SFWR ENG 4A03 Lab 2: PID Control of a Motor

Pre-Lab Exercises

1. Given that the model for the (open loop) transfer function for the angular position $\theta(t)$ of a motor is:
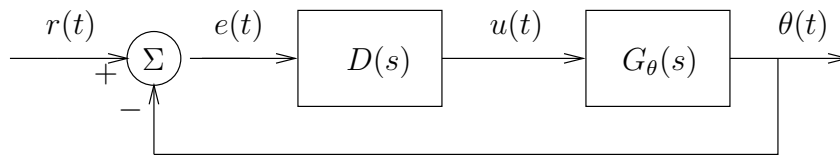
$$G_\theta(s) = \frac{\Theta(s)}{U(s)} = \frac{A}{s(\tau s + 1)}$$

where A and τ are positive, real valued constants.

a) What is the transfer function for the angular velocity $\dot\theta(t)$ denoted $G_{\dot\theta}(s) = \frac{\dot\Theta(s)}{U(s)}$?

b) What is the steady state value of $\dot\theta$ in open loop response to a step input:

$$u(t) = \begin{cases} U_o, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

2. Consider the motor given by $G_\theta(s)$ in Question 1 in the feedback control configuration:

where $D(s) = K$ (i.e. proportional control), with $K > 0$.

a) Sketch the root locus.

b) What is the closed loop transfer function:

$$G_{\theta_{cl}}(s) = \frac{\theta(s)}{R(s)}$$

c) Does the closed loop system track steps? i.e. what is the steady state error of the system:

$$e_\infty = \lim_{t\to\infty} e(t)$$

d) Suppose that the closed loop system is critically damped (i.e. characteristic equation has repeated roots) for $D(s) = K_*$. In this case, what is the value of τ in terms of K_* and A?

3. Repeat Question 2 (a) and (b) using Matlab to do the root locus for $A = 2$ and $\tau = 20$ with

   i) Integral control $D(s) = \frac{K}{s}$

   ii) Proportional Integral Control $D(s) = \frac{K(s+z)}{s}$

   iii) PID Control $D(s) = \frac{K(s+z_1)(s+z_2)}{s}$

For each of the cases indicate whether it is possible to stabilize the system. For (ii) and (iii) show how your choice of zeros can affect closed loop stability.

4. The Root Mean Square (RMS) value of a signal $f(t)$ that is periodic with period T is given by the equation

$$\sqrt{\frac{1}{T} \int_0^T (f(t))^2 dt}$$

RMS values are often easier to measure and more accurate than trying to determine peak amplitudes.

a) Show that the RMS value of $u(t) = A \sin \omega t$ is $\frac{A}{\sqrt{2}}$.

b) Suppose $u(t) = A \sin \omega t$ is the input to a transfer function $G(s) = \frac{Y(s)}{U(s)}$. Show that in the steady state the RMS value of $y(t)$ is $|G(j\omega)| \frac{A}{\sqrt{2}}$. Thus:

$$\frac{\text{RMS value of } y(t)}{\text{RMS value of } u(t)} = |G(j\omega)|$$

c) Another way of estimating τ is to experimentally determine the bandwidth of the closed loop system $G_{\theta_{cl}}(s)$ corresponding to the -3dB frequency ω_{bw}. Then we have:

$$|G_{\theta_{cl}}(j\omega_{bw})| = \frac{1}{\sqrt{2}}$$

In this case what is the value of τ in terms of K, A and ω_{bw} when the compensator in the closed loop system is $D(s) = K$?

Lab Exercise

To do the following exercises, login as user `rtlab` under Windows.

1. Run the `MAX` tool (click on the National Instrument Measurement & Automation icon on the desktop) and create:

   - A virtual input channel called `position` with units degrees with a range of $[-180, +180]$ degrees, sensor type `voltage` with range $[-5, +5]$ Volts. Be sure to specify that it is `range mapped` and it is a single-ended referenced signal.

   - A virtual output channel called `control` with units degrees with a range of $[-360, +360]$ degrees, with output voltage range $[-10, +10]$ Volts. Be sure to specify that it is `range mapped`.

Part I

2. Connect the potentiometer sensor from the motor to Analog Input Channel 0. Make sure that the `BNC/temp` switch is set to `BNC` and the `FS/GS` switch is set to `GS`.

3. Connect Analog Output Channel 0 to the input of the operational amplifier.

4. Test the connections from within `MAX` by right clicking on the `control` output and selecting `Test`. Change the output value to `60`. The motor should start to spin. Click on the `position` tab and you should see a graph of the position sensor changing linearly from -180 to +180. Click on the `control` sensor again and set the output to 0 to stop the motor before exiting `MAX`.

5. In Labview, load the PID demo and save it as `mypid.vi`. Edit this file to make the controller suitable for use with the motor. Note that the lower and upper limits on the output should be change to -360 and +360 respectively. Save the `mypid.vi` file and ask the TA to inspect it before running it. **Once the TA has approved your design, have the TA help you connect your design to the motor.**

    i) Set proportional gain K_p to a value of 1.2 and set the integral K_i and derivative K_d gains to 0. Test the different setpoints $-120, -90, -45, 45, +90, +120$ and record any steady state offset in the output from the setpoint.

    ii) Now set K_i to be greater than zero, between say 1 and 5. Check to see what the steady state errors at the previous set points are now. Record the steady state error values.

