Software Engineering/Mechatronics 3DX4

Slides 5: Time Response

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Material based on lecture notes by P. Taylor and M. Lawford, and *Control Systems Engineering* by N. Nise.

Introduction

- Once we have obtained a mathematical representation of a system, our next step is to analyze its transient and steady-state response.
- In this section, we will focus on how to analyze a system's transient response.
- We already know how to determine the output response by solving differential equations, or taking inverse Laplace transforms.
- These methods are laborous and time consuming.
- We want to develop a technique where we can get the desired information about a system's transient and steady-state response, basically by inspection.
- Our first topic will be how to analyze poles and zeros to determine a system's response.

Poles and Zeros

- Consider:

$$G(s) = \frac{b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \cdots + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_0} = \frac{N(s)}{D(s)}$$

- Poles of $G(s)$ are the roots of $D(s)$.

- Zeros of $G(s)$ are the roots of $N(s)$.

- Generally, at poles $G(s) = \infty$ unless the pole is cancelled by a matching zero.

- At zeros, $G(s) = 0$ unless the zero is cancelled by a matching pole.

Poles and Zeros of First Order System eg.

- A system's output response contains two parts:
    1. Forced or steady state response: this is caused by the poles of the input function, $R(s)$.
    2. Natural or homogeneous response: this is caused by the poles of the transfer function, $G(s)$.
- In example below, our transfer function is $G(s) = \dfrac{s+2}{s+5}$, and our input is $R(s) = \frac{1}{s}$.

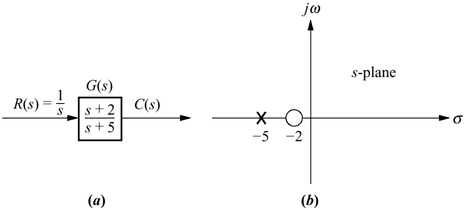


Figure 4.1.

Poles and Zeros of First Order System eg. - II

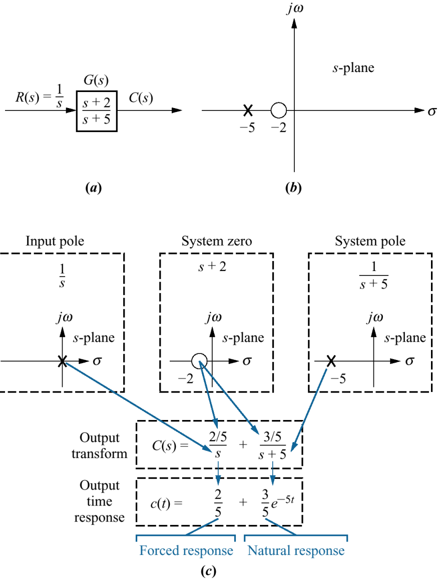


Figure 4.1.

Poles and Zeros of First Order System eg. - III

1. Pole of input function generated forced response, $u(t)$.
2. Pole of transfer function generated natural response, e^{-5t}.

   The above is not affected at all by the zero.
3. Pole on real axis, say at $-\alpha$, generates an exponential response, $e^{-\alpha t}$.

   Farther to the left on negative axis, the faster the response decays.
4. Both the poles and zeros contribute to the amplitude of the response (ie. the $\frac{2}{5}$ and $\frac{3}{5}$ factors).

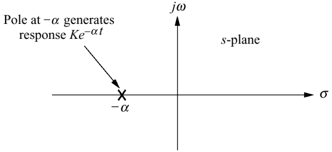


Figure 4.2.

Evaluating Response using Poles eg.

- Consider system shown below.

- From inspection, we can immediately determine:

$$C(s) \equiv \underbrace{\frac{K_1}{s}}_{\text{Forced}} + \underbrace{\frac{K_2}{s+2} + \frac{K_3}{s+4} + \frac{K_4}{s+5}}_{\text{Natural}}$$

- Using $\mathcal{L}\{\frac{1}{s+\alpha}\} = e^{-\alpha t}$ gives:

$$c(t) \equiv \underbrace{K_1}_{\text{Forced}} + \underbrace{K_2 e^{-2t} + K_3 e^{-4t} + K_4 e^{-5t}}_{\text{Natural}}$$

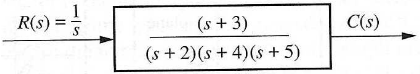


Figure 4.3.

