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SE 3E03, Fall 2005
Prerequisite Exercises 1

Design and Selection of Programming Languages

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Review Exercises: Discrete Mathematics

Exercise P.1 (Set Cardinality)

Calculate the cardinalities of the following sets:

a) $\{1\}$	e) $\{1, 2, 1\}$	i) $\{\}$	m) $\{0, \varnothing\}$
b) $\{3\}$	f) $\{1, \{2\}, 1\}$	j) $\{\{\}\}$	n) $\{\varnothing, \{0\}\}$
c) $\{c\}$	g) $\{1, 2, \{1\}\}$	k) $\{\{\}, \varnothing\}$	o) $\{\{\{\}, \{0\}\}\}$
d) $\{c, d\}$	h) $\{1, 2, \{1, 2\}\}$	l) $\{\varnothing, \{\varnothing\}\}$	p) $\{\{\{\}, \{0\}\}, \{\{2-2\}, \varnothing\}\}$

Which of these sets contain some *non-empty* set both as subset and as element?

Exercise P.2 (Set Operations)

Let a, b, c, d are names of different, but otherwise unspecified elements, and let the following sets be given: $A = \{a, b\}$ and $B = \{b, c, d\}$. Calculate the following sets, i.e., for each of the following sets, explicitly list all its elements:

a) $A \cap B$

b) $A \cup B$

c) $A \times B$

d) $\{A\} \times \{0, 1, 2\}$

e) $A \rightarrow \{0, 1\}$

f) $A \nrightarrow \{0, 1\}$

Exercise P.3 (Set Comprehension)

List the elements of each of the following sets:

a) $\{x : \mathbb{N}_1 \mid x^2 < 20 \bullet x^3\}$

b) $\{x, y : \mathbb{N}_1 \mid 5 \leq x + y \leq 6 \bullet x * y\}$

c) $\{s : \mathbb{P}\{1, 2, 3\} \mid \#s \geq 2\}$

d) $\{x, y : \mathbb{N}_1 \mid x^2 + y^2 < 20\}$

e) $\{s : \mathbb{P}\,\mathbb{P}\{1, 2\} \mid \#s > \#\bigcup s\}$

Exercise P.4 (Relations)

Let the sets $X = \{1,2,3\}$, $Y = \{4,5\}$ und $Z = \{6,7,8,9\}$ and the following relations be given:

$$
\begin{array}{llrcl}
R : X \leftrightarrow Y & \text{with} & R & = & \{(1,4),(2,4),(2,5),(3,5)\} \\
S : X \leftrightarrow Z & \text{with} & S & = & \{(1,6),(1,7),(3,7),(3,9)\} \\
T : Z \leftrightarrow Y & \text{with} & T & = & \{(7,4),(9,4),(9,5)\} \\
U : Y \leftrightarrow X & \text{with} & U & = & \{(4,3),(5,1)\}
\end{array}
$$

In addition, we consider the subsets $A = \{1,2\}$, $B = \{4\}$, and $C = \{6,7\}$. List the elements of each of the following sets:

a) $A \times Y$

b) $\mathrm{id}\, X$

c) $\mathsf{ran}\, S$

d) $\mathsf{dom}\, (\mathrm{id}\, Z)$

e) $T^{\smile}$

f) $S ⨾ T$

g) $R ⨾ T^{\smile} \cap S$

h) $U \cap (Y \times A)$

i) $(R \setminus (\mathsf{dom}\, S \times Y)) \cup U^{\smile}$

j) $U \times A$

k) $A \leftrightarrow B$

l) $A \rightarrowtail\!\!\!\!\mapsto C$

m) $X \rightarrowtail\!\!\!\!\twoheadrightarrow Y$

n) $(\mathbb{P}\, R) \cap (X \to Y)$

o) $(\mathbb{P}\, T) \cap (Z \twoheadrightarrow\!\!\!\!\!\!\twoheadrightarrow Y)$

p) $\#(\mathbb{P}(X \leftrightarrow Z))$

Exercise P.5 (Reasoning)

For each of the following statements, check whether it is true, and if it is false, give a counterexample (the variable x ranges over integers, and is implicitly universally quantified in each item).

a) If $x > 5$, then $2 * x > 4$.

b) If $x > 5$ and $2 * x > 4$, then $3 * x > 18$.

c) If $x > 5$ and $2 * x < 4$, then $x = 42$.

d) $x > 5$ if and only if $2 * x > 10$.

e) $x < 5$ or $x > 5$.

f) $6 * x < 31$ and $7 * x > 31$ holds, if and only if $(x + 1) * (x + 2) = 42$.

Exercise P.6 (Relation Properties)

For each of the following statements, check whether it is true, and if it is false, give a counterexample:

a) A transitive and symmetric relation is reflexive, too.

b) The composition of two orders cannot be an equivalence.

c) Intersecting an order with an equivalence yields an order, again.

d) The composition of an injective mapping with a surjective mapping is injective, again.

e) The composition of a transitive relation with its converse is again transitive.

f) The composition of an asymmetric relation with its converse is again asymmetric.

g) If an injective function $F : A \rightarrowtail\!\!\!\!\mapsto B$ is contained in a surjective mapping $G : A \twoheadrightarrow\!\!\!\!\!\!\twoheadrightarrow B$, then G is bijective.

Exercise P.7 (Finite-State Machines)

a) Give a formal definition of Finite-State Machines

b) Define the meaning of “a finite state machine M accepts the word w”.