SFWR ENG 2F04 Assignment 1: Propositional Semantics

Due: 1130 Tuesday September 19, 2000

1. Rubin p. 9-11 (25 marks total) A 1,3,9,10,12,16,23 (1 mark each); B 3,10; C 3; D 2; E 1,3 (3 marks each).

2. Rubin page 31-36 (30 marks total) A 2,4,6; B 2,6; C 3,5; E 2,4,6 (3 marks each).

3. Here NOR There: More World Building (35 marks)

   Another common type of logic gate that is compact and easy to build is the NOR gate. Just as P NAND Q was the negation of P and Q (i.e. $\neg(P \wedge Q)$), P NOR Q is the negation of $P \vee Q$ (i.e. $\neg(P \vee Q)$).

   a) Write down the truth table for P NOR Q.

   b) Check if NOR is:

   i) commutative

   ii) associative

   c) Write down the simplest contracdiction that uses only the propositional variable P and the binary operator NOR (i.e. find the simplest formula using P and NOR that is logically equivalent to $\bot$, the constant symbol for F).

   d) Write down a propositional formula using only P and NOR that is a tautology (i.e. find a formula using P and NOR that is equivalent to $\top$, the constant symbol for T).

   e) Show that you can "build the world" with NOR gates, that is, show that any propositional formula can be written in term of the NOR operator by showing how the operators $\neg, \vee, \wedge, \rightarrow$ and $\leftrightarrow$ can be written in terms of NOR.

   f) Write the formula $(\neg P \wedge Q) \vee Q \rightarrow R$ only using the propositional variables P, Q, R, and the NOR operator.

4. Verify that the following are tautologies: (10 marks)

   a) $\neg(\neg P) \rightarrow P$

   b) $P \rightarrow (Q \rightarrow P)$

   c) $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$

   d) $(P \vee Q \leftrightarrow R) \rightarrow (\neg A \wedge B \rightarrow (P \vee Q \leftrightarrow R))$ (Note: There is an easy way for (iv)!)