SFWR ENG 2F04 Assignment 2: Propositional Syntax

Due: 1730 Thursday October 5, 2000

All of your PVS work for this assignment should be done in a single file called `a2.pvs`. You will submit your PVS work electronically. Written work must be handed in during the lab before the submission deadline.

NOTE: For the latest decree on how to submit your PVS work to your professorial tormentor and his lackeys, please check out the URL:

`http://www.cas.mcmaster.ca/~lawford/2F04/e-submissions.html`

1. Consider Rubin p. 33 E and F.

   a) Do Rubin p. 33 E 2, 5, 7, 8 by hand.

   b) Do Rubin p. 33 F 2, 5, 7, 8 by hand.

   c) Check that your semantic evaluation of the arguments and syntactic manipulations agree. Why should this be the case?

   d) Create PVS formulas to check the validity of each of the arguments in E 2, 5, 7, 8. Name them `P33E2, P33E5, P33E7` and `P33E8`, respectively. Use the PVS syntactic manipulation commands `(FLATTEN)` and `(SPLIT x)` - here `x` is the formula number to split - to attempt proof of `P33E2` and `P33E5`. Use the PVS semantic evaluation command `(BDDSIMP)` to attempt proofs of `P33E7` and `P33E8`. Check that the results agree with your previous work. In the case where unprovable sequents result, verify that the counter example used in part (b) above falsifies at least one of the characteristic equations associated with the unprovable sequents.

2. Derive the "Rule of hypothetical syllogism" (Rubin p31 #7) by using "Rule of excluded middle" (#10), "De Morgan's" (#15) and any other rules you require.

3. Consider Rubin p. 48 B.

   a) Do Rubin p. 48 B 3, 7, 10, 12 by hand.

   b) Create PVS `PROPOSITION`s for problems 7 and 12 above with titles `P48B7` and `P48B12`, respectively. Use syntactic manipulation (i.e. `(FLATTEN)` and `(SPLIT)`) on 7 and semantic evaluation (i.e. `(BDDSIMP)`) on 12.

4. Understanding PVS (25 marks total)

   a) In the file `a2.pvs`, write down a PVS theorem called `C1` that you would attempt to prove to demonstrate the inconsistency of the set of premises:

   $$\Gamma_1 := \{P \to Q, \neg(Q \wedge \neg R), \neg R, P\}$$

   b) Invoke the PVS prover on theorem you stated in part (a), apply the (FLATTEN) command. You should obtain the sequent:

```
{-1}    (P IMPLIES Q)
{-2}    P
  |-------
{1}     (Q & NOT R)
{2}     R

Rule?
```

Show the step-by-step sequent transformations done by the (FLATTEN) command to transform your original theorem statement into the current sequent. Justify each step using tautologies, the deduction theorem and properties of sequents.

c) The PVS command (SPLIT) is applied to the first equation in the premises of the sequent in (b). What sequents result and why is one of them trivially true? Finish the proof using whatever PVS commands you desire.

d) Do a formal proof of the inconsistency of Γ_1 by hand (i.e. show $\Gamma_1 \vdash \bot$).

e) Make a copy of theorem `C1` and rename it `C2`. Modify `C2` to check the inconsistency of the set of premises:

$$\Gamma_2 := \{P \to Q, \neg(Q \wedge \neg S), \neg R, P\}$$

Attempting to prove `C2` should result in an unprovable sequent. Write down the characteristic equation for this unprovable sequent and find a counter example that makes the equation false.

f) Write down and prove a theorem called `C3` that demonstrates that the counter example from (d) satisfies all of the premises. What do you conclude about the set of premises Γ_2?