO [String]
getProgName :: IO String
getEnv :: String -> IO String
system :: String -> IO ExitCode
exitWith :: ExitCode -> IO a
exitFailure :: IO a

```

This library describes the interaction of the program with the operating system.
Any System operation could raise an isIllegaloperation, as described in Section 21.1; all other permissible errors are described below. Note that, in particular, if an implementation does not support an operation it must raise an isIllegaloperation.

The ExitCode type defines the exit codes that a program can return. ExitSuccess indicates successful termination; and ExitFailure code indicates program failure with value code. The exact interpretation of code is operating-system dependent. In particular, some values of code may be prohibited (for instance, 0 on a POSIX-compliant system).

Computation getArgs returns a list of the program's command line arguments (not including the program name). Computation getProgName returns the name of the program as it was invoked. Computation getEnv var returns the value of the environment variable var. If variable var is undefined, the isDoesNotExistError exception is raised.

Computation system \(c m d\) returns the exit code produced when the operating system processes the command cmd .

Computation exitWith code terminates the program, returning code to the program's caller. Before the program terminates, any open or semi-closed handles are first closed. The caller may interpret the return code as it wishes, but the program should return ExitSuccess to mean normal completion, and ExitFailure \(n\) to mean that the program encountered a problem from which it could not recover. The value exitFailure is equal to exitWith (ExitFailure exitfail), where exitfail is implementation-dependent. exitWith bypasses the error handling in the I/O monad and cannot be intercepted by catch.

If a program terminates as a result of calling error or because its value is otherwise determined to be \(\perp\), then it is treated identically to the computation exitFailure. Otherwise, if any program \(p\) terminates without calling exitWith explicitly, it is treated identically to the computation
```

(p >> exitWith ExitSuccess) `catch` \ _ -> exitFailure

```

\section*{Chapter 24}

\section*{Dates and Times}
```

module Time (
ClockTime,
Month(January,February,March,April, May, June,
July,August, September, October, November, December),
Day (Sunday,Monday, Tuesday,Wednesday, Thursday, Friday,Saturday),
CalendarTime(CalendarTime, ctYear, ctMonth, ctDay, ctHour, ctMin,
ctSec, ctPicosec, ctWDay, ctYDay,
ctTZName, ctTZ, ctIsDST),
TimeDiff(TimeDiff, tdYear, tdMonth, tdDay, tdHour,
tdMin, tdSec, tdPicosec),
getClockTime, addToClockTime, diffClockTimes,
toCalendarTime, toUTCTime, toClockTime,
calendarTimeToString, formatCalendarTime ) where
import Ix(Ix)
data ClockTime = ... -- Implementation-dependent
instance Ord ClockTime where ...
instance Eq ClockTime where ...
data Month = January | February | March | April
| January
deriving (Eq, Ord, Enum, Bounded, Ix, Read, Show)
data Day = Sunday | Monday | Tuesday | Wednesday | Thursday
| Friday | Saturday
deriving (Eq, Ord, Enum, Bounded, Ix, Read, Show)
data CalendarTime = CalendarTime {
ctYear :: Int,
ctMonth :: Month,
ctDay, ctHour, ctMin, ctSec :: Int,
ctPicosec :: Integer,
ctWDay :: Day,
ctYDay :: Int,
ctTZName :: String,
ctTZ :: Int,
ctIsDST :: Bool
} deriving (Eq, Ord, Read, Show)
data TimeDiff = TimeDiff {
tdYear, tdMonth, tdDay, tdHour, tdMin, tdSec :: Int,
tdPicosec :: Integer
} deriving (Eq, Ord, Read, Show)

```
```

-- Functions on times
getClockTime :: IO ClockTime
addToClockTime :: TimeDiff -> ClockTime -> ClockTime
diffClockTimes :: ClockTime -> ClockTime -> TimeDiff
toCalendarTime :: ClockTime -> IO CalendarTime
toUTCTime :: ClockTime -> CalendarTime
toClockTime :: CalendarTime -> ClockTime
calendarTimeToString :: CalendarTime -> String
formatCalendarTime :: TimeLocale -> String -> CalendarTime -> String

```

The Time library provides standard functionality for clock times, including timezone information. It follows RFC 1129 in its use of Coordinated Universal Time (UTC).

ClockTime is an abstract type, used for the system's internal clock time. Clock times may be compared directly or converted to a calendar time CalendarTime for I/O or other manipulations. Calendartime is a user-readable and manipulable representation of the internal ClockTime type. The numeric fields have the following ranges.
\begin{tabular}{|lcl|}
\hline\(\underline{\text { Value }}\) & \(\underline{\text { Range }}\) & \(\underline{\text { Comments }}\) \\
ctYear & -maxInt \(\ldots\) maxInt & Pre-Gregorian dates are inaccurate \\
ctDay & \(1 \ldots 31\) & \\
ctHour & \(0 \ldots 23\) & \\
ctMin & \(0 \ldots 59\) & \\
ctSec & \(0 \ldots 61\) & Allows for two Leap Seconds \\
ctPicosec & \(0 \ldots\left(10^{12}\right)-1\) & \\
ctYDay & \(0 \ldots 365\) & 364 in non-Leap years \\
ctTZ & \(-89999 \ldots 89999\) & Variation from UTC in seconds \\
\hline
\end{tabular}

The ctTZName field is the name of the time zone. The ctIsDST field is True if Daylight Savings Time would be in effect, and False otherwise. The TimeDiff type records the difference between two clock times in a user-readable way.

Function getClockTime returns the current time in its internal representation. The expression addToClockTime \(d t\) adds a time difference \(d\) and a clock time \(t\) to yield a new clock time. The difference \(d\) may be either positive or negative. The expression diffclockTimes t1 t2 returns the difference between two clock times \(t 1\) and \(t 2\) as a TimeDiff.

Function tocalendarTime \(t\) converts \(t\) to a local time, modified by the timezone and daylight savings time settings in force at the time of conversion. Because of this dependence on the local environment, toCalendarTime is in the IO monad.

Function toUTCTime \(t\) converts \(t\) into a CalendarTime in standard UTC format. toClockTime \(l\) converts \(l\) into the corresponding internal ClockTime ignoring the contents of the \(c t W D a y, c t Y D a y, c t T Z N a m e\), and \(c t I s D S T\) fields.

Function calendarTimeToString formats calendar times using local conventions and a formatting string.

\subsection*{24.1 Library Time}
```

module Time (
ClockTime,
Month(January,February,March,April,May, June,
July,August,September, October,November,December),
Day(Sunday,Monday,Tuesday,Wednesday,Thursday,Friday,Saturday),
CalendarTime(CalendarTime, ctYear, ctMonth, ctDay, ctHour, ctMin,
ctSec, ctPicosec, ctWDay, ctYDay,
ctTZName, ctTZ, ctIsDST),
TimeDiff(TimeDiff, tdYear, tdMonth, tdDay,
tdHour, tdMin, tdSec, tdPicosec),
getClockTime, addToClockTime, diffClockTimes,
toCalendarTime, toUTCTime, toClockTime,
calendarTimeToString, formatCalendarTime ) where
import Ix(Ix)
import Locale(TimeLocale(..),defaultTimeLocale)
import Char ( intToDigit )
data ClockTime = ... -- Implementation-dependent
instance Ord ClockTime where ...
instance Eq ClockTime where ...

```

