Name: ..........................................
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# Software Engineering 4A03

DAY CLASS — Dr. Mark Lawford
DURATION OF EXAMINATION: 2 Hours
McMaster University Midterm Examination — October 26, 2001

THIS EXAMINATION PAPER INCLUDES 3 PAGES AND 4 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

Special Instructions: The use of your course notes, and the required text books for this course are permitted during this exam. You may use any calculator. Answer all questions in the provided answer booklets. Fill in your name and student number and sign each booklet you use.

**1. $\mathcal{Z}$-Transforms & Difference Equations** (20 marks)

Consider the difference equation:

$$y(k) + y(k-2) = u(k)$$

**a)** (5 marks) Find the transfer function $\frac{Y(z)}{U(z)}$.

**b)** (10 marks) Assuming zero initial conditions (i.e. $y(-1) = y(-2) = 0$), compute the system's step response.

**c)** (5 marks) The system is not BIBO stable. Find a bounded continuous input that, when sampled and used as input for $u(k)$, would cause the system's output to become unbounded.

**NOTE:** You do NOT have to compute what the output is, only suggest a bounded input that will produce an unbounded output.

**2. Basic Discrete Approximations** (30 marks)

In all of the following, unless otherwise stated, assume the sampling period is $T$.

**a)** Consider the continuous controller:

$$D(s) = \frac{U(s)}{E(s)} = \frac{s+a}{s+b}$$

**i)** (5 marks) Find a difference equation to implement this controller using the Forward Rectangular Rule.

**ii)** (5 marks) Assuming zero initial conditions, find the transfer function $D(z)$, for this difference equation.

**iii)** (5 marks) Suppose $a = 1$ and $b = 10$. For what range of values of sampling period $T$ is $D(z)$ guaranteed to be Bounded Input Bounded Output (BIBO) stable?

**iv)** (5 marks) Assume the sample period $T$ is fixed. For what range of values for $b$ is $D(z)$ BIBO stable?

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**b)** Consider the continuous controller:

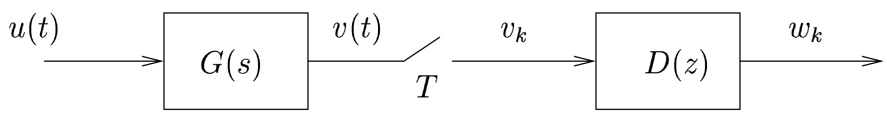
$$D(s) = \frac{U(s)}{E(s)} = \frac{K_{ii}}{s^2} + K_p$$

**i)** (5 marks) Find a difference equation to implement this controller using the Forward Rectangular Rule.

**ii)** (5 marks) Find the transfer function $D(z)$ for this difference equation (assume zero initial conditions).

**3. Frequency Response** (15 marks)

Consider the system shown below:




**a)** (6 marks) Consider the ideal sampling operator shown above that converts the continuous signal $v(t)$ to the discrete signal $v_k = v(kT)$. Is the operator:

**i)** Linear?

**ii)** Time Invariant?

**iii)** Causal?

Provide justification for each of the above answers to obtain full marks.

**b)** (9 marks) Assume the sampling period $T = \frac{1}{2}$ second, input $e(t) = 3\cos(2\pi t)$,

$$G(S) = \frac{1}{s+1} \text{ and } D(z) = \frac{W(z)}{V(z)} = \frac{z}{z+0.5}$$

Consider only the **steady state** behavior of the system

**i)** What is $v(t)$ in steady state?

**ii)** What is $v_k$ in steady state?

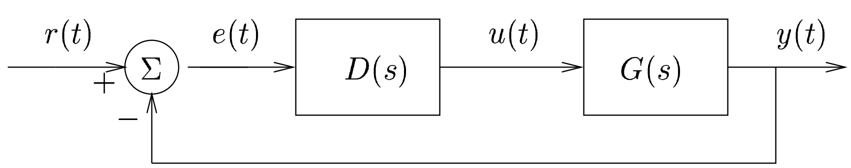
**iii)** What is $w_k$ in steady state?

**4. Root Locus Design & Sampled Data Systems** (35 marks)

Consider a motor that is modeled by the transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{3}{s(s+3)}$$

in the feedback loop:




**a)** (4 marks) Sketch the root locus for the system when we use proportional feedback $D(s) = K$. For what values of $K$ is the closed loop systems stable?

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**b)** (2 marks) Compute the closed loop transfer function $G_{cl}(s) = \frac{Y(s)}{R(s)}$ in the case when $D(s) = 1$.

**c)** (3 marks) When $D(s) = 1$, what is the steady state error in response to a unit step input $r(t) = 1(t)$?

**d)** (7 marks) We wish to do a simple digital implementation of the controller without having to worry about things so we wish to sample at 30 times the closed loop bandwidth of the system. What is the closed loop bandwidth of the continuous control system when $D(s) = 1$? What value should we use for $T$ in the digital version?

**e)** (3 marks) Approximately how will the phase margin be reduced by when using this digital implementation?

**f)** (8 marks) The configuration for the digital control system is shown in figure 1 Compute the zero

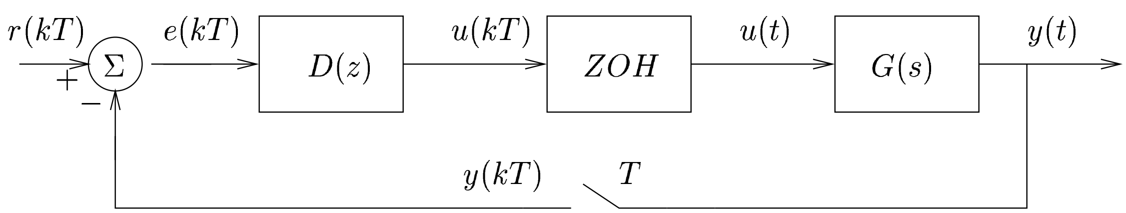


Figure 1: Block Diagram for Sampled Data Control System

order hold equivalent $G_{ZOH}(z) = \frac{Y(z)}{U(z)}$ (the transfer function from $u_k = u(kT)$ to $y_k = y(kT)$ in the digital control diagram) for our motor model $G(s) = \frac{3}{s(s+3)}$. Keep this in terms of $T$ i.e., do not plug in a specific value for $T$ (since you probably got it wrong above anyway ;-).

**g)** (4 marks) For the data acquisition cards used in the lab the A/D conversion hardware is 16-bit while the D/A hardware is 12-bit.

- **i)** Assuming the analog input to the card is between $-5$ and $+5$ Volts, what is the resolution in Volts of the A/D?
- **ii)** Assuming the analog output from the card is between $-10$ and $+10$ Volts, what is the resolution in Volts of the D/A?

**h)** (4 marks) In practice we notice a steady state error in the order of $\pm 0.3$ Volts in the lab.

- **i)** Explain the cause of this error by referring to specific signals in in Figure 1 and the properties of the actual motor versus the model of the motor.
- **ii)** Why is the steady state error eliminated by adding an integral term to our controller?

Bonus Question: What does NTL stand for?

"What's that sum thing with the $z^{-k}$?" - An infrequent attendee of 4A03 lectures

The End