Part II

6. Connect the motor's position encoder to the digital inputs for the counter (the TA will do this for you).

7. The encoder has 1024 pulses/revolution. Load the BNC-2120 encoder example from the previous lab. Save the file as `MotorEncoder.vi` and edit the example so that it read the correct angle for the motor's position.

8. Create a VI called `velocity.vi` to do open loop control of the motor (i.e., have some sort of way of setting the analog output channel `control` to a desired value. Your VI should read the encoder for the position of the motor and use this to estimate the value of $\dot\theta \approx \frac{\Delta\theta}{\Delta t}$. The VI should plot the estimate of the derivative and have a numeric display of the sliding average of the derivative over a user setable number of samples.

   HINT: You might want to see the internals of the PID block for their method of computing the derivative.

9. **Once the TA has approved your design, have the TA help you connect your design to the motor.** Test the different setpoints $-120, -90, -45, 0, 45, +90, +120$ and for each setpoint record the steady state value of your estimate of $\dot\theta$.

Part III

10. Open your `mypid.vi` file and save it as a file called `pidbode.vi`. Edit this file to replace the setpoint control with a basic function generator. Add in controls to allow you to change the amplitude and frequency of a reference signal into the motor control loop. Add plots of the RMS value of the reference signal and the motor position as well as a numeric display of these values.

11. **Once the TA has approved your design, have the TA help you connect your design to the motor.** Using this set up, make $K_i = K_d = 0$ and then tune the value of K_p until the system is critically damped (i.e. just before overshoot occurs). Record the value. Use a square wave reference signal of a reasonably low frequency to help you tune the system. Note that this is the value of K_* in Question 2 for your motor.

12. Leave $K_i = K_d = 0$ and $K_P = K_*$. Change the input to your system to a low frequency sinusoid of amplitude 90. Increase the frequency of the sinusoid until the RMS value of the output is $\frac{1}{\sqrt{2}}$ of the RMS value of the input. Record this frequency as ω_{bw} of Question 4.

Post-Lab and Other Exercises

1. Analyze your data to obtain estimates of A and τ. You should have two estimates of τ from Part III of the lab. How do the two estimates compare? What do you conclude about your results? For the value of τ obtained from ω_{bw} in Part III and the value of A obtained from Part II of the lab, what should be the value of K_*? What is your best estimate of the open loop transfer function $G_\theta(s)$?

2. Use Matlab to simulate the closed loop step response of the transfer function you obtained in Question 1 when $D(s) = 1.2$. Explain any discrepancies from the data you obtained in Part I of the lab and the simulation results and mathematical results of Pre-Lab Question 2(c).

3. In the lab we used two different sensors to estimate the motor's position. In Part I we used a potentiometer with readings between [-5,+5] V being read by a 16-bit A/D on a Data Acquisition (DAQ) card. In Part II we used an encoder with 1024 pulses/revolution and A and B channels going into a 24-bit up/down counter also on the DAQ card. Both of these sensors have their readings scaled in software to give [-180,+180] degrees for one full revolution of the motor.

4. What is the resolution in both percentage and degrees of

   i) The potentiometer read by the 16-bit A/D?
   ii) The encoder?

5. The analog potentiometer has a discontinuity from +5 to -5 or vice versa as the system moves beyond 180 degrees of rotation. While it is possible to take this discontinuity into account in software, what might still be a problem?

6. Assuming that the motor starts with the 24-bit counter initialized to zero, how many revolutions would it take for the encoder to roll over the counter possibly resulting in a large discontinuity in the estimated position (if the software does not take this into account)? What would be the approximate size of the discontinuity?

7. What are the advantages and disadvantages of each type of sensor?

8. Discrete systems with poles on the unit circle are not BIBO stable. As an example to illustrate this fact, consider the following system:
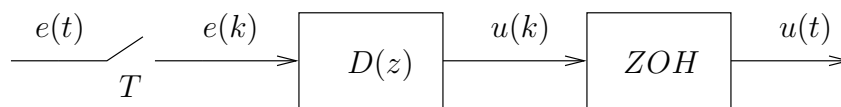
Assume the sampling period $T = \frac{1}{2}$ second, input $e(t) = \cos(2\pi t)$ and

$$D(z) = \frac{U(z)}{E(z)} = \frac{z}{z+1}$$

We will show that $u(k)$ (and hence $u(t)$) grows without bound in this case.

   a) Sketch $e(k)$ for $k = 0$ to 4.
   b) Compute $E(z) = \mathcal{Z}\{e(k)\}$.
   c) Compute $u(k)$. Is it clear that $|u(k)|$ is not bounded?
   d) Sketch $u(t)$, the output of the ZOH, for $0 \le t < 2$.
   e) What other bounded inputs of the form $e(t) = \cos(\omega t)$ will cause an unbounded output from $D(z)$?