```

data Day = Sunday | Monday | Tuesday | Wednesday | Thursday
| Friday | Saturday
deriving (Eq, Ord, Enum, Bounded, Ix, Read, Show)

```
```

data CalendarTime = CalendarTime {
ctYear :: Int,
ctMonth :: Month,
ctDay, ctHour, ctMin, ctSec :: Int,
ctPicosec :: Integer,
ctWDay :: Day,
ctYDay :: Int,
ctTZName :: String,
ctTZ :: Int,
ctIsDST :: Bool
} deriving (Eq, Ord, Read, Show)
data TimeDiff = TimeDiff {
tdYear, tdMonth, tdDay, tdHour, tdMin, tdSec :: Int,
tdPicosec :: Integer
} deriving (Eq, Ord, Read, Show)
getClockTime :: IO ClockTime
getClockTime = ... -- Implementation-dependent
addToClockTime :: TimeDiff -> ClockTime -> ClockTime
addToClockTime td ct = ... -- Implementation-dependent
diffClockTimes :: ClockTime -> ClockTime -> TimeDiff
diffClockTimes ct1 ct2 = ... -- Implementation-dependent
toCalendarTime :: ClockTime -> IO CalendarTime
toCalendarTime ct = ... -- Implementation-dependent
toUTCTime :: ClockTime -> CalendarTime
toUTCTime ct = ... -- Implementation-dependent
toClockTime :: CalendarTime -> ClockTime
toClockTime cal = ... -- Implementation-dependent
calendarTimeToString :: CalendarTime -> String
calendarTimeToString = formatCalendarTime defaultTimeLocale "%c"

```
```

formatCalendarTime :: TimeLocale -> String -> CalendarTime -> String
formatCalendarTime l fmt ct@(CalendarTime year mon day hour min sec sdec
wday yday tzname _ _) =
doFmt fmt
where doFmt ('%':c:cs) = decode c ++ doFmt cs
doFmt (c:cs) = c : doFmt cs
doFmt "" = ""
to12 :: Int -> Int
to12 h = let h' = h `mod` 12 in if h' == 0 then 12 else h'
decode 'A' = fst (wDays l !! fromEnum wday)
decode 'a' = snd (wDays l !! fromEnum wday)
decode 'B' = fst (months l !! fromEnum mon)
decode 'b' = snd (months l !! fromEnum mon)
decode 'h' = snd (months l !! fromEnum mon)
decode 'C' = show2 (year 'quot' 100)
decode 'c' = doFmt (dateTimeFmt l)
decode 'D' = doFmt "%m/%d/%y"
decode 'd' = show2 day
decode 'e' = show2' day
decode 'H' = show2 hour
decode 'I' = show2 (to12 hour)
decode 'j' = show3 yday
decode 'k' = show2' hour
decode 'l' = show2' (to12 hour)
decode 'M' = show2 min
decode 'm' = show2 (fromEnum mon+1)
decode 'n' = "\n"
decode 'p' = (if hour < 12 then fst else snd) (amPm l)
decode 'R' = doFmt "%H:%M"
decode 'r' = doFmt (time12Fmt l)
decode 'T' = doFmt "%H:%M:%S"
decode 't' = "\t"
decode 'S' = show2 sec
decode 's' = ... -- Implementation-dependent
decode 'U' = show2 ((yday + 7 - fromEnum wday) `div` 7)
decode 'u' = show (let n = fromEnum wday in
if n == 0 then 7 else n)
decode 'V' =
let (week, days) =
(yday + 7 - if fromEnum wday > 0 then
fromEnum wday - 1 else 6) `divMod` 7
in show2 (if days >= 4 then

```
```

            week+1
            else if week == 0 then 53 else week)
    decode 'W' =
        show2 ((yday + 7 - if fromEnum wday > 0 then
                fromEnum wday - 1 else 6) `div` 7)
    decode 'w' = show (fromEnum wday)
    decode 'X' = doFmt (timeFmt l)
    decode ' }\textrm{x}\mathrm{ ' = doFmt (dateFmt l)
    decode 'Y' = show year
    decode 'Y' = show2 (year 'rem' 100)
    decode ' Z' = tzname
    decode '%' = "%"
    decode c = [c]
    show2, show2', show3 :: Int -> String
show2 x = [intToDigit (x `quot` 10), intToDigit (x `rem` 10)]
show2' x = if x < 10 then [ ' ', intToDigit x] else show2 x
show3 x = intToDigit (x `quot` 100) : show2 (x `rem` 100)

```

\section*{Chapter 25}

\section*{Locale}
```

module Locale(TimeLocale(..), defaultTimeLocale) where
data TimeLocale = TimeLocale {
wDays :: [(String, String)], -- full and abbreviated week days
months :: [(String, String)], -- full and abbreviated months
amPm :: (String, String), -- AM/PM symbols
dateTimeFmt, dateFmt, -- formatting strings
timeFmt, time12Fmt :: String
} deriving (Eq, Ord, Show)
defaultTimeLocale :: TimeLocale

```

The Locale library provides the ability to adapt to local conventions. At present, it supports only time and date information as used by calendarTimeToString from the Time library.

\subsection*{25.1 Library Locale}
```

module Locale(TimeLocale(..), defaultTimeLocale) where
data TimeLocale = TimeLocale {
wDays :: [(String, String)], -- full and abbreviated week days
months :: [(String, String)], -- full and abbreviated months
amPm :: (String, String), -- AM/PM symbols
dateTimeFmt, dateFmt, -- formatting strings
timeFmt, time12Fmt :: String
} deriving (Eq, Ord, Show)
defaultTimeLocale :: TimeLocale
defaultTimeLocale = TimeLocale {
wDays = [("Sunday", "Sun"), ("Monday", "Mon"),
("Tuesday", "Tue"), ("Wednesday", "Wed"),
("Thursday", "Thu"), ("Friday", "Fri"),
("Saturday", "Sat")],
months = [("January", "Jan"), ("February", "Feb"),
("March", "Mar"), ("April", "Apr"),
("May", "May"), ("June", "Jun"),
("July", "Jul"), ("August", "Aug"),
("September", "Sep"), ("October", "Oct"),
("November", "Nov"), ("December", "Dec")],
amPm = ("AM", "PM"),
dateTimeFmt = "%a %b %e %H:%M:%S %Z %Y",
dateFmt = "%m/%d/%y",
timeFmt = "%H:%M:%S",
time12Fmt = "%I:%M:%S %p"
}

```

\section*{Chapter 26}

\section*{CPU Time}
```

