SFWR ENG 2F03 Assignment 1: Propositional Semantics

Due: 0830 Thursday September 23, 2004

1. **Propositional Semantics**

   The exclusive OR operator $\oplus$ has the following truth table:

p	q	$p \oplus q$
F	F	F
F	T	T
T	F	T
T	T	F

   a) How does this differ from the standard (non-exludsive) OR operator $\vee$?

   b) Check if $\oplus$ is:

   i) commutative

   ii) associative

   c) Write down the simplest contradiction that uses only the propositional variable p and the binary operator $\oplus$ (i.e. find the simplest formula ψ using p and $\oplus$ such that $\psi \equiv \bot$).

   d) Write down the simplest propositional formula ϕ that *only uses* the $\oplus$ operator, and one or more instances of the propositional constants $\top$ and/or $\bot$ such that $\phi \equiv \neg p$.

   e) What is the full Disjunctive Normal Form (DNF) for $p \oplus (q \oplus r)$?

   f) What is the full Conjunctive Normal Form (CNF) for $(p \oplus q) \oplus r$?

   g) Using what we already know from the lectures and assignments, is it possible to to "Build the World" using only $\oplus$, $\top$ and $\wedge$? Briefly justify your answer.

   h) Consider the formula:

   $$(p \vee q) \wedge (r \vee s)$$

   It is in Conjunctive Normal Form so it has only two "levels" of gates, i.e., travelling from a propositional variable (atom) in the parse tree to the root of the parse tree, we encounter 2 operator nodes first $\vee$, then $\wedge$.

   NOR gates are particularly well suited to directly implement formulas that are in Conjunctive Normal Form (CNF). Write down a formula with two levels of NOR gates that is equivalent to $(p \vee q) \wedge (r \vee s)$.