SFWR ENG 2F04 Assignment 1: Propositional Semantics

Due: 1720 Tuesday September 24, 2002

1. Huth+Ryan p. 5-6 4, 7, 10 (c),(f),(g)

2. Huth+Ryan p. 42-44 1(d),(k);3, 6, 10, 12

3. Huth+Ryan p. 49-51 1, 3, 5, 6

4. Huth+Ryan p. 75-76 5, 7, 8

5. Hilbert Systems

   Hilbert style systems for propositional logic write all the other operators in terms of $\rightarrow$ and $\bot$ and make use of tautologies (c)(i)-(iii) below and all their substitution instances together with the single rule of inference modus ponens to obtain a consistent and complete proof system.

   a) **i)** Write down the truth table for $p \rightarrow Q$.

   ii) The constant symbols $\top$ and $\bot$ are interpreted as T (true) and F (false) respectively. Find the simplest propositional logic formula that uses only $\bot$ and $\rightarrow$ and is logically equivalent to $\top$.
   NOTE: No propositional variables or operators other than $\rightarrow$ can occur in the formula so you are NOT allowed to use $\neg\bot$ as your answer!

   b) Show that you can "build the world" using $\rightarrow$ with the help of the propositional constant $\bot$. To do this first show that $\neg p$ can be written in terms of $\rightarrow$ and $\bot$. Next show that $p \vee q$ can be written using $\neg$ and $\rightarrow$. Finally show that $p \wedge q$ can be written in terms of $\vee$ and $\neg$. Since we know that any formula has a logically equivalent DNF formula that only uses $\neg, \vee$ and $\wedge$ we are done!

   c) Verify that the following are tautologies:

   i) $\neg(\neg p) \rightarrow p$
   ii) $p \rightarrow (q \rightarrow p)$
   iii) $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$
   iv) $(p \vee q \leftrightarrow r) \rightarrow (\neg a \wedge b \rightarrow (p \vee q \leftrightarrow r))$ (Note: There is an easy way for (iv)!)