module CPUTime ( getCPUTime, cpuTimePrecision ) where
getCPUTime :: IO Integer
cpuTimePrecision :: Integer

```

Computation getCPUTime returns the number of picoseconds of CPU time used by the current program. The precision of this result is given by cpuTimePrecision. This is the smallest measurable difference in CPU time that the implementation can record, and is given as an integral number of picoseconds.

\section*{Chapter 27}

\section*{Random Numbers}
```

module Random (
RandomGen(next, split, genRange),
StdGen, mkStdGen,
Random( random, randomR,
randoms, randomRs,
randomIO, randomRIO ),
getStdRandom, getStdGen, setStdGen, newStdGen
) where
---------------- The RandomGen class ----------------------------
class RandomGen g where
genRange :: g -> (Int, Int)
next :: g -> (Int, g)
split :: g -> (g, g)
---------------- A standard instance of RandomGen -------------
data StdGen = ... -- Abstract
instance RandomGen StdGen where ...
instance Read StdGen where ...
instance Show StdGen where ...
mkStdGen :: Int -> StdGen
class Random a where
randomR :: RandomGen g => (a, a) -> g -> (a, g)
random :: RandomGen g => g -> (a, g)
randomRs :: RandomGen g => (a, a) -> g -> [a]
randoms :: RandomGen g => g -> [a]
randomRIO :: (a,a) -> IO a
randomIO :: IO a
instance Random Int where ...
instance Random Integer where ...
instance Random Float where ...
instance Random Double where ...
instance Random Bool where ...
instance Random Char where ...
---------------- The global random generator -------------------
newStdGen :: IO StdGen
setStdGen :: StdGen -> IO ()
getStdGen :: IO StdGen
getStdRandom :: (StdGen -> (a, StdGen)) -> IO a

```

The Random library deals with the common task of pseudo-random number generation. The library makes it possible to generate repeatable results, by starting with a specified initial random number generator; or to get different results on each run by using the system-initialised generator, or by supplying a seed from some other source.

The library is split into two layers:
- A core random number generator provides a supply of bits. The class RandomGen provides a common interface to such generators.
- The class Random provides a way to extract particular values from a random number generator. For example, the Float instance of Random allows one to generate random values of type Float.

\subsection*{27.1 The RandomGen class, and the StdGen generator}

The class RandomGen provides a common interface to random number generators.
```

class RandomGen g where
genRange :: g -> (Int,Int)
next :: g -> (Int, g)
split :: g -> (g, g)
-- Default method
genRange g = (minBound,maxBound)

```
- The genRange operation yields the range of values returned by the generator.

It is required that:
- If \((a, b)=\) genRange \(g\), then \(a<b\).
- genRange \(\perp \neq \perp\).

The second condition ensures that genRange cannot examine its argument, and hence the value it returns can be determined only by the instance of RandomGen. That in turn allows an implementation to make a single call to genRange to establish a generator's range, without being concerned that the generator returned by (say) next might have a different range to the generator passed to next.
- The next operation returns an Int that is uniformly distributed in the range returned by genRange (including both end points), and a new generator.
- The split operation allows one to obtain two independent random number generators. This is very useful in functional programs (for example, when passing a random number generator down to recursive calls), but very little work has been done on statistically robust implementations of split ([1,4] are the only examples we know of).

The Random library provides one instance of RandomGen, the abstract data type StdGen:
```

data StdGen = ... -- Abstract
instance RandomGen StdGen where ...
instance Read StdGen where ...
instance Show StdGen where ...
mkStdGen :: Int -> StdGen

```

The StgGen instance of RandomGen has a genRange of at least 30 bits.
The result of repeatedly using next should be at least as statistically robust as the "Minimal Standard Random Number Generator" described by [2,3]. Until more is known about implementations of split, all we require is that split deliver generators that are (a) not identical and (b) independently robust in the sense just given.

The Show/Read instances of StdGen provide a primitive way to save the state of a random number generator. It is required that read (show \(g\) ) \(==g\).

In addition, read may be used to map an arbitrary string (not necessarily one produced by show) onto a value of type StdGen. In general, the read instance of StdGen has the following properties:
- It guarantees to succeed on any string.
- It guarantees to consume only a finite portion of the string.
- Different argument strings are likely to result in different results.

The function mkStdGen provides an alternative way of producing an initial generator, by mapping an Int into a generator. Again, distinct arguments should be likely to produce distinct generators.

Programmers may, of course, supply their own instances of RandomGen.
Implementation warning. A superficially attractive implementation of split is
```

instance RandomGen MyGen where
split g = (g, variantOf g)

```

Here, split returns \(g\) itself and a new generator derived from \(g\). But now consider these two apparently-independent generators:
```

g1 = snd (split g)
g2 = snd (split (fst (split g)))

```

If split genuinely delivers independent generators (as specified), then \(g 1\) and \(g 2\) should be independent, but in fact they are both equal to variantof \(g\). Implementations of the above form do not meet the specification.

\subsection*{27.2 The Random class}

With a source of random number supply in hand, the Random class allows the programmer to extract random values of a variety of types:
```

class Random a where
randomR :: RandomGen g => (a, a) -> g -> (a, g)
random :: RandomGen g => g >> (a, g)
randomRs :: RandomGen g => (a, a) -> g -> [a]
randoms :: RandomGen g => g -> [a]
randomRIO :: (a,a) -> IO a
randomIO :: IO a
-- Default methods
randoms g = x : randoms g'
where
(x, g') = random g
randomRs = ...similar...
randomIO = getStdRandom random
randomRIO range = getStdRandom (randomR range)
instance Random Int where ...
instance Random Integer where ...
instance Random Float where ...
instance Random Double where ...
instance Random Bool where ...
instance Random Char where ...

```
- randomR takes a range \((l o, h i)\) and a random number generator \(g\), and returns a random value uniformly distributed in the closed interval [lo, hi], together with a new generator. It is unspecified what happens if \(l o>h i\). For continuous types there is no requirement that the values \(l o\) and \(h i\) are ever produced, but they may be, depending on the implementation and the interval.
- random does the same as randomR, but does not take a range.
- For bounded types (instances of Bounded, such as Char), the range is normally the whole type.
- For fractional types, the range is normally the semi-closed interval \([0,1)\).
- For Integer, the range is (arbitrarily) the range of Int.
- The plural versions, randomRs and randoms, produce an infinite list of random values, and do not return a new generator.
- The IO versions, randomRIO and randomIO, use the global random number generator (see Section 27.3).

\subsection*{27.3 The global random number generator}

There is a single, implicit, global random number generator of type StdGen, held in some global variable maintained by the IO monad. It is initialised automatically in some system-dependent fashion, for example, by using the time of day, or Linux's kernel random number generator. To get deterministic behaviour, use setStdGen.
```

