SFWR ENG 2F04 Assignment 3: Resolution Theorem Proving & Predicate Logic Syntax, Interpretations and Proofs

Due: 1130 Thursday October 26, 2000

1. Resolution Theorem Proving (25 marks total)

   a) (5 marks) Indirect proof or "proof by refutation" is when one shows that some set of premises $\Gamma = \{P_1, P_2, \ldots, P_n\}$ is a valid argument for Q by showing that $\Gamma, \neg Q$ is inconsistent. Use the deduction theorem to show that $\Gamma, \neg Q \vdash \bot$ iff $\Gamma \vdash Q$.

   b) (10 marks) Resolution rules of inference:

   i) Show $P \vee Q, \neg Q \vee R \models P \vee R$ using truth tables. Therefore we have valid rule of inference Rule R1: "If $\Gamma \vdash P \vee Q$ and $\Gamma \vdash \neg Q \vee R$ then $\Gamma \vdash P \vee R$."

   ii) Show $P \vee \neg Q, Q \vee R \vdash P \vee R$ by formal proof.

   iii) Given the proof $P \vee \neg Q, Q \vee R \vdash P \vee R$, why can we conclude that $P \vee \neg Q, Q \vee R \models P \vee R$? This provides the valid rule of inference Rule R2: $\Gamma \vdash P \vee \neg Q$ and $\Gamma \vdash Q \vee R$ then $\Gamma \vdash P \vee R$."

   These two rules together with the commutativity of $\vee$, Rule P for stating a premise and the instances of Rule T making use of the tautologies $P \leftrightarrow P \vee \bot$ and $P \wedge \neg P \leftrightarrow \bot$, comprise the complete set of rules of inference for resolution (refutation) theorem proving.

   c) (10 marks) You will use resolution theorem proving to prove the following:

   $$P \rightarrow Q, \neg(Q \wedge \neg R), \neg R \models \neg P$$

   i) Replace $P \rightarrow Q$ and $\neg(Q \wedge \neg R)$ by logically equivalent formulas P_1 and P_2 that only use $\vee$ and $\neg$.

   ii) Using only the rules of inference mentioned in (b) above, formally prove that prove that

   $$P_1, P_2, \neg R, P \vdash \bot$$

2. Predicate logic syntax: Do Rubin (52 Marks / 2 Marks each one)

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p.116 A:5,10,20,25 ;
      B:2,5,7,11,16 ;
      C:3,4,5,21,25,28
      E:1,3,5,7
      F:3,5,7,13,16,21
      G:2
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3. Predicate logic interpretation: Do Rubin (24 Marks/ 4 Marks each one)

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p.138 A:2,4,8
      B:2,6,8
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4. Predicate logic proof: Do Rubin (24 Marks / 4 Marks each one)

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p.151 A:3,7
      D:5,7
      F:3,7
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