# SFWR ENG 2F04 Assignment 3: Resolution Theorem Proving & Predicate Logic Syntax, Interpretations and Proofs

Due: 1130 Tuesday October 30, 2001

1. Resolution Theorem Proving

   **a)** (5 marks) A version of indirect proof known as *reductio ad absurdum* (RAA) is when one shows that some sequence of premises $\Gamma = \{\phi_1, \phi_2, \ldots, \phi_n\}$ is a valid argument for $\psi$ by showing that $\Gamma, \neg\psi$ is inconsistent. Show that this is a derived rule by showing that if $\Gamma, \neg\psi \vdash \bot$ then $\Gamma \vdash \psi$.

   **b)** (10 marks) Resolution rules of inference:

   **i)** Show $p \vee q, \neg q \vee r \models p \vee r$ using truth tables. Therefore we have valid rule of inference Rule R1: "If $\Gamma \vdash \phi \vee \psi$ and $\Gamma \vdash \neg\psi \vee \chi$ then $\Gamma \vdash \phi \vee \chi$." We summarize this rule as follows:

   $$\frac{\phi \vee \psi \qquad \neg\psi \vee \chi}{\phi \vee \chi}\text{R1}$$

   **ii)** Show $p \vee \neg q, q \vee r \vdash p \vee r$ by formal proof using only the rules from the lecture slides on propositional logic.

   **iii)** Given the proof $p \vee \neg q, q \vee r \vdash p \vee r$, why can we conclude that $p \vee \neg q, q \vee r \models p \vee r$? This provides the valid rule of inference Rule R2: $\Gamma \vdash \phi \vee \neg\psi$ and $\Gamma \vdash \psi \vee \chi$ then $\Gamma \vdash \phi \vee \chi$."

   $$\frac{\phi \vee \neg\psi \qquad \psi \vee \chi}{\phi \vee \chi}\text{R2}$$

   These two rules together with the commutativity of $\vee$, the rule Premise for stating a premise, $\neg e$ and the additional rule

   $$\frac{\phi \vee \bot}{\phi}\bot e_2$$

   comprise the complete set of rules of inference for resolution (refutation) theorem proving.

   **c)** (10 marks) You will use resolution theorem proving to prove the following:

   $$p \to q, \neg(q \wedge \neg r), \neg r \models \neg p$$

   **i)** Replace $p \to q$ and $\neg(q \wedge \neg r)$ by logically equivalent formulas $\phi_1$ and $\phi_2$ that only use $\vee$ and $\neg$.

   **ii)** Using only the rules of inference mentioned in (b) above, formally prove that prove that

   $$\phi_1, \phi_2, \neg r, p \vdash \bot$$

2. Huth+Ryan p. 101 2, 3, 4, 5, 6

3. Huth+Ryan p. 108 1 (a)-(e), 2

4. Huth+Ryan p. 135 1, 2, 3, 5, 6

5. Huth+Ryan p. 139 1, 2, 5, 6