setStdGen :: StdGen -> IO ()
getStdGen :: IO StdGen
newStdGen :: IO StdGen
getStdRandom :: (StdGen -> (a, StdGen)) -> IO a

```
- getStdGen and setStdGen get and set the global random number generator, respectively.
- newStdGen applies split to the current global random generator, updates it with one of the results, and returns the other.
- getStdRandom uses the supplied function to get a value from the current global random generator, and updates the global generator with the new generator returned by the function. For example, rollDice gets a random integer between 1 and 6:
```

rollDice :: IO Int
rollDice = getStdRandom (randomR (1,6))

```

\section*{References}
[1] FW Burton and RL Page, "Distributed random number generation", Journal of Functional Programming, 2(2):203-212, April 1992.
[2] SK Park, and KW Miller, "Random number generators - good ones are hard to find", Comm ACM 31(10), Oct 1988, pp1192-1201.
[3] DG Carta, "Two fast implementations of the minimal standard random number generator", Comm ACM, 33(1), Jan 1990, pp87-88.
[4] P Hellekalek, "Don't trust parallel Monte Carlo", ACM SIGSIM Simulation Digest 28(1), pp8289, July 1998.

The Web site http://random.mat.sbg.ac.at/is a great source of information.

\section*{Bibliography}
[1] J. Backus. Can programming be liberated from the von Neumann style? A functional style and its algebra of programs. CACM, 21(8):613-641, August 1978.
[2] H.B. Curry and R. Feys. Combinatory Logic. North-Holland Pub. Co., Amsterdam, 1958.
[3] L. Damas and R. Milner. Principal type schemes for functional programs. In Proceedings of the 9th ACM Symposium on Principles of Programming Languages, pages 207-212, Albuquerque, N.M., January 1982.
[4] K-F. Faxén A static semantics for Haskell Journal of Functional Programming, 2002.
[5] J.R. Hindley. The principal type scheme of an object in combinatory logic. Transactions of the American Mathematical Society, 146:29-60, December 1969.
[6] P. Hudak, J. Fasel, and J. Peterson. A gentle introduction to Haskell. Technical Report YALEU/DCS/RR-901, Yale University, May 1996.
[7] Mark P. Jones. A system of constructor classes: overloading and implicit higher-order polymorphism. Journal of Functional Programming, 5(1), January 1995.
[8] Mark P. Jones. Typing Haskell in Haskell. Haskell Workshop, Paris, October 1999.
[9] P. Penfield, Jr. Principal values and branch cuts in complex APL. In APL ' 81 Conference Proceedings, pages 248-256, San Francisco, September 1981.
[10] S.L. Peyton Jones. The Implementation of Functional Programming Languages. Prentice-Hall International, Englewood Cliffs, New Jersey, 1987.
[11] Unicode Consortium. The Unicode Standard, Version 3.0. Addison Wesley, Reading, MA, 2000.
[12] P. Wadler and S. Blott. How to make ad hoc polymorphism less ad hoc. In Proceedings of the 16th ACM Symposium on Principles of Programming Languages, pages 60-76, Austin, Texas, January 1989.

\section*{Index}

Index entries that refer to nonterminals in the Haskell syntax are shown in an italic font. Code entities are shown in typewriter font. Ordinary index entries are shown in a roman font.
```

!, 45,173,174,176,177
!!, 55, 115, 116
\$, 55, 81, 104, 110
\$!, 81, 104, 110
%,151,153
\&\&, 55, 79, 104, 110
( , ),80
(, , ),80
(), see trivial type and unit expression
*, 55, 90, 91, 104, 105