First Order Systems

- We will now examine first order systems without zeros so we can define performance specifications.

- We use systems of the form $G(s) = \frac{a}{s+a}$ as our base form for our definitions.

- If our input is the step function, $R(s) = \frac{1}{s}$, we get

$$c(t) = c_f(t) + c_n(t) = 1 - e^{-at}$$

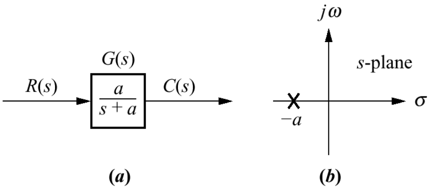


Figure 4.4.

Time Constant

- We will now examine first order systems without zeros so we can define performance specifications.
- Our first specification is the system's time constant, $\tau = \frac{1}{a}$.
- The time constant is the time required for step response to rise to 63% percent of its final value.

$$c(\tau) = 1 - e^{-a\tau} = 1 - 0.37 = 0.63$$

- As $\frac{dc(t)}{dt} = ae^{-at}$, we thus have a equal to the slope at $t = 0$.
- We call a the exponential frequency.

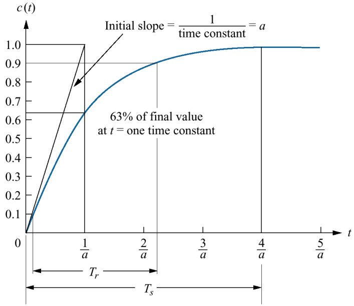


Figure 4.5

Rise and Settling Time

- Rise time, T_r, is the time for the output to go from 10% to 90% of its final value.
- Can show that $T_r = \frac{2.2}{a}$.
- Settling time, T_s, is time required for the output to reach 98% of its final value.

- Setting $c(T_s) = 0.98$, we find that $T_s = \frac{4}{a}$.

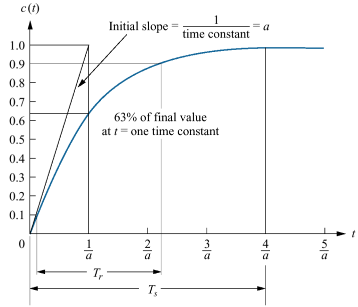


Figure 4.5

Using Testing to Determine Transfer Function

- It is quite often not possible or practical to determine a system's transfer function by analytical means.
- In general, gain of system at $s = 0$ (D.C. gain) is not unity.
- A more general model would be $G(s) = \dfrac{K}{s+a}$.
- Step response is thus

$$C(s) = \frac{K}{s(s+a)} = \frac{\frac{K}{a}}{s} - \frac{\frac{K}{a}}{s+a}$$

- We thus have

$$c(t) = \frac{K}{a}(1 - e^{-at})$$

- How can we experimentally determine the values of K and a?

Using Testing to Determine Transfer Function - II

- For system to be a first-order system, its unit step response should have *no overshoot* and should have a *nonzero initial slope*, as in diagram below.

- From diagram, we can see the final value is about 0.72, thus 63% of that is $0.63 \times 0.72 = 0.45$.

- From diagram, the output reaches 0.45 at about $\tau = 0.13$ (time constant).

- We thus have $a = \dfrac{1}{\tau} = 7.7$.
- We next note that $c(\infty) = \dfrac{K}{a}(1 - e^{-at})|_{t\to\infty} = \dfrac{K}{a}$
- Thus $K = a \cdot c(\infty) = (7.7)(0.72) = 5.54$
- Thus $G(s) = \dfrac{5.54}{s + 7.7}$

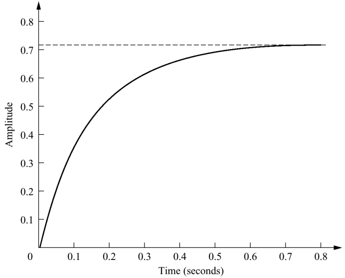


Figure 4.6

Second-Order Systems

- For first order systems, varying the systems parameters only changed the speed of the response.
- Form of a second order system we will analyze is $G(s) = \frac{b}{s^2+as+b}$.
- Changes in these parameters can actually change the *form* of the system's response.
- May see responses similar to first-order system, damped oscillations, or undamped oscillations.

