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SFWR ENG 3E03
Exercise Sheet 5
Solution Hints

Design and Selection of Programming Languages

12 October 2005

Exercise 5.1 — Haskell Evaluation (36% of Midterm 1, 2004)

Assume the following Haskell definitions to be given:

```
succ n = n+1                          -- reduce in one step, e.g.: succ 5
 → 6

take :: Int -> [a] -> [a]
take 0 _       =  []
take _ []      =  []
take n (x:xs)  =  x : take (n-1) xs

feed h q y =  q : feed h (q + y) (h y)
```

Simulate Haskell evaluation for the following expression (write down the sequence of intermediate expressions):

```
take 3 (feed succ 0 1)
```

Note: You may introduce **abbreviations for repeated subexpressions**, or use **repetition marks for material that is unchanged from the previous line**. In particular, ***write*** "*s*" ***instead of*** "*succ*"!

Solution Hints

13 steps, 1 contractible arith

take 3 (*feed succ* 0 1)
= *take* 3 (0 : *feed succ* (0 + 1) (*succ* 1))
= 0 : *take* (3−1) (*feed succ* (0 + 1) (*succ* 1))
= 0 : *take* 2 (*feed succ* (0 + 1) (*succ* 1))
= 0 : *take* 2 ((0 + 1) : *feed succ* ((0 + 1) + *succ* 1) (*succ* (*succ* 1)))
= 0 : (0 + 1) : *take* (2−1) (*feed succ* ((0 + 1) + *succ* 1) (*succ* (*succ* 1)))
= 0 : 1 : *take* (2−1) (*feed succ* (1 + *succ* 1) (*succ* (*succ* 1)))
= 0 : 1 : *take* 1 (*feed succ* (1 + *succ* 1) (*succ* (*succ* 1)))
= 0 : 1 : *take* 1 ((1 + *succ* 1) : *feed* (+) *succ* ((1 + *succ* 1) + *succ* (*succ* 1)) (*succ* (*succ* (*succ* 1)
)))
= 0 : 1 : (1 + *succ* 1) : *take* (1−1) (*feed succ* ((1 + *succ* 1) + *succ* (*succ* 1)) (*succ* (*succ* (*succ* 1)
)))
= 0 : 1 : (1 + 2) : *take* (1−1) (*feed succ* ((1 + 2) + *succ* (*succ* 1)) (*succ* (*succ* 2)))
= 0 : 1 : 3 : *take* (1−1) (*feed succ* (3 + *succ* (*succ* 1)) (*succ* (*succ* 2)))
= 0 : 1 : 3 : *take* 0 (*feed succ* (3 + *succ* (*succ* 1)) (*succ* (*succ* 2)))

$= 0 : 1 : 3 : [\,]$

3% per necessary step:
- 1% for reducing the right redex
- 2% for performing the reduction correctly
- -1% for not writing down

Exercise 5.2 — Haskell Typing (19% of Midterm 1, 2004)

Provide **detailed derivations** of the Haskell types of the following functions:

```
swibble x y = [ ( x , y ) , ( x ++ "'", y + 1 ) ]
swoon g h = [ g ( (1 + ) . h ) ]
```

Solution Hints

Type classes have not been taught yet, only mentioned: Numeric types can be defaulted to *Integer* or *Int*.

$swibble :: (Num\ n) \Rightarrow String \to n \to [(String, n)]$

Assuming $1 :: Integer$, we must have $y :: Integer$ because of $y + 1$.

Since $"" :: String$, we also have $x :: String$ because of $x +\!\!+ "'" :: String$.

Then $(x, y) :: (String, Integer)$, and the type of *swibble* follows easily.

$swoon :: (Num\ n) \Rightarrow ((a \to n) \to b) \to (a \to n) \to [b]$

Assuming $1 :: Integer$, we have $(1\,+\,) :: Integer \to Integer$, and because of the composition, we must have

$h :: a \to Integer$ for some type a.

Therefore, we have $((1+\,) \circ h) :: a \to Integer$, and may assume $g :: (a \to Integer) \to b$ for some type b.

Then we have $[\,g\,((1+\,) \circ h)\,] :: b$, and therefore

$swoon\ g :: (a \to Integer) \to b$

and

$swoon :: ((a \to Integer) \to b) \to (a \to Integer) \to b.$

Exercise 5.3 — Defining Haskell Functions (20% of Midterm 1, 2004)

Define the follwing Haskell functions (the solutions are independent of each other):

(a) $sum :: [Integer] \to Integer$

such that $sum\ xs$ evaluates to the sum of all elements of the list xs.

