SFWR 2F03 Midterm

Date: Wednesday October 21, 1998
Duration: 2 hours
Answer all questions in the provided answer booklets. Fill in your name and student number and sign each booklet you use. No calculators allowed.

1. Hilbert Systems (25 marks total)

   Hilbert style systems for propositional logic write all the other operators in terms of $\rightarrow$ and $\perp$ and make use of tautologies (c)(i)-(iii) below and all their substitution instances together with the single rule of inference modus ponens to obtain a consistent and complete proof system.

a) **i)** (2 marks) Write down the truth table for $P \rightarrow Q$.

ii) (3 marks) The constant symbols $\top$ and $\perp$ are interpreted as T (true) and F (false) respectively. Find the simplest propositional logic formula that uses only $\perp$ and $\rightarrow$ and is logically equivalent to $\top$.
NOTE: No propositional variables or operators other than $\rightarrow$ can occur in the formula so you are NOT allowed to use $\neg\perp$ as your answer!

b) (10 marks) Show that you can "build the world" using $\rightarrow$ with the help of the propositional constant $\perp$. To do this first show that $\neg P$ can be written in terms of $\rightarrow$ and $\perp$. Next show that $P \vee Q$ can be written using $\neg$ and $\rightarrow$. Finally show that $P \wedge Q$ can be written in terms of $\vee$ and $\neg$. Since we know that any formula has a logically equivalent DNF formula that only uses $\neg, \vee$ and $\wedge$ we are done!

c) (10 marks) Verify that the following are tautologies:

i) $\neg(\neg P) \rightarrow P$

ii) $P \rightarrow (Q \rightarrow P)$

iii) $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$

iv) $(P \vee Q \leftrightarrow R) \rightarrow (\neg A \wedge B \rightarrow (P \vee Q \leftrightarrow R))$ (Note: There is an easy way for (iv)!)

2. Resolution Theorem Proving (25 marks total)

a) (5 marks) Indirect proof or "proof by refutation" is when one shows that some set of premises $\Gamma = \{P_1, P_2, \ldots, P_n\}$ is a valid argument for Q by showing that $\Gamma, \neg Q$ is inconsistent. Use the deduction theorem to show that $\Gamma, \neg Q \vdash \perp$ iff $\Gamma \vdash Q$.

b) (10 marks) Resolution rules of inference:

i) Show $P \vee Q, \neg Q \vee R \models P \vee R$ using truth tables. Therefore we have valid rule of inference Rule R1: "If $\Gamma \vdash P \vee Q$ and $\Gamma \vdash \neg Q \vee R$ then $\Gamma \vdash P \vee R$."

ii) Show $P \vee \neg Q, Q \vee R \vdash P \vee R$ by formal proof.

iii) Given the proof $P \vee \neg Q, Q \vee R \vdash P \vee R$, why can we conclude that $P \vee \neg Q, Q \vee R \models P \vee R$? This provides the valid rule of inference Rule R2: $\Gamma \vdash P \vee \neg Q$ and $\Gamma \vdash Q \vee R$ then $\Gamma \vdash P \vee R$."

These two rules together with the commutativity of $\vee$, Rule P for stating a premise and the instances of Rule T making use of the tautologies $P \leftrightarrow P \vee \perp$ and $P \wedge \neg P \leftrightarrow \perp$, comprise the complete set of rules of inference for resolution (refutation) theorem proving.

c) (10 marks) You will use resolution theorem proving to prove the following:

$$P \rightarrow Q, \neg(Q \wedge \neg R), \neg R \models \neg P$$

i) Replace $P \rightarrow Q$ and $\neg(Q \wedge \neg R)$ by logically equivalent formulas P_1 and P_2 that only use $\vee$ and $\neg$.

ii) Using only the rules of inference mentioned in (b) above, formally prove that prove that

$$P_1, P_2, \neg R, P \vdash \perp$$

(Continued on back . . .)

3. PVS (25 marks total)

a) (5 marks) Write down a PVS theorem called "C1" that you would attempt to prove to demonstrate the inconsistency of the set of premises:

$$\{P \rightarrow Q, \neg(Q \wedge \neg R), \neg R, P\}$$

Assume that the theory and propositional constant declarations already exist (i.e. You just have to write down the theorem).

b) (10 marks) After invoking the PVS prover on the theorem you stated in part (a), you apply the (FLATTEN) command and obtain the sequent: (continued on back)

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Rule? (flatten)
Applying disjunctive simplification to flatten sequent,
this simplifies to:
C1 :

{-1}    (P IMPLIES Q)
{-2}    P
  |-------
{1}     (Q & NOT R)
{2}     R

Rule?
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Show the step-by-step sequent transformations done by the (FLATTEN) command to transform your original theorem statement into the current sequent. Justify each step using tautologies, the deduction theorem and properties of sequents.

c) (10 marks) The PVS command (SPLIT) is applied to the first equation in the premises of the sequent in (b). What sequents result and why is one of them trivially true?

4. Predicate logic (25 marks total)

a) (10 marks) Consider the set of premises Γ defined as follows:

$$\Gamma = \{(\forall x)(Px \vee Qx), (\neg Py \rightarrow \neg Qy)\}$$

Find an interpretation structure **S** for Γ that:

i) Makes both formulas in Γ false.

ii) Makes the second formula in Γ neither true nor false.

iii) Is a model for Γ (i.e. $\mathbf{S} \models \Gamma$).

b) (10 marks) Let $\Gamma' = \{(\forall x)(Px \vee Qx), (\forall y)(\neg Py \rightarrow \neg Qy), (\forall x)\neg Px\}$. Formally prove that:

$$\Gamma' \vdash (\forall x)(Qx \wedge \neg Qx)$$

c) (5 marks) Dilbert is initially given Γ as a specification for his latest project. He has nearly completed the project when his pointy haired boss returns from a meeting with the marketing department where it was decided that the product would be much more marketable if they could use the slogan: "You'll never P again!" As a result the specification is revised to Γ'. Upon hearing the news, Dilbert decides that his time would be more productively spent playing Quake II rather than working to implement the new design. Why?