Second-Order System Examples

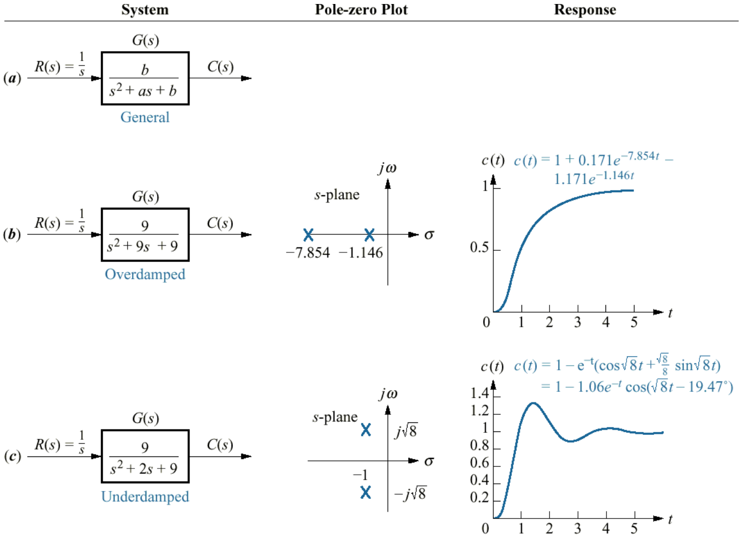


Figure 4.7.

Second-Order System Examples -II

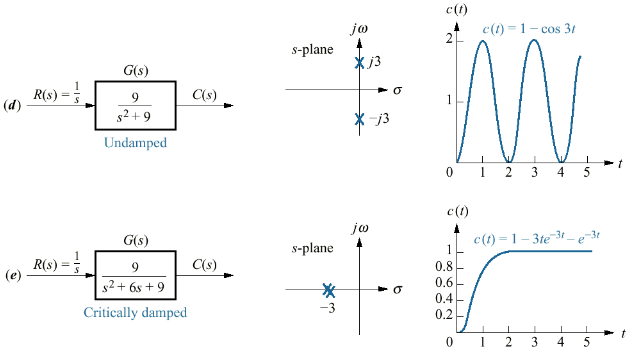


Figure 4.7.

- Like for first-order system, we want to determine information about system's steady state and transient response by examination.

Overdamped Response

- For overdamped response, we have a system with two non equal real poles.
- The unit step response to system below is

$$C(s) = \frac{9}{s(s^2+9s+9)} = \frac{9}{s(s+7.854)(s+1.146)}$$

- From inspection of poles, we know form of system's response will be:

$$c(t) = K_1 + K_2 e^{-\sigma_1 t} + K_3 e^{-\sigma_2 t}$$

where $-\sigma_1 = -7.854$ and $-\sigma_2 = -1.146$, are our two real poles.

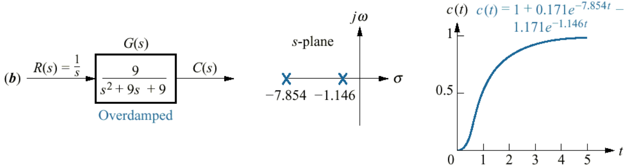


Figure 4.7.

Critically Damped Response

- For critically damped response, we have a system with two equal real poles.

- The unit step response to system below is

$$C(s) = \frac{9}{s(s^2+6s+9)} = \frac{9}{s(s+3)^2} = \frac{K_1}{s} + \frac{K_2}{(s+3)} + \frac{K_3}{(s+3)^2}$$

- From inspection of poles, we know form of system's response will be:

$$c(t) = K_1 + K_2 e^{-\sigma_1 t} + K_3 t e^{-\sigma_1 t}$$

where $-\sigma_1 = -3$ is our pole location.

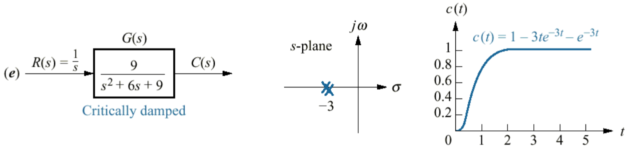


Figure 4.7.

Underdamped Response

- For underdamped response, we have a system with two complex conjugate poles (non zero real and imaginary parts).

