

Software Engineering/Mechatronics 3DX4

Slides 3: Reduction of Multiple Subsystems

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Material based on lecture notes by P. Taylor and M. Lawford, and *Control Systems Engineering* by N. Nise.

Introduction

- ▶ So far we have represented systems as a single block (transfer function), with its inputs and outputs.
- ▶ Many systems are much more complicated and represented by many interconnected subsystems.
- ▶ As its straightforward to calculate the response of a single transfer function, we want to be able to convert multiple subsystems into an equivalent single transfer function.
- ▶ We will use **block diagram algebra** to do the reduction.
- ▶ This then allows us to apply the techniques we have already developed to the resulting single subsystem.

Block Diagrams

- ▶ When interconnecting multiple subsystems, we need more elements than just a single block with inputs and outputs.
- ▶ We add the elements:
 - Summing Junctions:** they combine two or more signals, producing the algebraic sum as output.

Pickoff Points: it breaks input signal into multiple copies to be sent to different destinations.

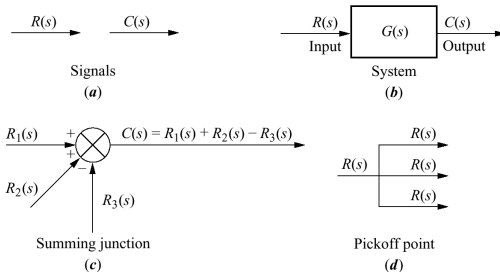


Figure 5.2

Cascade Form

- ▶ First common interconnection method we look at is called the **cascade form**.
- ▶ Consists of two or more subsystems connected in a serial fashion.
- ▶ Equivalent to a single block with transfer function equal to product of the individual block's transfer functions.

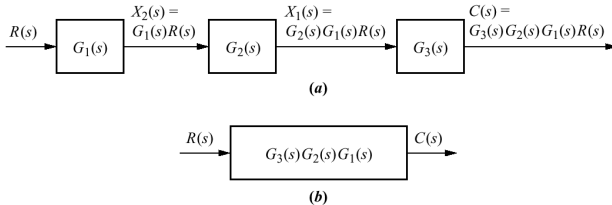
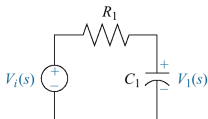


Figure 5.3.

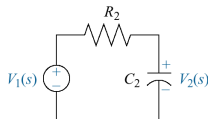
Loading in Cascaded Subsystems

- ▶ Formula for combining cascaded subsystems is invalid when a given subsystem **loads** its preceding subsystem.
- ▶ A given subsystem is not loaded by the next subsystem if its output is unchanged by connecting the following subsystem.



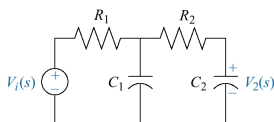
$$G_1(s) = \frac{V_1(s)}{V_i(s)}$$

(a)



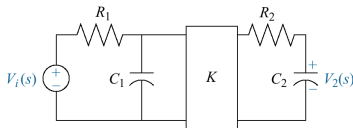
$$G_2(s) = \frac{V_2(s)}{V_1(s)}$$

(b)



$$G_T(s) = \frac{V_2(s)}{V_i(s)} \neq G_2(s)G_1(s)$$

(c)



$$G_T(s) = \frac{V_2(s)}{V_i(s)} = KG_2(s)G_1(s)$$

(d)

Parallel Form

- ▶ For subsystems connected in parallel:
 - ▶ They all have same input.
 - ▶ The output of the group is the sum of each individual subsystem's output.
- ▶ The equivalent transfer function is the sum of the individual transfer functions.

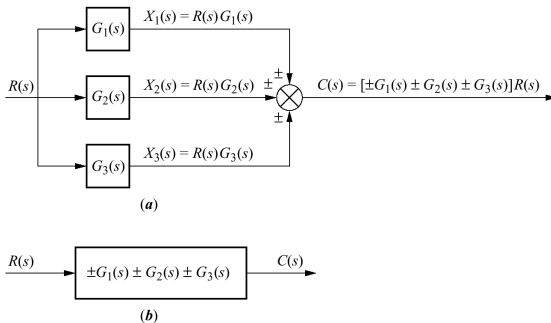


Figure 5.5.

