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SFWR ENG 3E03
Exercise Sheet 8

Design and Selection of Programming Languages

4 November 2005

Exercise 8.1 — Using Operational Semantics to Prove Incorrectness

The following Hoare triples do not hold.

For each of these Hoare triples, present a derivation in the operational semantics that proves a counterexample to the statement.

(a) $\{x \geq -5\}\ z := 5 - x\ \{z \leq 11 \wedge x \geq -3\}$

(b) $\{x \geq -5\}\ z := 5 - x\ \mathbf{;}\ x := z + 2\ \{z \leq 11 \wedge x \geq -3\}$

"Proving a counterexample" for the Hoare triple

$$\{pre\}Prog\{post\}$$

means to derive an assertion

$$\sigma_1(Prog) \Rightarrow \sigma_2$$

involving

– a state σ_1 for which *pre* **holds**, and

– a state σ_2 for which *post* **does not hold**.

Exercise 8.2 — Partial Correctness

For each of the following Hoare triples, determine whether it holds; if yes, prove it using the rules of axiomatic semantics, and if no, prove a counter-example using the rules of operational semantics (**you may abbreviate each *expression* evaluation into a single step**).

(a) $\{x \geq -5\}\ z := 5 - x\ \{z \leq 11\}$

(b) $\{x \geq -5\}\ z := 5 - x\ \{z \leq 11 \wedge x \geq -7\}$

(c) $\{x \geq -5\}\ z := 5 - x\ \{z \leq 11 \wedge x \geq -3\}$

(d) $\{x \geq -5\}\ z := 5 - x\ \mathbf{;}\ x := x + 2\ \{z \leq 11 \wedge x \geq -3\}$

(e) $\{x \geq -5\}\ z := 5 - x\ \mathbf{;}\ x := x + z\ \{z \leq 11 \wedge x = 2\}$

(f) $\{z = abs(x)\}$ **if** $x \geq 0$ **then** $z := -z$ **fi** $\{xz = -x^2\}$

(g) $\{z = 0\}$ **if** $x = 0$ **then** $w :=$ True **else** $z := 1/x$ **fi** $\{\neg w \rightarrow xz = 1\}$

Exercise 8.3 — Partial Correctness Proof — 50% of Midterm 4, 2003

Consider the following program in a language providing a Java-like printing statement:

```
s := 1;
r := 0;
while s ≤ n do
    r := r + 1;
    s := s + 2 * r + 1;
    println(r + " " + s)
od
```

(a) What is the output of this program for $n = 30$?

(b) Give an equation relating the values of r and s in each **println** statement.

(c) For this program **without the println** statement, **prove partial correctness** with respect to the **precondition** $\{ n \geq 0 \}$ and the **postcondition** $\{ r^2 \leq n \ \wedge \ n < (r+1)^2 \}$.

Hint: ***Use the equation*** from (b) **as part of the invariant!**