McMaster University
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SFWR ENG 3E03
Rules
Fall 2005

Design and Selection of Programming Languages

Operational Semantics Rules

Assignment: $$\frac{\sigma(e) \Rightarrow v}{\sigma(x := e) \Rightarrow \sigma \oplus \{x \mapsto v\}}$$

Sequence: $$\frac{\sigma_1(s_1) \Rightarrow \sigma_2 \qquad \sigma_2(s_2) \Rightarrow \sigma_3}{\sigma_1(s_1;\ s_2) \Rightarrow \sigma_3}$$

Conditional: $$\frac{\sigma(b) \Rightarrow \mathsf{True} \qquad \sigma(s_1) \Rightarrow \sigma_1}{\sigma(\textbf{if } b \textbf{ then } s_1 \textbf{ else } s_2 \textbf{ fi}) \Rightarrow \sigma_1} \qquad \frac{\sigma(b) \Rightarrow \mathsf{False} \qquad \sigma(s_2) \Rightarrow \sigma_2}{\sigma(\textbf{if } b \textbf{ then } s_1 \textbf{ else } s_2 \textbf{ fi}) \Rightarrow \sigma_2}$$

while: $$\frac{\sigma(b) \Rightarrow \mathsf{True} \qquad \sigma(s) \Rightarrow \sigma_1 \qquad \sigma_1(\textbf{while } b \textbf{ do } s \textbf{ od}) \Rightarrow \sigma_2}{\sigma(\textbf{while } b \textbf{ do } s \textbf{ od}) \Rightarrow \sigma_2} \qquad \frac{\sigma(b) \Rightarrow \mathsf{False}}{\sigma(\textbf{while } b \textbf{ do } s \textbf{ od}) \Rightarrow \sigma}$$

Axiomatic Semantics Rules

Assignment: $$\{\ P[x_1 \setminus e_1, \ldots, x_n \setminus e_n]\ \}\ (x_1, \ldots, x_n) := (e_1, \ldots e_n)\ \{\ P\ \}$$

Logical consequence: $$\frac{P \Rightarrow P' \qquad \{\ P'\ \}\ S\ \{\ Q'\ \} \qquad Q' \Rightarrow Q}{\{\ P\ \}\ S\ \{\ Q\ \}}$$

Sequence: $$\frac{\{\ P\ \}\ S_1\ \{\ R\ \} \qquad \{\ R\ \}\ S_2\ \{\ Q\ \}}{\{\ P\ \}\ S_1 \textbf{ ; } S_2\ \{\ Q\ \}}$$

Conditional: $$\frac{\{\ P \wedge b\ \}\ S_1\ \{\ Q\ \} \qquad \{\ P \wedge \neg b\ \}\ S_2\ \{\ Q\ \}}{\{\ P\ \}\ \textbf{if } b \textbf{ then } S_1 \textbf{ else } S_2 \textbf{ fi}\ \{\ Q\ \}}$$

while-Loop: $$\frac{\{\ INV \wedge b\ \}\ S\ \{\ INV\ \}}{\{\ INV\ \}\ \textbf{while } b \textbf{ do } S \textbf{ od}\ \{\ INV \wedge \neg b\ \}}$$

Assignment Tactics:

$$\frac{\dfrac{\textit{<< Reasoning >>}}{P\ \Rightarrow\ Q[x \setminus e]} \qquad \dfrac{\textit{True}}{\{\ Q[x \setminus e]\ \}\ x := e\ \{\ Q\ \}}\ \text{(Assignment Ax .)}}{\{\ P\ \}\ x := e\ \{\ Q\ \}}\ \text{(Left Cons .)}$$