(b) $all :: (a \to Bool) \to [a] \to Bool$

such that $all\ p\ xs$ evaluates to **True** if p considered as a predicate holds for all elements of x, and

to **False** if there is at least one element in *xs* for which *p* does not hold.

E.g., *all* (> 1) [2..10] = **True**

(c) *selMod* :: *Integer* → [*Integer*] → [*Integer*]

such that *selMod k xs* selects from the list *xs* all those elements that are equivalent to *k* modulo *k* + 1, e.g.,

selMod 2 [2, 3, 8, 1, 2, 5] = [2, 8, 2, 5]

(d) *sources* :: *Eq a* ⇒ [(*a*, *a*)] → [*a*]

such that *sources ps* returns the *sources* of the graph *ps*.

Here, the list *ps* of pairs is considered as representing a simple graph by representing each edge from node *x* to node *y* by the pair (*x*, *y*).

The *context* "*Eq a* ⇒" just means that you may use the equality test for elements of type *a*, i.e., (==) :: *a* → *a* → *Bool*.

Example: *sources* [(2,3), (3,4), (1,4), (1,5), (2,5)] = [2,1]

(The order is irrelevant.)

Solution Hints

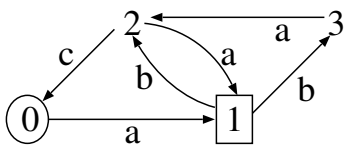
```
sum = foldl (+) 0

sum = foldr (+) 0

sum [ ] = 0
sum (x:xs) = x + sum xs


all = foldr (&&) True

all p [ ] = True
all p (x:xs) = p x && all p xs


selMod :: Integer → [ Integer ] → [ Integer ]
selMod k xs = [ x | x ← xs , x `mod` (k+1) ≡ k ]


sources, sources' :: Eq a ⇒ [ (a, a) ] → [ a ]
sources ps = let (srcs, trgs) = unzip ps
        in filter (`notElem` trgs) srcs

sources' ps = let trgs = [ snd p | p ← ps ]
        in = [ x | (x, y) ← ps, x `notElem` trgs ]
```

Exercise 5.4 — Finite-State Machines (25% of Midterm 1, 2004)

Let the following type synonyms be given, as in the presentation in the first lecture:

```
type State = Int
type Symbol = Char
type TransRel = [(State, Symbol, State)]
type FSM = (State, TransRel, [State])
```

(a) Define *fsm1* :: *FSM* such that it represents the following finite-state machine (with start state circled and end states in boxes):



(b) Define the Haskell function *isDet* :: *FSM* → *Bool*

such that *isDet fsm* evaluates to the Boolean value indicating whether the finite-state machine *fsm* is deterministic or not.

For example, *isDet fsm1* = **False** since leaving state 1, there are two *b*-edges directed towards different nodes.

Hint: Define auxiliary functions! For example:

- Calculate all start nodes of transitions in a *TransRel*.
- Given a state, calculate all edges leaving that state in a *TransRel*.
- Given a *Symbol* and a *TransRel*, find all target nodes of edges with that symbol.
- Given a *State* and a *TransRel*, find out whether any edges leaving that state violate determinacy.

Other functions may be useful, too. **Document your functions!**

Solution Hints

```
type State = Int
type Symbol = Char
type TransRel = [(State, Symbol, State)]

type FSM = (State, TransRel, [State])

fsm1 :: FSM              -- 6%
fsm1 = (0, tr1, [1])
 where
  tr1 =
   [(0,'a',1)
   ,(1,'b',2)
   ,(1,'b',3)
   ,(2,'a',1)
```

```
  ,(2,'c',0)
  ,(3,'a',2)
  ]

edgeStarts tr = [ s | (s, c, t) ← tr ]   -- 3%

outEdges tr s = [(c,t) | (s',c,t) ← tr, s' ≡ s ] -- 3%

isUnique es (c,t) = all (t ≡) [ t' | (c',t') ← es, c' ≡ c ] -- 5%

isDetState tr s = all (isUnique es) es -- 4%
 where es = outEdges tr s

isDet (s0, tr, fin) = all (isDetState tr) (edgeStarts tr) -- 4%
```