Software Engineering 4A03

DAY CLASS — Dr. Mark Lawford
DURATION OF EXAMINATION: 1 Hour 50 min
McMaster University Midterm Examination — October 25, 2002

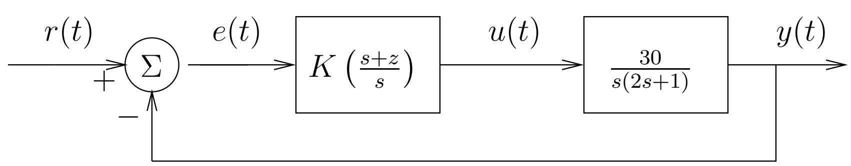
THIS EXAMINATION PAPER INCLUDES 2 PAGES AND 3 QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

Special Instructions: The use of notes, and text books is not permitted during this exam. You may use the McMaster Standard Calculator. Answer all questions in the provided answer booklets. Fill in your name and student number and sign each booklet you use.

Tables of Transforms will be handed out separately.

1. Root Locus Design & Basic Discrete Equivalents (35 marks)

Consider the feedback control system below where the controller zero $z \in \mathbb{R}$ will be chosen by the system designer:



a) (5 marks) Compute the closed loop transfer function $G_{cl}(s) = \frac{Y(s)}{R(s)}$.

b) (10 marks) For what values of z is the closed loop systems stable for all values of $K > 0$? Justify your answer with Root Locus sketches. (Hint: Think asymptotes.)

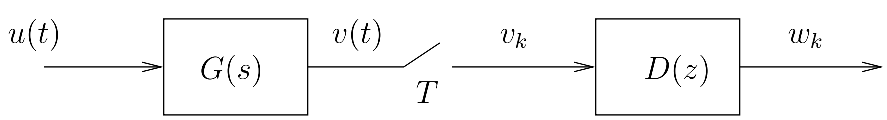
c) (5 marks) What is the steady state error $e(t)$ in response to a unit ramp input $r(t) = t$ when z and K are chosen so that $G_{cl}(s)$ is strictly stable?

d) (8 marks) Find a difference equation to implement a digital version of the controller $D(s) = K\frac{s+z}{s}$ using the Trapezoid Rule with sampling period T.

e) (7 marks) Assuming zero initial conditions, find the transfer function $D(z)$, for this difference equation.

2. Frequency Response (30 marks)

Consider the system shown below:



a) (5 marks) Assume that $G(s)$ is a stable, strictly causal, linear, time invariant transfer function of a continuous system. Use the Final Value Theorem to prove that for a step input $u(t) = 1(t)$ then

$$\lim_{t\to\infty} v(t) = G(0)$$

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b) (10 marks) Assume the sampling period $T = \frac{1}{2}$ second, input $u(t) = 1(t)$,

$$G(S) = \frac{1}{s+3} \text{ and } D(z) = \frac{W(z)}{V(z)} = \frac{z}{z+0.5}$$

Consider only the **steady state** behaviour of the system

i) What is $v(t)$ in steady state?

ii) What is v_k in steady state?

iii) What is w_k in steady state?

c) (15 marks) Repeat the previous question with $u(t) = 3 - 2\sin(2\pi t)$.

3. Sampled Data Systems (35 marks)

You have been assigned to work on the controls software for a new car that is powered by an electric motor. After considerable effort the automotive engineers have come up with a simplified model for the engine:

$$G(s) = \frac{3}{s(s+3)}$$

You have been asked to do a digital control design for the setting shown in Figure 1.

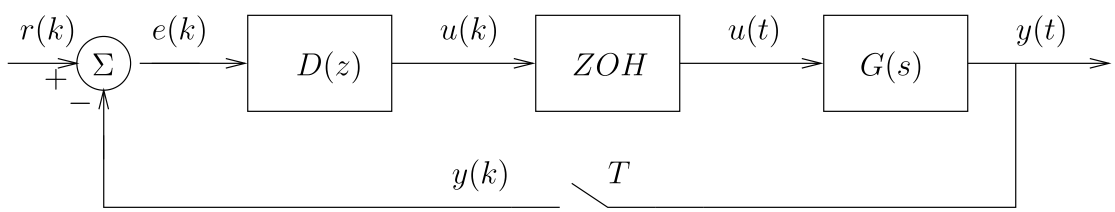

Figure 1: Block diagram for Question 3.

a) (10 marks) Assuming a sampling rate of T, find the ZOH discrete equivalent $G_{ZOH}(z)$ system for $G(s)$ where:

$$G_{ZOH}(z) = (1 - z^{-1})\mathcal{Z}\{\frac{G(s)}{s}\}$$

This represents the discrete transfer function from $u(k)$ to $y(k)$.

b) (5 marks) What is the transfer function for $G_{ZOH}(z)$ when $T = \frac{1}{30}$?

c) (10 marks) What is the (open loop) impulse response of $G_{ZOH}(z)$ when $T = \frac{1}{30}$?

d) (5 marks) What is the (open loop) response of the system $G_{ZOH}(z)$ for sampling rate $T = \frac{1}{30}$ when the input is:

$$u_k = \begin{cases} 2, & k = 1 \\ -2, & k = 2 \\ 0, & \text{otherwise} \end{cases}$$

(HINT: The discrete system we are considering is LTI!)

e) (5 marks) Let $D(z) = K$. Compute the closed loop transfer function for the discrete system transfer function from $r(k)$ to $y(k)$ for sampling rate $T = \frac{1}{30}$?

"Where's Mo?" - A mistaken 4A03 lecturer

The End