* *, 55, 91, 104, 106
+, 55, 90, 91, 104, 105, see also }n+k\mathrm{ pattern
++, 55, 115
-, 55, 90, 91, 104, 105, see also negation
., 55, 81, 104, 110
/, 55, 90, 91, 104, 106
/ /, 173, 175-177
/=, 55, 82, 104, 142
:, 55,80
:+, 155,156
: :, 30
<, 55, 84, 104, 142
<=, 55, 84, 104, 142
=<<, 88, 104, 109
==, 55, 82, 104, 142
>, 55, 84, 104, 142
>=, 55, 84, 104, 142
>>, 55, 88, 97, 104, 109
>>=,55, 88, 97, 104, 109
@, see as-pattern
[] (nil), 80
<br>, 180, 186, 187
\, see lambda abstraction
\&,12
<br>,182
\a,12
\b, 12
\f, 12

```
\n, 12
\r, 12
\t, 12
\v, 12
\(\perp, 17,230\)
^, 55, 91, 92, 104, 109
^ ^, 55, 91, 92, 104, 109
_, see wildcard pattern
||, 55, 79, 104, 110
~, see irrefutable pattern
abbreviated module, 66
abs, 91, 92, 105
AbsoluteSeek, 217
abstract datatype, 44, 76
accum, 173, 175, 177
accumArray, 173, 174, 177
acos, 91, 106
acosh, 91, 106
addToClockTime, 233, 235
aexp, 16, 20-23, 139
algebraic datatype, 43, 67, 141
all, 119
alt, 25, 139
alts, 25, 139
ambiguous type, 51, 62
and, 119
ANY, 9, 129
any, 9, 129
any, 119
ANYseq, 9, 129
ap, 202, 206
apat, 31, 140
appendFile, 97, 126
AppendMode, 214
application, 19
function, see function application operator, see operator application
approxRational, \(92,93,151,153\)
arithmetic operator, 90
arithmetic sequence, 23,80
Array (datatype), 174
Array (module), 161, 173, 176, 197
array, 173
accumulated, 174
derived, 176
array, 173, 174, 177
as-pattern (@), 31, 33
ascDigit, 9, 129
ascii, 12, 130
ASCII character set, 7
ascLarge, 9, 129
ascSmall, 9, 129
ascSymbol, 9, 129
asin, 91, 106
asinh, 91, 106
assocs, 173, 174, 177
asTypeOf, 114
atan, 91, 106
atan2, 92, 93, 108
atanh, 91, 106
atype, 40, 137
basic input/output, 95
binding, 37
function, see function binding
pattern, see pattern binding
simple pattern, see simple pattern binding
body, 66, 136
Bool (datatype), 79, 110
boolean, 79
Bounded (class), 89, 105
derived instance, 51, 143
instance for Char, 111
bounds, 173, 174, 177
bracket, 211, 221
bracket_, 211, 221
break, 118
btype, 40, 137
BufferMode (datatype), 210
CalendarTime (datatype), 232, 235
calendarTimeToString, 233, 235
case expression, 25
catch, \(98,125,230\)
catMaybes, 193, 194
cdecl, 38, 47, 137
cdecls, 38, 47, 137
ceiling, 92, 93, 107
changing the directory, 226
Char (datatype), 79, 110
Char (module), 121, 161, 195, 197, 234
char, 12, 130
character, 79
literal syntax, 12
character set
ASCII, see ASCII character set
transparent, see transparent character set
charesc, 12, 130
chr, 195, 198
cis, 155, 156
class, 38, 47
class, 41, 137
class assertion, 41
class declaration, 47, 67
with an empty where part, 48
class environment, 42
class method, \(38,39,48,49\)
clock time, 232
ClockTime (datatype), 232, 234
closecom, 9, 129
closing a file, 215
closure, 74
cname, 69, 136
cntrl, 12, 130
coercion, 93
comment, 9
end-of-line, 9
nested, 9
comment, 9, 129
compare, 84, 104, 142
Complex (module), 155, 156
con, 17, 18, 140
concat, 115
concatMap, 115
conditional expression, 21
conid, 10, 11, 130
conjugate, 155, 156
conop, 17, 18, 140
const, 81,110
constr, 43, 138
constrs, 43, 138
constructor class, 38
constructor expression, 40
consym, 10, 130
context, 41
context, 41, 137
context reduction, 59
cos, 91, 106
cosh, 91, 106
cosine, 93
CPU time, 241
CPUTime (module), 241
cpuTimePrecision, 241
createDirectory, 224, 225
creating a file, 214
ctDay, 232, 235
ctHour, 232, 235
ctIsDST, 232, 235
ctMin, 232, 235
current directory, 226
curry, 80, 114
Curry, Haskell B., vii
cycle, 117
dashes, 9, 129
data constructor, 43
data declaration, 27, 43
datatype, 43
abstract, see abstract datatype
algebraic, see algebraic datatype
declaration, see data declaration
recursive, see recursive datatype
renaming, see newtype declaration
dateFmt, 239, 240
dateTimeFmt, 239, 240
Day (datatype), 232, 234
dclass, 43, 138
decimal, 11, 130
decl, 38, 56, 137
declaration, 37
class, see class declaration
datatype, see data declaration
default, see default declaration
fixity, see fixity declaration
import, see import declaration
instance, see instance declaration
within a class declaration, 47
within a let expression, 24
within an instance declaration, 49
declaration group, 58
decls, 38, 137
decodeFloat, 92, 93, 108
default class method, 48, 50, 145
default declaration, 51
defaultTimeLocale, 239, 240
delete, 180, 182, 187
deleteBy, 180, 184, 187
deleteFirstsBy, 180, 184, 187
deleting directories, 225
deleting files, 225
denominator, 151, 153
dependency analysis, 58
derived instance, 51, 141, see also instance declaration
deriving, 43, 138
diffClockTimes, 233, 235
digit, 9, 129
digitToInt, 195, 197
directories, 213, 224
Directory (module), 224
div, 55, 90, 91, 104, 106
divMod, 91, 106
do expression, 26, 97
doDiv, 163
doesDirectoryExist, 224
doesFileExist, 224
Double (datatype), 89, 92, 112
drop, 118
dropWhile, 118
e, 167
Either (datatype), 81, 111
either, 81, 111
elem, 55, 115, 119
elemIndex, 180, 182, 186
elemIndices, 180, 182, 186
elems, 173, 174, 177
encodeFloat, 92, 93, 108
end of file, 215
entity, 65
Enum (class), 51, 86, 105
derived instance, 51, 142
instance for Char, 111
instance for Double, 113
instance for Float, 113
instance for Ratio, 154
superclass of Integral, 106
enumeration, 170
enumFrom, 86, 105
enumFromThen, 86, 105
enumFromThenTo, 86, 105