Feedback Form

- ▶ Feedback topology is basis of control systems theory.

- ▶ In Simplified model (Fig. 5.6(b)), we see that:

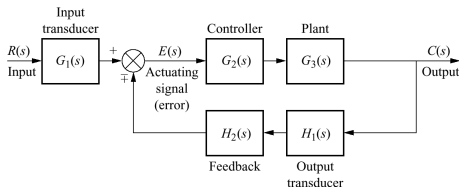
$$E(s) = R(s) \mp C(s)H(s)$$

- ▶ We also see $C(s) = E(s)G(s)$
thus $E(s) = \frac{C(s)}{G(s)}$.

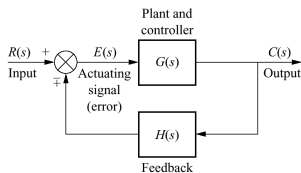
- ▶ Substituting in above gives:

$$G_e(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

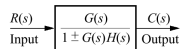
- ▶ We call $G(s)H(s)$ the **open loop transfer function** or **loop gain**.



(a)



(b)



(c)

Figure 5.6.

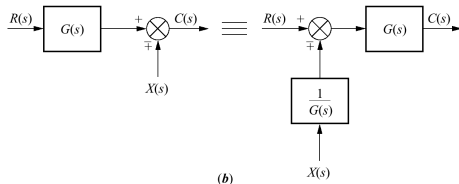
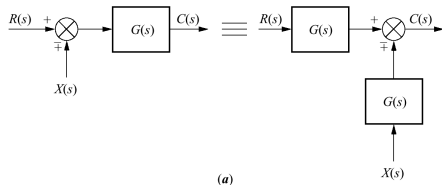
Moving Blocks to Create Familiar Forms

- ▶ Have examined three different topologies so far.
- ▶ In physical systems, we will find them combined into complex arrangements.
- ▶ Recognizing these structures will be key to reducing more complex systems to a single transfer function.
- ▶ Unfortunately, these forms may be present, but not always obvious.
- ▶ We will learn how to move blocks forward or backwards past summing junctions and pickoff points.

Moving Blocks Through Summing Junctions

- ▶ Top figure shows the equivalent diagram when block moved to the left of junction.
- ▶ Can see they are equivalent by noting that on left,
 $C(s) = [R(s) \mp X(s)]G(s) = R(s)G(s) \mp X(s)G(s)$.

- ▶ Bottom figure shows equivalent system when moving block to the right of junction.



$$\begin{aligned} C(s) &= \left[R(s) \mp \frac{X(s)}{G(s)} \right] G(s) \\ &= R(s)G(s) \mp X(s) \end{aligned}$$

Figure 5.7.

Moving Blocks Through Pickoff Points

- ▶ Top figure shows the equivalent diagram when block moved to the left of pickoff point.
- ▶ Bottom figure shows equivalent system when moving block to the right of pickoff point.

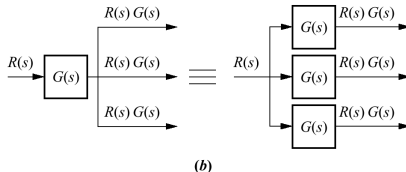
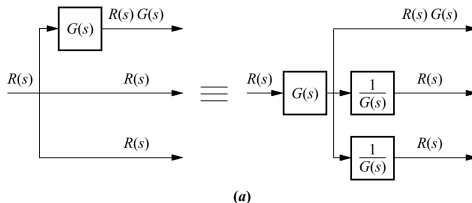


Figure 5.8.

Reduction Via Familiar Forms eg.

- ▶ Reduce block diagram to a single transfer function.

