SFWR ENG 2F04 Assignment 2: Propositional Syntax & Proof

Due: 1720 Tuesday October 8, 2002

All of your PVS work for this assignment should be done in a single file called `a2.pvs` in a subdirectory where you want to keep your 2F04 PVS material. For this assignment, to receive credit for your PVS work in questions 4-6, you must demonstrate your PVS work in the lab to a TA. Your written work can be handed in as usual at the end of your tutorial on the due date.

1. Huth+Ryan p. 36-39 1(d), 2(a)(d)(f), 3(d), 4
2. Huth+Ryan p. 61 2(a)(c)
3. Using only the proof rules from the slides in class, show that: $\vdash (s \wedge m \rightarrow \neg l) \leftrightarrow (s \rightarrow \neg(m \wedge l))$.
4. Use the PVS `(BDDSIMP)` command to prove Huth+Ryan p.36 1(d), 2(a) and use the PVS commands `(FLATTEN)` and `(SPLIT)` to prove p.36 2(f). Names these theorems `P36_1d`, `P36_2a` and `P36_2f`. For the proof of `P36_2f` use the "x-prove" command from the PVS "Prover Invocation" submenu to see the structure of the proof.
5. A sequence of premises Γ is *inconsistent* if $\Gamma \vdash \bot$. In this case, by soundness of our proof system $\Gamma \models \bot$ which tells us that there are no rows in the truth table for Γ where all of the premises are true (i.e., it is impossible to simultaneously satisfy all of the premises). On the other hand, we say that the sequence of premises Γ is *consistent* if $\Gamma \not\models \bot$, i.e., there is at least one row of the truth table where all of the premises are true. In this case, by the completeness of out proof system we have $\Gamma \nvdash \bot$ (i.e. no proof of $\bot$ from Γ exists).

   Use PVS to determine if Γ is inconsistent for

   a)

   $$\Gamma := a \wedge c \rightarrow d, b \wedge c \rightarrow d, \neg(a \vee b) \rightarrow e \vee f, g \rightarrow \neg e, \neg f \vee h, c \wedge \neg d, g \rightarrow h$$

   Call this `Q5a: PROPOSITION` ....

   b)

   $$\Gamma := a \rightarrow (b \rightarrow c), \neg a \rightarrow d \wedge \neg e, a \wedge b \rightarrow \neg c, d \rightarrow f \vee g, b \vee (g \rightarrow h), g \rightarrow e \vee h, g \wedge \neg h$$

   Call this `Q5b: PROPOSITION` ....

   In any cases when the proof of $\Gamma \vdash \bot$ fails, write down the characteristic equation of one of the unprovable sequents, determine a truth assignment that falsifies the characteristic equation and then use that to obtain a row in the truth table that shows that $\Gamma \not\models \bot$.
6. We would like to determine if the following is a valid argument:

   $$a \rightarrow (b \leftrightarrow c), b \rightarrow d, b \vee c \rightarrow \neg d \overset{?}{\models} \neg d \rightarrow \neg a$$

   a) Create a theorem called `Q6a`) in the file `a2.pvs` that you wll try to prove in PVS to determine if the above is a valid argument.

   b) Applying the command `(BDDSIMP)` to the `Q6a`) proposition. Write down the sequent that results from this.

   c) Use this sequent to find a counter example that shows that the argument is not valid and check by hand that it is indeed a counter example.

   d) State and prove a theorem called `Q6d` that shows that the argument is not valid (i.e. use PVS to do the checking that you just did by hand).

The End