- The unit step response to system below is

$$C(s) = \frac{9}{s(s^2+2s+9)} = \frac{9}{s(s+1+j\sqrt{8})(s+1-j\sqrt{8})}$$
$$= \frac{K_1}{s} + \frac{\alpha + j\beta}{s+1+j\sqrt{8}} + \frac{\alpha - j\beta}{s+1-j\sqrt{8}}$$

- Thus the form of system's response will be:

$$c(t) = K_1 + e^{-\sigma_d t}[2\alpha\,\cos\omega_d t + 2\beta\,\sin\omega_d t]$$

where $-\sigma_d \pm j\omega_d = -1 \pm j\sqrt{8}$.

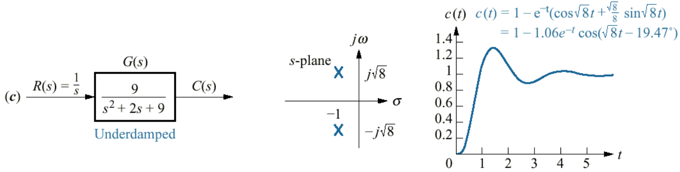


Figure 4.7.

Underdamped Response - II

- For system with poles at $s = -\sigma_d \pm j\omega_d$, the real part ($\sigma_d$) determines the exponential frequency (decay rate) for the exponential envelope.

- The imaginary part, ω_d, determines the oscillation frequency of the sinusoids, and is called the damped frequency of oscillation.

- Can show that

$$e^{-\sigma_d t}[2\alpha \cos\omega_d t + 2\beta \sin\omega_d t]$$
$$= K_4 e^{-\sigma_d t}\cos(\omega_d t - \phi)$$

where $\phi = \tan^{-1}(\frac{\beta}{\alpha})$ and $K_4 = \sqrt{(2\alpha)^2 + (2\beta)^2}$

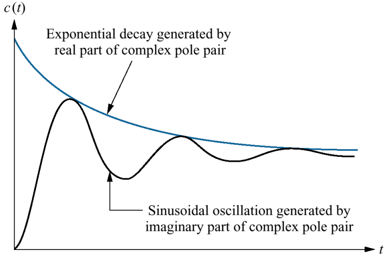


Figure 4.8

Undamped Response

- For undamped response, we have a system with two imaginary poles (zero real part).

- The unit step response to system below is

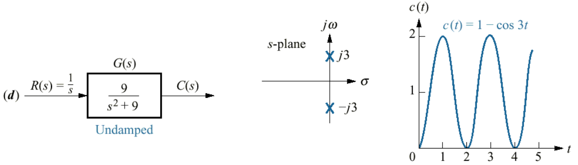
$$C(s) = \frac{9}{s(s^2+9)} = \frac{9}{s(s+j3)(s-j3)} = \frac{K_1}{s} + \frac{\alpha + j\beta}{s+j3} + \frac{\alpha - j\beta}{s-j3}$$

- Thus the form of system's response will be:

$$\begin{aligned} c(t) &= K_1 + e^{-(0)t}[2\alpha \cos\omega_d t + 2\beta \sin\omega_d t] \\ &= K_1 + 2\alpha \cos\omega_d t + 2\beta \sin\omega_d t \\ &= K_1 + K_4 \cos(\omega_d t - \phi) \end{aligned}$$

- where
  $\pm j\omega_d = \pm j3.$

Figure 4.7



Second-order System's Step Responses

- Critically damped case represents the transition between the underdamped and overdamped cases.

- Critically damped case is the fastest response without overshoot.

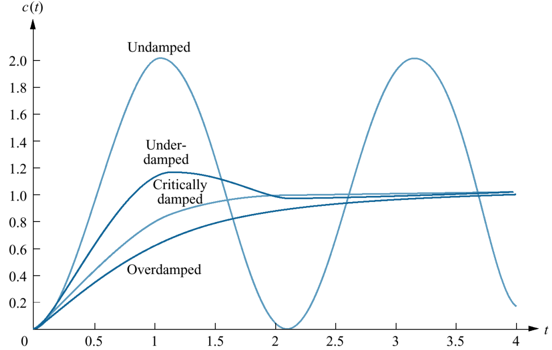


Figure 4.10.

General Second-Order Systems

- We now generalize our discussion of second-order systems and develop specifications to describe the response of the system.

