Name: ....
Student Number ....

Software Engineering 2F04

DAY CLASS — Dr. Mark Lawford
DURATION OF EXAMINATION: 2 Hours
McMaster University Midterm Examination — November 2, 2000

THIS EXAMINATION PAPER INCLUDES 4 PAGES AND 5 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

Special Instructions: The use of calculators, notes, and text books is not permitted during this exam. Answer all questions in the provided answer booklets. Fill in your name and student number and sign each booklet you use.

1. Propositional Logic Basics (25 marks total)

Here you will state some of the fundamental results of propositional logic and demonstrate your understanding of their use.

a) (5 marks) Show the following:

$$\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$$

What two valid rules of inference does this tautology define?

b) (3 marks) State the Deduction Theorem.

c) (6 marks) Syntax and Semantics

Let Γ be a set of propositional formulas (the premises) and Q be some propositional formula (the conclusion). Give one or two sentence answers to the following questions:

i) Syntax entailment: What does $\Gamma \vdash Q$ mean?

ii) Semantic entailment: What does $\Gamma \models Q$ mean?

iii) Completeness and consistency: What is the relationship between these two symbols and what does this relationship have to do with valid rules of inference?

d) (10 marks) Let $\Gamma := \{A \to (B \to \neg C), \neg(\neg C \wedge D), \neg E \to D\}$. Formally show:

$$\Gamma \vdash B \to (A \to E)$$

e) (1 mark) Is Γ a valid argument for $B \to (A \to E)$? Justify your answer.

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2. More Propositional Logic (20 Marks)

a) (10 marks) Determine if the following set of premises is inconsistent:

$$\Gamma := \{A \rightarrow (B \leftrightarrow C), B \rightarrow D, \neg D \rightarrow B \vee C, A \wedge \neg D\}$$

Justify your answer.

b) (3 marks) Write down PVS input (the definitions and theorem) that you would enter to use PVS to check the above question.

c) (7 marks) What sequent would result if you applied the `(FLATTEN)` command to the theorem from (b) above? Justify you answer step by step using propositional rules of inference.

3. Automated Verification of Propositional Logic (20 Marks Total)

Consider the set of premises:

$$\begin{aligned}\Gamma \;:=\; & \{(A \rightarrow (B \rightarrow C)), (\neg A \rightarrow D \wedge \neg E), (A \wedge B \rightarrow \neg C), (D \rightarrow F \vee G), \\ & (\neg B \rightarrow (G \rightarrow H)), (E \vee H \vee \neg G)\}\end{aligned}$$

We wish to determine if $\Gamma \models G \wedge H$

a) (5 marks) Applying the (BDDSIMP) command when trying to prove the PVS formula:

```
Q4:PROPOSITION
(A=>(B=>C))&(NOT A => D&NOT E)&(A&B=>NOT C)&(D=>F OR G) &
(NOT B =>(G=>H)) & (E OR H OR NOT G)
IMPLIES G&H
```

results in 3 unprovable sequents. They are shown below:

```
Sequent 1          Sequent 2          Sequent 3

{-1}   A           {-1}   A           {-1}   D
{-2}   F             |-------         {-1}   F
  |-------         {1}    B             |-------
{1}    B           {2}    D           {1}    A
{2}    G           {3}    G           {2}    E
                                      {3}    G
```

b) (3 marks) Write down the characteristic equation of each of the sequents.

c) (3 marks) For each sequent, find a counter example that falsifies the sequent's characteristic equation.

d) (2 marks) If you substituted any of these counter examples back into the premises in Γ, what would you find? What would you find for the conclusion $G \wedge H$?

e) (4 marks) Use your counter example based upon `Sequent 3` to create a counter example that shows that $\Gamma \not\models G \wedge H$. Create a one line truth table to verify the counter example.

f) (3 marks) For the set of premises Γ as defined above, what value must G have for any counter example to the argument $\Gamma \models G \wedge H$. Why?

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4. Predicate Logic Basics (25 Marks Total)

The formal language of Predicate Logic makes use of the connectives

$$\wedge, \rightarrow, \neg, \leftrightarrow, \vee$$

and quantifiers $\forall, \exists$.

To reduce the number of parentheses required when writing down formulas, one typically assigns an order of precedence to the connectives and quantifiers from those with the highest precedence (do first!) to those with the lowest precedence (do last!).

a) (4 marks) What is the order of precedence employed by:

i) The textbook by Rubin?

ii) The automated proof assistant PVS?

b) (6 marks) For the following formula:

$$(\exists x)Px \rightarrow Px$$

i) What is ϕ_i, the fully parenthesized formula for the above using Rubin's order of precedence?

ii) What is ϕ_{ii}, the fully parenthesized formula for the above using PVS' order of precedence?

iii) To show that the fully parenthesized formulas are not equivalent, find an interpretation structure $\mathbf{S}$ such that $\mathbf{S} \not\models \phi_i$ but $\mathbf{S} \models \phi_{ii}$.

c) (2 marks) Translate the following sentence into predicate logic using Rubin's order of precedence:

"It is always the case that the simplest explanation is the most likely." (Sx, Lx)

d) (3 marks) Assuming the order of precedence of operations from Rubin, what are the free variables of the following formulas?

i) $\phi : Ix \rightarrow (\exists y)(Iy \wedge y > x)$

ii) $\psi : (\forall x)Px \rightarrow Qx$

iii) Is $z \in FV(\{\phi, \psi\})$?

e) (4 marks) For ϕ and ψ as defined above, which of the following are valid substitutions?

(i) $\phi[u|x]$ (ii) $\phi[f(y)|x]$ (iii) $\phi[u|y]$ (iv) $\psi[y|x]$

f) (6 marks) Show:

$$(\forall y)\neg Qxf(y) \vdash (\forall x)(\forall y)(Qf(x)f(y) \rightarrow Qxy) \rightarrow (\forall y)\neg Qf(x)f(f(y))$$

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5. Predicate Logic Formulas as Specifications and Programs as Models: (10 marks)

Consider the following set of formulas:

$$\begin{aligned}\Gamma \quad := \quad & \{(\forall x)\neg Qxf(x), \\ & (\forall x)(\forall y)(Qf(x)f(y) \to Qxy), \\ & (\exists x)(\forall y)\neg Qxf(y)\}\end{aligned}$$

The intended interpretation is given by a structure of the form $\mathbf{S} := \langle \mathbf{U}, \mathbf{Q}, \mathbf{f} \rangle$ where:

$$\begin{aligned}\mathbf{U} \quad &:= \quad \mathbb{N} = \{0, 1, 2, \ldots\} \\ \mathbf{Q} \quad &:= \quad \{(x, y) \in \mathbb{N} | x = y\}\end{aligned}$$

and the interpretation of f, denoted $\mathbf{f}$ is yet to be determined.

In your summer job as a software developer[1] you are given Γ as a formal specification and told to create two different versions of $\mathbf{f}$.

a) (3 marks) First, find an $\mathbf{f}$ such that for this interpretation of f we have $\mathbf{S} \not\models \Gamma$.

b) (7 marks) Next, find an $\mathbf{f}$ such that for this interpretation of f we have $\mathbf{S} \models \Gamma$.

"Its not logic, but it makes so much more sense." - 2F04 student

-- The End --

[1]You aren't "real" software engineers until we get accredited, then you graduate, get the required experience, and the pass the Law & Ethics exams.