enumFromTo, 86, 105
environment
class, see class environment
type, see type environment
environment variables, 229
EQ, 81
Eq (class), 82, 89, 104
derived instance, 51,142
instance for Array, 177
instance for Char, 110
superclass of Num, 105
superclass of Ord, 104
error, 4, 17
error, 17, 114, 230
escape, 12, 130
even, 91, 108
exception handling, 98
executable, 224
execution time, 241
ExitCode (datatype), 229
ExitFailure, 229
exitFailure, 229
ExitSuccess, 229
exitWith, 229
\(\exp ^{i}, 16,139\)
\(\exp , 16,19,21,24-26,30,139\)
exp, 91, 106
exponent, 92, 94, 108
exponentiation, 91
export, 67, 136
export list, 66
exports, 67, 136
expression, 4, 15
case, see case expression
conditional, see conditional expression
let, see let expression
simple case, see simple case expression
type, see type expression
unit, see unit expression
expression type-signature, 30, 52
expt, 162
expts, 163
f, 167
fail, \(88,99,109,212\)
False, 79
fbind, 28, 139
fexp, 16, 19, 139
FFFormat (datatype), 164
field label, see label, 44
construction, 28
selection, 27
update, 29
fielddecl, 43, 138
file, 213
file buffering, 215
file system, 213
FilePath (type synonym), 97, 125
filter, 115
filterM, 202, 207
find, 180, 182, 186
findIndex, 180, 186
findIndices, 180, 186
fixity, 18
fixity, 38, 54, 137
fixity declaration, 48, 49, 54
flip, 81,110
Float (datatype), 89, 92, 112
float, 12
floatDigits, 92, 93, 108
Floating (class), 89, 91, 106
instance for Complex, 158
superclass of RealFloat, 108
floating literal pattern, 33
floatRadix, 92, 93, 108
floatRange, 92, 93, 108
floatToDigits, 159, 166
floor, 92, 93, 107
flushing a file buffer, 217
fmap, \(87,109,176\)
foldl, 116
foldl1, 116
foldM, 202, 207
foldr, 117
foldr1, 117
formal semantics, 3
formatCalendarTime, 233, 236
formatRealFloat, 165
formfeed, 9, 129
fpat, 31, 140
fpats, 31, 140
Fractional (class), 18, 89, 91, 106
instance for Complex, 157
instance for Ratio, 153
superclass of Floating, 106
superclass of RealFrac, 107
fromEnum, 86, 105
fromInteger, 18, 90, 91, 105
fromIntegral, 92, 94, 109
fromJust, 193, 194
fromMaybe, 193, 194
fromRat, 159, 162
fromRat', 162
fromRational, 18, 90, 91, 106
fst, 80,114
function, 81
function binding, 55, 56
function type, 40, 41
functional language, vii
Functor (class), 87, 109
instance for [ ] , 113
instance for Array, 177
instance for IO, 111
instance for Maybe, 111
functor, 87
funlhs, 138
gap, 12, 130
gcd, 91, 92, 108
gcon, 18, 140
gconsym, 18
\(g d, 25,56,138\)
gdpat, 25, 139
gdrhs, 56, 138
gendecl, 38, 47, 54, 137
generalization, 59
generalization preorder, 42
generator, 23
genericDrop, 181, 189
genericIndex, 181, 190
genericLength, 181, 189
genericReplicate, 181, 190
genericSplitAt, 181, 189
genericTake, 181, 189
genRange, 244, 245
get the contents of a file, 219
getArgs, 229
getChar, 96, 125
getClockTime, 233, 235
getContents, 96, 125
getCPUTime, 241
getCurrentDirectory, 224,225
getDirectoryContents, 224, 225
getEnv, 229
getLine, 96, 125
getModificationTime, 224
getPermissions, 224
getProgName, 229
getStdGen, 244, 248
getstdRandom, 244, 248
graphic, 9, 129
group, 181, 183, 187
groupBy, 181, 184, 188
GT, 81
gtycon, 40, 49, 137
guard, 23, 25, 34
guard, 202, 206
Handle (datatype), 210
HandlePosn (datatype), 210
handles, 213
Haskell, vii, 3
Haskell kernel, 4
hClose, 210, 215
head, 115
hexadecimal, 11, 130
hexit, 9, 129
hFileSize, 211, 215
hFlush, 211, 217
hGetBuffering, 211, 217
hGetChar, 211, 218
hGetContents, 211, 214, 219
hGetLine, 211, 218
hGetPosn, 211, 217
hiding, 70
Hindley-Milner type system, 4, 38, 58
hIsClosed, 211, 218
hIsEOF, 211, 215
hIsOpen, 211, 218
hIsReadable, 211, 218
hIsSeekable, 211, 218
hIsWritable, 211, 218
hLookAhead, 211, 216, 219
hPrint, 211, 219
hPutChar, 211, 219
hPutStr, 211, 219
hPutStrLn, 211
hReady, 211, 218
hSeek, 211, 217
hSetBuffering, 211, 216
hSetPosn, 211, 217
hWaitForInput, 211, 218

I/O, 210
I/O errors, 212
id, 81, 110
idecl, 38, 49, 137
idecls, 38, 49, 137
identifier, 9
if-then-else expression, see conditional expression
imagPart, 155, 156
impdecl, 69, 136
impdecls, 66, 136
import, 69, 136
import declaration, 69
impspec, 69, 136
index, 169, 171, 172
indices, 173, 174, 177
init, 116
inits, 181, 183, 188
inlining, 147
input/output, 210
input/output examples, 219
inRange, 169, 171, 172
insert, 181, 183, 188
insertBy, 181, 184, 189
inst, 49, 138
instance declaration, 49, 50, see also derived instance
importing and exporting, 71
with an empty where part, 49
Int (datatype), 89, 92, 112
Integer (datatype), 92, 112
integer, 12
integer literal pattern, 33
integerLogBase, 163
Integral (class), 89, 91, 106
interact, 96, 126
intersect, 181, 182, 187
intersectBy, 181, 184, 187
intersperse, 181, 183, 187
intToDigit, 195, 198
IO, 210
IO (datatype), 81, 111
IO (module), 210, 221
ioeGetErrorString, 211, 212
ioeGetFileName, 211, 212
ioeGetHandle, 211, 212
IOError (datatype), 81, 125, 212
ioError, 98, 125
IOMode (datatype), 210, 214
irrefutable pattern, 24, 33, 34, 57
isAlpha, 195, 197
isAlphaNum, 195, 197
isAlreadyExistsError, 211, 212
isAlreadyInUseError, 211, 212, 215
isAscii, 195, 197
isControl, 195, 197
isDigit, 195, 197
isDoesNotExistError, 211, 212, 215
isEOF, 211, 215
isEOFError, 211, 212
isFullerror, 211, 212
isHexDigit, 195, 197
isIllegalOperation, 211, 212
isJust, 193, 194
```