1. The **natural frequency**, ω_n, of a second-order system is the frequency of oscillation of the system with damping removed.

2. The **damping ratio** of a second-order system is a way to describe a system's damped oscillation, independent of time scale.

   We define *damping ratio*, ζ, to be

$$\zeta = \frac{\text{Exponential decay frequency}}{\text{Natural frequency(rad/sec)}}$$

Deriving Parameters - ω_n

- We want to rewrite the second-order system shown below in terms of ω_n and ζ

$$G(s) = \frac{b}{s^2 + as + b} \tag{1}$$

- The quadratic equation tells us the poles are:

$$s_{1,2} = \frac{-a \pm \sqrt{a^2 - 4b}}{2} = \frac{-a}{2} \pm \frac{\sqrt{a^2 - 4b}}{2} \tag{2}$$

- To determine ω_n, we need an undamped system; thus $a = 0$, $G(s) = \frac{b}{s^2+b}$.

- Our poles are thus $s_{1,2} = \pm j\sqrt{b}$, giving $\omega_n = \sqrt{b}$ and $b = \omega_n^2$.

Deriving Parameters - ζ

$$G(s) = \frac{b}{s^2 + as + b} \qquad s_{1,2} = \frac{-a}{2} \pm \frac{\sqrt{a^2 - 4b}}{2}$$

- For an underdamped system, the poles must have a real part, $\sigma = \dfrac{-a}{2}$.
- The *exponential decay frequency* is equal to the absolute value of σ.

$$\zeta = \frac{\text{Exponential decay frequency}}{\text{Natural frequency(rad/sec)}} = \frac{|\sigma|}{\omega_n} = \frac{a/2}{\omega_n} \tag{3}$$

- We thus have:

$$a = 2\zeta\omega_n \tag{4}$$

- We can now rewrite our system as

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{5}$$

Relating Parameters to Poles

- Substituting into the poles equation gives:

$$s_{1,2} = \frac{-a}{2} \pm \frac{\sqrt{a^2 - 4b}}{2} = \frac{-(2\zeta\omega_n)}{2} \pm \frac{\sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2}$$
$$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} \tag{6}$$

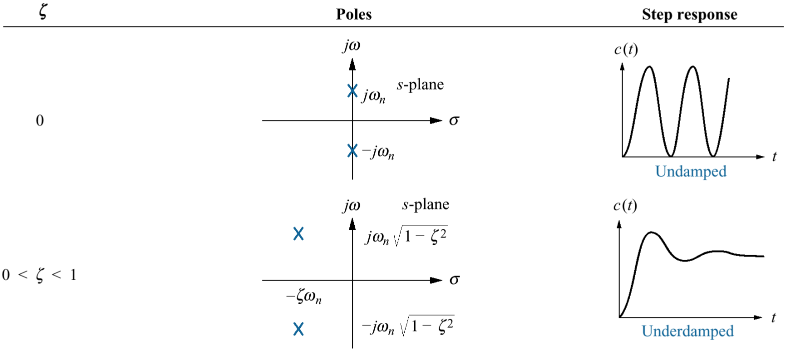


Figure 4.11.

Relating Parameters to Poles - II

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

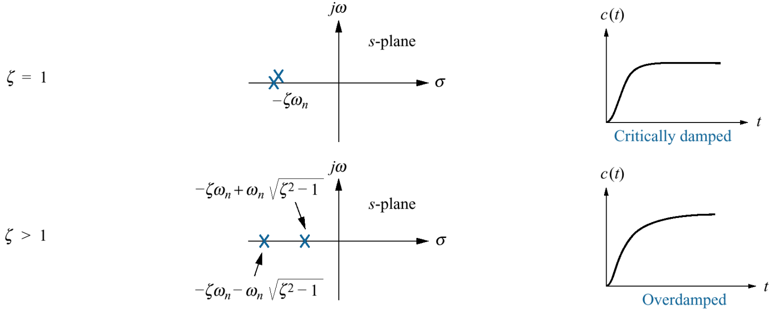


Figure 4.11.

