

SFWR ENG 2F04 Assignment 1: Propositional Semantics

Due: 1330 Tuesday September 28, 1999

1. Rubin p. 9-11 (25 marks) A 2, 4, 6, 8, 19, 22, 25;
B 4, 6, 8; C 2; D 1, E 3, 4
2. Rubin p. 31-36 (25 marks) A 5, 7, 11; B 5, 7; C 1, 3; E 1, 3
3. More World Building (25 marks)

Consider the “Building the World with NAND” example from class.

- a) Recall that we used the symbol T (representing True) in the definition of some of the operators (e.g. $\neg P \Leftrightarrow T \text{ NAND } P$). Write down a logically equivalent formula for $\neg P$ that only makes use of NAND and the propositional variable P .
- b) Write down the simplest tautology that uses only the propositional variable P and the binary operator NAND (i.e. find the simplest formula using P and NAND that is equivalent to T).
- c) Write down the simplest propositional formula using only P and NAND that is a contradiction (i.e. find the simplest formula using P and NAND that is equivalent to F - HINT: Use parts (a) and (b)).
- d) Write the formula $(\neg P \wedge Q) \vee Q \rightarrow R$ in only use the propositional variables P, Q, R, T and/or F and the NAND operator. (NOTE: $P \rightarrow Q \Leftrightarrow \neg P \vee Q$)
- e) Write the formula $(\neg P \wedge Q) \vee Q \rightarrow R$ in only use the propositional variables P, Q, R , and the NAND operator.

4. Hilbert Systems (25 marks total)

Hilbert style systems for propositional logic write all the other operators in terms of \rightarrow and \perp and make use of tautologies (c)(i)-(iii) below and all their substitution instances together with the single rule of inference modus ponens to obtain a consistent and complete proof system.

- a)
 - i) (2 marks) Write down the truth table for $P \rightarrow Q$.
 - ii) (3 marks) The constant symbols \top and \perp are interpreted as T (true) and F (false) respectively. Find the simplest propositional logic formula that uses only \perp and \rightarrow and is logically equivalent to \top .
NOTE: No propositional variables or operators other than \rightarrow can occur in the formula so you are NOT allowed to use $\neg\perp$ as your answer!
- b) (10 marks) Show that you can “build the world” using \rightarrow with the help of the propositional constant \perp . To do this first show that $\neg P$ can be written in terms of \rightarrow and \perp . Next show that $P \vee Q$ can be written using \neg and \rightarrow . Finally show that $P \wedge Q$ can be written in terms of \vee and \neg . Since we know that any formula has a logically equivalent DNF formula that only uses \neg, \vee and \wedge we are done!
- c) (10 marks) Verify that the following are tautologies:
 - i) $\neg(\neg P) \rightarrow P$
 - ii) $P \rightarrow (Q \rightarrow P)$
 - iii) $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$
 - iv) $(P \vee Q \leftrightarrow R) \rightarrow (\neg A \wedge B \rightarrow (P \vee Q \leftrightarrow R))$ (Note: There is an easy way for (iv)!)