isLatin1, 195,197
isLower,195,197
isNothing, 193,194
isOctDigit, 195,197
isPermissionError, 211, 212, 215
isPrefixOf, 181, 188
isPrint,195,197
isSpace, 195,197
isSuffixOf, 181, 188
isUpper, 195,197
isUserError,211,212
iterate,117
Ix (class), 169, 171, 172
derived instance, }17
instance for Char, }17
instance for Integer, 172
instance for Int, 172
Ix (module), 169, 172, 173, 176, 210, 232,
234
ixmap, 173, 176,177
join, 202, 206
Just, 81

```
kind, 39, 40, 44, 45, 50, 64
kind inference, 40, 44, 45, 50, 64
label, 27
lambda abstraction, 19
large, 9, 129
last, 116
layout, 13, 130, see also off-side rule
\(1 \mathrm{~cm}, 91,92,109\)
Left, 81
length, 116
let expression, 24
in do expressions, 26
in list comprehensions, 23
lex, 86, 122
lexDigits, 159, 168
lexeme, 9, 129
lexical structure, 7
lexLitChar, 195, 199
lexp \({ }^{i}, 16,139\)
libraries, 74
liftM, 202, 207
liftM2, 202, 207
liftM3, 202, 207
liftM4, 202, 207
liftm5, 202, 207
linear pattern, 19, 31, 56
linearity, 19, 31, 56
lines, 118
List (module), 176, 180, 186
list, 21, 40, 80
list comprehension, 23, 80
list type, 41
listArray, 173, 174, 177
listToMaybe, 193, 194
literal, 9, 129
literal pattern, 32
literate comments, 134
Locale (module), 234, 239, 240
locale, 239
log, 91, 106
logarithm, 91
logBase, 91, 106
lookahead, 219
lookup, 119
lpat \({ }^{i}\), 31, 140
LT, 81
magnitude, 92
magnitude, 155, 156
Main (module), 65
main, 65
making directories, 225
map, 115
mapAccumL, 181, 183, 188
mapAccumR, 181, 183, 188
mapAndUnzipM, 202, 206
mapm, 88, 109
mapMaybe, 193, 194
mapM_, 88, 109
match, 198
max, 84, 104, 142
maxBound, 89, 105, 143
maximal munch rule, \(9,12,127\)
maximum, 119
maximumBy, 181, 184, 189
Maybe (datatype), 81, 111

Maybe (module), 186, 193, 194
maybe, 81,111
maybeToList, 193, 194
method, see class method
min, \(84,104,142\)
minBound, 89, 105, 143
minimum, 119
minimumby, 181, 184, 189
mkPolar, 155, 156
mkStdGen, 244, 246
mod, 55, 90, 91, 104, 106
modid, 11, 66, 130, 136
module, 65
module, 66, 136
Monad (class), 27, 88, 109
instance for [ ] , 113
instance for IO, 111
instance for Maybe, 111
superclass of MonadPlus, 206
Monad (module), 202, 206
monad, 26, 88, 95
MonadPlus (class), 202, 206
instance for [ ] , 206
instance for Maybe, 206
monomorphic type variable, \(34,60,61\)
monomorphism restriction, 61
Month (datatype), 232, 234
moving directories, 225
moving files, 225
mplus, 202, 206
msum, 202, 206
mzero, 202, 206
\(n+k\) pattern, 32, 33, 57
name
qualified, see qualified name
special, see special name
namespaces, 5, 10
ncomment, 9, 129
negate, 19, 90, 91, 105
negation, 16, 19, 20
newconstr, 46, 138
newline, 9, 129
newStdGen, 244, 248
newtype declaration, 32-34, 46
next, 244, 245
nonnull, 168
not, 79,110
notElem, 55, 115, 119
Nothing, 81
nub, 180, 182, 186
nubBy, 180, 184, 187
null, 116
Num (class), 18, 52, 89, 91, 105
instance for Complex, 157
instance for Ratio, 153
superclass of Fractional, 106
superclass of Real, 105
number, 89
literal syntax, 11
translation of literals, 18
numerator, 151, 153
Numeric (module), 121, 159, 161, 197
numeric type, 90
numericEnumFrom, 113
numericEnumFromThen, 113
numericEnumFromThenTo, 113
numericEnumFromTo, 113
octal, 11, 130
octit, 9, 129
odd, 91, 108
off-side rule, 13, see also layout
op, 17, 54, 140
opencom, 9, 129
openFile, 210, 214
opening a file, 214
operating system commands, 230
operator, \(9,10,19\)
operator application, 19
ops, 38, 54, 137
or, 119
Ord (class), 84, 89, 104
derived instance, 51, 142
instance for Array, 178
instance for Char, 111
instance for Ratio, 153
superclass of Real, 105
ord, 195, 198
Ordering (datatype), 81, 112
otherwise, 79, 110
overloaded functions, 38
overloaded pattern, see pattern-matching
overloading, 47
defaults, 51
partition, 181, 183, 187
pat \({ }^{i}\), 31, 140
pat, 31, 140
path, 225
pattern, 25, 30
@, see as-pattern
_, see wildcard pattern
constructed, see constructed pattern
floating, see floating literal pattern
integer, see integer literal pattern
irrefutable, see irrefutable pattern
linear, see linear pattern
\(n+k\), see \(n+k\) pattern
refutable, see refutable pattern
pattern binding, 55, 57
pattern-matching, 30
overloaded constant, 34
Permissions (datatype), 224
phase, 155, 156
physical file, 213
pi, 91, 106
polar, 155, 156
polling a handle for input, 218
polymorphic recursion, 54
polymorphism, 4
pragmas, 147
precedence, 43, see also fixity
pred, 105
Prelude
implicit import of, 75
Prelude (module), 75, 103
PreludeBuiltin (module), 103, 125
PreludeIO (module), 103, 125
PreludeList (module), 103, 115
PreludeText (module), 103, 121
principal type, 42, 53
print, 96, 125
product, 119
program, 9, 129
program arguments, 229
program name, 229
program structure, 3
properFraction, 92, 93, 107
putChar, 96, 125
putStr, 96, 125
putStrLn, 96, 125
qcon, 18, 140
qconid, 11, 130
qconop, 18, 140
qconsym, 11, 130
qop, 18, 19, 140
qtycls, 11, 130
qtycon, 11, 130
qual, 23, 139
qualified name, 11, 70, 72
qualifier, 23
quantification, 41
quot, \(90,91,104,106\)
quotRem, 91, 106
quar, 18, 140
quarid, 11, 130
quarop, 18,140
quarsym, 11, 130
Random (class), 247
Random (module), 244
random, 244, 247
random access files, 217
RandomGen, 245
randomIO, 244,247
randomR, 244, 247
randomRIO, 244, 247
randomRs, 244, 247
randoms, 244,247
range, \(169,171,172\)
rangeSize, 169, 171, 172
Ratio (datatype), 151
Ratio (module), 103, 151, 153, 161
Rational (type synonym), 151, 153
rational numbers, 151
Read (class), 85, 121
derived instance, 51, 143
instance for [a], 124
instance for Array, 178
instance for Char, 124
instance for Double, 123
instance for Float, 123
instance for Integer, 123
instance for Int, 123
instance for Ratio, 154
read, \(85,86,122\)
readable, 224
readDec, 159, 164
readEsc, 198
readFile, \(97,126,214\)
readFloat, 159, 168
readHex, 159, 164
reading a directory, 226
reading from a file, 218
readInt, 159,164
readIO, 96, 126