Underdamped Second-order Systems

- We are mostly interested in underdamped systems as they give us the fastest response.
- Need to examine behavior more closely for analysis and design.
- Will now define transient specifications for underdamped responses.
- The step response is:

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} = \frac{K_1}{s} + \frac{K_2 s + K_3}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (7)$$

- Assuming $\zeta < 1$ (ie. underdamped case), partial fractions gives:

$$C(s) = \frac{1}{s} - \frac{(s + \zeta\omega_n) + \dfrac{\zeta}{\sqrt{1-\zeta^2}}\omega_n\sqrt{1-\zeta^2}}{(s+\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)} \quad (8)$$

Underdamped Second-order Systems - II

$$C(s) = \frac{1}{s} - \frac{(s+\zeta\omega_n) + \frac{\zeta}{\sqrt{1-\zeta^2}}\omega_n\sqrt{1-\zeta^2}}{(s+\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}$$

- We can now use the inverse Laplace transform below that we derived earlier

$$\mathcal{L}\{K_1 e^{-at} cos\omega t + K_2 e^{-at} sin\omega t\} = \frac{K_1(s+a) + K_2\omega}{(s+a)^2 + \omega^2}$$

- This gives:

$$c(t) = 1 - e^{-\zeta\omega_n t}(cos\omega_n\sqrt{1-\zeta^2}\,t + \frac{\zeta}{\sqrt{1-\zeta^2}} sin\omega_n\sqrt{1-\zeta^2}\,t)$$

$$= 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} cos(\omega_n\sqrt{1-\zeta^2}\,t - \phi) \quad (9)$$

where $\phi = \tan^{-1}(\frac{\zeta}{\sqrt{1-\zeta^2}})$.

Underdamped Second-order Systems - III

$$c(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} cos(\omega_n \sqrt{1-\zeta^2}\, t - \phi)$$

where $\phi = \tan^{-1}(\frac{\zeta}{\sqrt{1-\zeta^2}})$.

- We can now plot the output with the time axis normalized to the natural frequency.

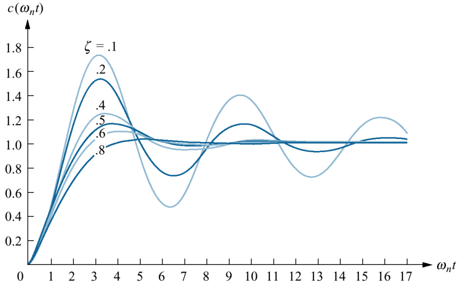


Figure 4.13.

Underdamped Response Specifications

Let $c_{final} = \lim_{t\to\infty} c(t)$.

1. **Rise time**, T_r, is the time for the output to go from 10% ($0.1c_{final}$) to 90% ($0.9c_{final}$) of its final value.
2. **Peak time**, T_p, is the time required to reach the first and largest peak, c_{max}.
3. **Percent overshoot**, **%OS**, is the percentage that the output overshoots the final value at $t = T_p$.

$$\%\text{OS} = \frac{c_{max} - c_{final}}{c_{final}} \times 100\%$$

4. **Settling time**, T_s, is time required for the output to reach and stay within $\pm 2\%$ of c_{final}.

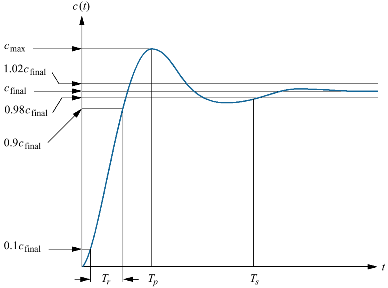


Figure 4.14

Calculating Peak time

- We can determine T_p by differentiating $c(t)$ in equation 9 and finding the first time it equals zero after $t = 0$.

$$\mathcal{L}\{\dot{c}(t)\} = sC(s) = \frac{\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)} \tag{10}$$

- As we want in form of $\mathcal{L}\{sinwt\}$, we complete the square for the denominator giving

$$\begin{aligned}\mathcal{L}\{\dot{c}(t)\} = sC(s) &= \frac{\omega_n^2}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)} \\ &= \frac{\frac{\omega_n}{\sqrt{1-\zeta^2}}\omega_n\sqrt{1-\zeta^2}}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)}\end{aligned} \tag{11}$$

- We thus have

$$\dot{c}(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n\sqrt{1-\zeta^2}\, t) \tag{12}$$