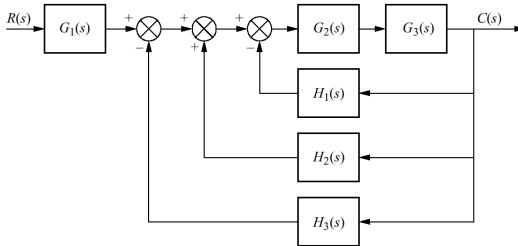


Figure 5.9.

- ▶ We start by noting that the three summations are just doing algebraical sums and can be combined

Reduction Via Familiar Forms eg. - II

- ▶ Combining the summations gives Fig. (a).
- ▶ Applying parallel and cascade rule gives Fig. (b).
- ▶ Applying feedback rule, followed by cascade rule to combine with $G_1(s)$, gives Fig. (c).

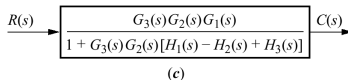
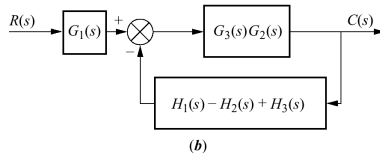
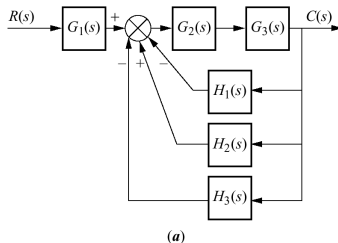


Figure 5.10.

Reduction by Moving Block

- Reduce block diagram to a single transfer function.

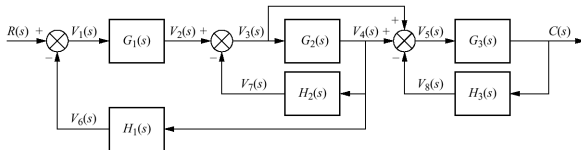


Figure 5.11.

1. Move G_2 to left of pickoff point creating parallel form.
2. Reduce feedback system (G_3, H_3).

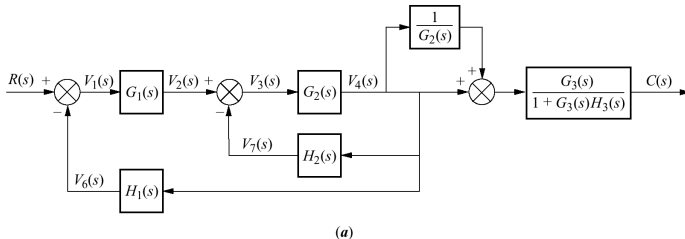


Figure 5.12.

Reduction by Moving Block eg. - II

3. Reduce parallel form containing $\frac{1}{G_2(s)}$ and unity.
4. Push $G_1(s)$ to the right past summing junction. Creates parallel form (H_1 and $[\frac{1}{G_1}, H_2]$).
5. Combine serial forms (G_1, G_2) and $(\frac{1}{G_1}, H_2)$.

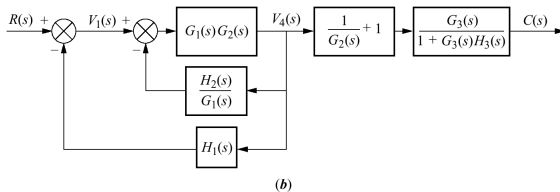
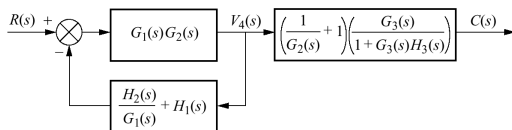


Figure 5.12.

6. Collapse summing junctions, and combine parallel form.
7. Combine serial form on right.



Reduction by Moving Block eg. - III

8. Collapse feedback form.

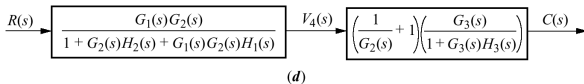


Figure 5.12.

9. Combine the two cascade blocks.

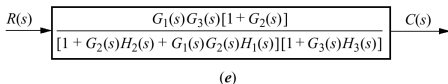


Figure 5.12.