readList, 85, 121, 143
readLitChar, 195, 198
readLn, 96, 126
ReadMode, 214
readOct, 159,164
readParen, 122
ReadS (type synonym), 85, 121
reads, \(85,86,122\)
readSigned, 159,163
readsPrec, \(85,121,143\)
ReadWriteMode, 214
Real (class), 89, 91, 105
instance for Ratio, 153
superclass of Integral, 106
superclass of RealFrac, 107
Realfloat (class), 92, 93, 108
RealFrac (class), 92, 107
instance for Ratio, 154
superclass of RealFloat, 108
realPart, 155, 156
realToFrac, 92, 94, 109
recip, 91, 106
recursive datatype, 46
refutable pattern, 33
RelativeSeek, 217
rem, 55, 90, 91, 104, 106
removeDirectory, 224, 225
removeFile, 224, 225
removing directories, 225
removing files, 225
renameDirectory, 224, 225
renameFile, 224, 225
renaming directories, 225
renaming files, 225
repeat, 117
replicate, 117
reservedid, 10, 130
reservedop, 10, 130
return, 88,109
reverse, 119
rexp \({ }^{i}\), 16, 139
rhs, 56, 138
Right, 81
round, 92, 93, 107
roundTo, 166
rpat \({ }^{i}\), 31, 140
scaleFloat, 92, 108
scaleRat, 162
scanl, 116
scanl1, 116
scanr, 117
scanr1, 117
scontext, 137
searchable, 224
section, 10, 20, see also operator application
SeekFromEnd, 217
seeking a file, 217
SeekMode (datatype), 210, 217
semantics
formal, see formal semantics
semi-closed handles, 214
separate compilation, 76
seq, \(81,104,110\)
sequence, 88,109
sequence_, 88, 109
setCurrentDirectory, 224, 225
setPermissions, 224
setStdGen, 244, 248
setting the directory, 226
Show (class), 85, 121
derived instance, 51, 143
instance for [a], 124
instance for Array, 178
instance for Char, 124
instance for Double, 123
instance for Float, 123
instance for HandlePosn, 210
instance for Integer, 123
instance for Int, 123
instance for Ratio, 154
superclass of Num, 105
show, \(85,86,121\)
show2, 237
show2', 237
show3, 237
showChar, 122
showEFloat, 159,164
showFFloat, 159, 164
showFloat, 159,164
showGFloat, 159, 164
showHex, 159, 163
showInt, 159,163
showIntAtBase, 159, 163
showList, \(85,121,143\)
showLitChar, 195, 199
showOct, 159, 163
showParen, 122
Shows (type synonym), 85, 121
shows, \(85,86,122\)
showSigned, 159, 163
showsPrec, \(85,121,143\)
showString, 122
sign, 92
signature, see type signature
signdecl, 53
significand, 92, 94, 108
signum, 91, 92, 105
simple pattern binding, 57, 62
simpleclass, 41, 137
simpletype, 43, 45, 46, 138
sin, 91, 106
sine, 93
sinh, 91, 106
size of file, 215
small, 9, 129
snd, 80,114
sort, 181, 183, 188
sortBy, 181, 184, 188
space, 9, 129
span, 118
special, 9, 129
split, 244, 245
splitAt, 118
sqrt, 91, 106
standard handles, 213
standard prelude, 74 , see also Prelude
stderr, 210, 213
StdGen (datatype), 244, 246
stdin, 210, 213
stdout, 210, 213
stmt, 26, 139
stmts, 26, 139
strictness flag, 45
strictness flags, 82
String (type synonym), 80, 111
string, 79
literal syntax, 12
transparent, see transparent string
string, 12, 130
subtract, 108
succ, 105
sum, 119
superclass, 48, 49
symbol, 9, 129, 130
synonym, see type synonym
syntax, 127
System (module), 229
system, 229
tab, 9, 129
tail, 115
tails, 181, 183, 188
take, 117
takeWhile, 118
tan, 91, 106
tangent, 93
tanh, 91, 106
tdDay, 232, 235
tdHour, 232, 235
tdMin, 232, 235
tdMonth, 232, 235
tdPicosec, 232, 235
tdYear, 232, 235
terminating a program, 230
the file system, 224
Time (module), 224, 232, 234
time, 232
time of day, 232
time12Fmt, 239, 240
TimeDiff (datatype), 232, 235
timeFmt, 239, 240
TimeLocale (datatype), 239, 240
to12,236
toCalendarTime, 233, 235
toClockTime, 233, 235
toEnum, 86, 105
toInteger, 106
toLower, 195, 198
topdecl (class), 47
topdecl (data), 43
topdecl (default), 51
topdecl (instance), 49
topdecl (newtype), 46
topdecl (type), 45
topdecl, 38, 136
topdecls, 38, 66, 136
toRational, 91, 93, 105
toUpper, 195, 198
toUTCTime, 233, 235
transpose, 181, 183, 187
trigonometric function, 93
trivial type, 22, 40, 81
True, 79
truncate, 92, 93, 107
try, 211, 221
tuple, 22, 40, 80
tuple type, 41
tycls, 11, 41, 130
tycon, 11, 130
type, \(4,39,42\)
ambiguous, see ambiguous type
constructed, see constructed type
function, see function type
list, see list type
monomorphic, see monomorphic type
numeric, see numeric type
principal, see principal type
trivial, see trivial type
tuple, see tuple type
type, 40, 137
type class, 4,38 , see class
type constructor, 43
type environment, 42
type expression, 40
type renaming, see newtype declaration
type signature, \(42,49,53\)
for an expression, see expression typesignature
type synonym, 45, 49, 67, see also datatype
recursive, 46
tyvar, 11, 41, 130
uncurry, 80, 114
undefined, 17, 114
unfoldr, 181, 188
Unicode character set, 7, 12
UnicodePrims (module), 103, 197
uniDigit, 9, 129
uniLarge, 9, 129
union, 180, 182, 187
unionBy, 180, 184, 187
uniSmall, 9, 129
uniSymbol, 9, 129
unit datatype, see trivial type
unit expression, 22
uniWhite, 9, 129
unless, 202, 206
unlines, 119
until, 81, 114
unwords, 119
unzip, 120
unzip3, 120
unzip4, 181, 185, 190
unzip5, 181, 191
unzip6, 181, 191
unzip7, 181, 191
userError, 98, 125
valdefs, 49
value, 4
var, 17, 18, 140
varid, 10, 11, 130
varop, \(17,18,140\)
vars, 38, 53, 137
varsym, 10,130
vertab, 9, 129
when, 202, 206
whitechar, 9, 129
whitespace, 9, 129
whitestuff, 9, 129
wildcard pattern (_), 31
words, 118
writable, 224
writeFile, 97, 126, 214
WriteMode, 214
zip, 80, 120
zip3,120
zip4, 181, 185, 190
zip5, 181, 190
zip6,181,190
zip7,181, 190
zipWith, 120
zipWith3,120
zipWith4,181, 185, 190
zipWith5, 181, 190
zipWith6,181, 190
zipWith7,181, 190
zipWithM, 202, 207
zipWithM_, 202, 207```


[^0]:    ${ }^{1}$ Miranda is a trademark of Research Software Ltd.

[^1]:    ${ }^{1}$ A similar transformation is described in Peyton Jones' book [10].

[^2]:    ${ }^{1}$ There are two minor exceptions to this statement. First, default declarations scope over a single module (Section 4.3.4). Second, Rule 2 of the monomorphism restriction (Section 4.5.5) is affected by module boundaries.