Calculating Peak time - II

- Setting $\dot{c}(t) = 0$ gives

$$\omega_n \sqrt{1-\zeta^2}\, t = n\pi$$

thus

$$t = \frac{n\pi}{\omega_n \sqrt{1-\zeta^2}} \tag{13}$$

- At $n = 1$, we get first time derivative equals zero after $t = 0$. We thus have:

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} \tag{14}$$

Calculating %OS

- The definition of percent overshoot is:

$$\%\text{OS} = \frac{c_{max} - c_{final}}{c_{final}} \times 100\%$$

- To determine c_{max}, we need to evaluate $c(T_p)$ by substituting equation 14 into equation 9:

$$c(t) = 1 - e^{-\zeta\pi/\sqrt{1-\zeta^2}}(cos\pi + \frac{\zeta}{\sqrt{1-\zeta^2}}sin\pi)$$

$$c(t) = 1 + e^{-\zeta\pi/\sqrt{1-\zeta^2}} \qquad (15)$$

- From equation 9, it is easy to see that $c(\infty) = c_{final} = 1$.

- Substituting into the %OS formula gives:

$$\%\text{OS} = \frac{1 + e^{-\zeta\pi/\sqrt{1-\zeta^2}} - 1}{1} \times 100\%$$

$$\%\text{OS} = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\% \qquad (16)$$

Calculating %OS -II

$$\%\text{OS} = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\%$$

- However, what if we knew which %OS we wanted?
- We can use the equation above to solve for ζ in terms of %OS. This gives:

$$\zeta = \frac{-\ln(\%\text{OS}/100)}{\sqrt{\pi^2 + \ln^2(\%\text{OS}/100)}} \tag{17}$$

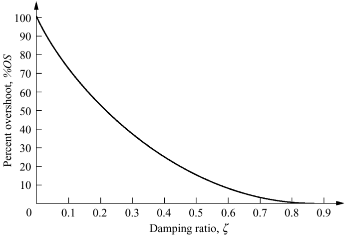


Figure 4.15.

Calculating 2% Settling Time

- Settling time is when the ouput reaches and stays within 2% of its final value.
- This occurs at latest when the exponential envelope of equation 9 reaches the value of 0.02. This gives:

$$\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} = 0.02 \tag{18}$$

- Solving for t gives:

$$e^{-\zeta\omega_n t} = 0.02\sqrt{1-\zeta^2}$$

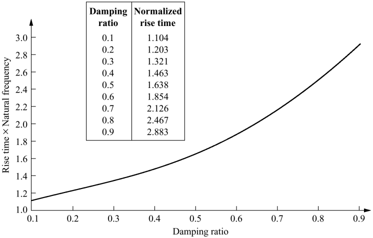
$$-\zeta\omega_n t = \ln(0.02\sqrt{1-\zeta^2})$$

$$T_s = \frac{-\ln(0.02\sqrt{1-\zeta^2})}{\zeta\omega_n} \tag{19}$$

$$T_s \approx \frac{4}{\zeta\omega_n} \tag{20}$$

Calculating Rise Time

- There does not exist a precise relationship between rise time and damping ratio.
- Instead, we use a computer and equation 9 to solve for $c(\omega_n t_1) = 0.1 c_{final}$ and $c(\omega_n t_2) = 0.9 c_{final}$, normalizing for the natural frequency.
- We then calculate the *normalized rize time*, $\omega_n T_r$, as $\omega_n T_r = (\omega_n t_2) - (\omega_n t_1)$
- Then, we can use charts below to solve for T_r given a specific ζ and ω_n.



Damping ratio	Normalized rise time
0.1	1.104
0.2	1.203
0.3	1.321
0.4	1.463
0.5	1.638
0.6	1.854
0.7	2.126
0.8	2.467
0.9	2.883

Figure 4.16.

System Response with Additional Poles

- In last section, we analyzed second-order systems.
- The formulas we have derived for percent overshoot, settling time, and peak time are only directly valid for systems with two complex poles and no zeros.
- However, sometimes we can approximate a higher-order system as a second-order system containing the dominant poles.
- The dominant poles are the two poles farthest to the right.

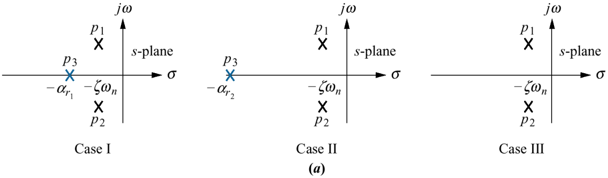


Figure 4.23.

