SFWR ENG 2F04 Assignment 2: Propositional Syntax

Due: 1730 Tuesday October 9, 2001

All of your PVS work for this assignment should be done in a single file called `a2.pvs`. You will submit your PVS work electronically. Written work must be handed in during the lab before the submission deadline.

NB: For the latest edict on how to submit your PVS work to *The Powers That Be* (TPTB^{Tm}), proceed to the URL:

`http://www.cas.mcmaster.ca/~lawford/2F04/e-submissions.html`

1. Huth+Ryan p. 36-39 1(c), 2(a)(b)(i), 3(a),4, 6, 7

2. Use the PVS `(BDDSIMP)` command to prove p.36 1(c), 2(a) and use the PVS commands `(FLATTEN)` and `(SPLIT)` to [prove p.36 2(i). Names these theorems `P36_1c`, `P36_2a` and `P36_2i`.

3. Huth+Ryan p. 61 2(b)(c)

4. A sequence of premises Γ is *inconsistent* if $\Gamma \vdash \bot$. In this case, by soundness of our proof system $\Gamma \models \bot$ which tells us that there are no rows in the truth table for Γ where all of the premises are true (i.e., it is impossible to simultaneously satisfy all of the premises). On the other hand, we say that the sequence of premises Γ is *consistent* if $\Gamma \not\models \bot$, i.e., there is one (or more) rows of the truth table where all of the premises are true. In this case, by the completeness of out proof system we have $\Gamma \nvdash \bot$ (no proof of $\bot$ from Γ exists).

   Use PVS to determine if Γ is inconsistent for

   a)

   $$\Gamma := a \wedge c \rightarrow d, b \wedge c \rightarrow d, \neg(a \vee b) \rightarrow e \vee f, g \rightarrow \neg e, \neg f \vee h, c \wedge \neg d, g \rightarrow h$$

   Call this Q4a: PROPOSITION ....

   b)

   $$\Gamma := a \rightarrow (b \rightarrow c), \neg a \rightarrow d \wedge \neg e, a \wedge b \rightarrow \neg c, d \rightarrow f \vee g, b \vee (g \rightarrow h), g \rightarrow e \vee h, g \wedge \neg h$$

   Call this Q4b: PROPOSITION ....

   In any cases when the proof of $\Gamma \vdash \bot$ fails, write down the characteristic equation of one of the unprovable sequents, determine a truth assignment that falsifies the characteristic equation and then use that to obtain a row in the truth table that shows that $\Gamma \not\models \bot$.

5. Show that the proof rules $\rightarrow 2\vee$ and $\vee 2 \rightarrow$ are derived rules by coming up with a formal proof of the following sequent:

   $$\vdash (p \rightarrow q) \leftrightarrow (\neg p \vee q)$$

   using only the rules on p.31 of Huth+Ryan along with $\leftrightarrow e$ and $\leftrightarrow i$.

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6. Understanding PVS (25 marks total)

a) In the file `a2.pvs`, write down a PVS theorem called `C1` that you would attempt to demonstrate that the following argument is valid:

$$p \to q, \neg(q \wedge \neg r), p \models r$$

b) Invoke the PVS prover on theorem you stated in part (a), apply the (FLATTEN) command. You should obtain the sequent:

```
{-1}    (p IMPLIES q)
{-2}    p
  |-------
{1}     (q & NOT r)
{2}     r

Rule?
```

Show the step-by-step sequent transformations done by the (FLATTEN) command to transform your original theorem statement into the current sequent. Justify each step using our proof rules, tautologies, valid arguments and properties of sequents.

c) The PVS command (SPLIT -1) is applied to the first equation in the premises of the sequent in (b). What sequents result and why is one of them trivially true? Finish the proof using whatever PVS commands you desire.

d) Do a formal proof of the valid argument by hand i.e., show

$$p \to q, \neg(q \wedge \neg r), p \models r$$

by showing that

$$p \to q, \neg(q \wedge \neg r), p \vdash r$$

e) Make a copy of theorem `C1` and rename it `C2`. Modify `C2` to check the validity of the argument

$$p \to q, \neg(q \wedge \neg s), p \models r$$

Attempting to prove `C2` should result in an unprovable sequent. Write down the characteristic equation for this unprovable sequent and find a counter example that makes the equation false.

f) Write down and prove a theorem called `C3` that demonstrates that the counter example from (e) satisfies all of the premises, but does not satisfy the conclusion r. What do you conclude about the validity of the argument

$$p \to q, \neg(q \wedge \neg s), p \overset{?}{\models} r$$