System Response with Additional Poles - II

- How far away do the additional poles have to be?.
- Depends on the accuracy you want.
- Text assumes that if a pole is five times more to the left than the dominant poles, then system is represented by the dominant poles.
- If above met, you would design using the second-order approximation, then simulate final system to make sure it satisfies the design specifications such as %OS, and T_s etc.

Example Three Pole Systems

- Compare step responses of systems below:

$$T_1(s) = \frac{24.542}{s^2 + 4s + 24.542}$$
$$T_2(s) = \frac{24.542}{(s+10)(s^2 + 4s + 24.542)}$$
$$T_3(s) = \frac{24.542}{(s+3)(s^2 + 4s + 24.542)}$$

$$c_1(t) = 1 - 1.09e^{-2t}\cos(4.532t - 23.8^\circ)$$
$$c_2(t) = 1 - 0.29e^{-10t} - 1.189e^{-2t}\cos(4.532t - 53.34^\circ)$$
$$c_3(t) = 1 - 1.14e^{-3t} + 0.707e^{-2t}\cos(4.532t + 78.63^\circ)$$

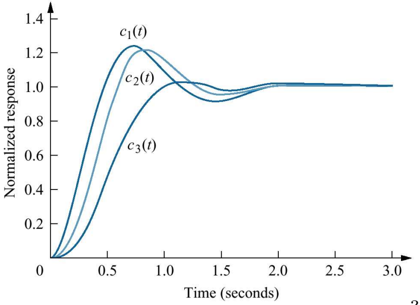


Figure 4.24.

Justification for Ignoring Nondominant Poles

- As long as the nondominant pole is far enough to the left, then its contribution to the output will be negligible.
- Easy to see that this will cause it to decay quickly, but what about its amplitude?
- Consider the third order system below

$$C(s) = \frac{bc}{s(s^2 + as + b)(s + c)} = \frac{A}{s} + \frac{Bs + C}{s^2 + as + b} + \frac{D}{s + c}$$

- If we assume steady state response is unity, and that the nondominant pole is at $s = -c$, we can then solve for the following constants using partial fractions:

$$A = 1; \qquad B = \frac{ca - c^2}{c^2 + b - ca}$$

$$C = \frac{ca^2 - c^2a - bc}{c^2 + b - ca}; \qquad D = \frac{-b}{c^2 + b - ca} \tag{21}$$

Justification for Ignoring Nondominant Poles - II

$$A = 1; \qquad B = \frac{ca - c^2}{c^2 + b - ca}$$

$$C = \frac{ca^2 - c^2 a - bc}{c^2 + b - ca}; \qquad D = \frac{-b}{c^2 + b - ca}$$

- If we let $c \rightarrow \infty$, we find:

$$A = 1; \qquad B = -1$$

$$C = -a; \qquad D = 0. \tag{22}$$

System Response with Zeros

- We now examine systems with Zeros.
- As we saw before, zeros don't change the type of system, but can affect the constants found during partial fraction expansion.
- Consider the system below

$$G(s) = \frac{(s+a)}{(s+b)(s+c)} = \frac{A}{s+b} + \frac{B}{s+c}$$

where by partial fractions we have:

$$A = \frac{-b+a}{-b+c}; \qquad B = \frac{-c+a}{-c+b} \tag{23}$$

System Response with Zeros -II

$$A = \frac{-b+a}{-b+c}; \qquad\qquad B = \frac{-c+a}{-c+b}$$

- When the zero is far to the left, it will be much larger than the poles, thus:

$$A \approx \frac{a}{-b+c}; \qquad\qquad B \approx \frac{a}{-c+b}$$

- Our system then becomes

$$G(s) \approx a\left[\frac{\frac{1}{-b+c}}{s+b} + \frac{\frac{1}{-c+b}}{s+c}\right] = \frac{a}{(s+b)(s+c)}$$

Nonminium-Phase System

- What if the zero is in the right half plane (ie. $a < 0$)?
- If $C(s)$ is the response of a system, then after adding zero at $-a$, we get:

$$(s + a)C(s) = sC(s) + aC(s)$$

- If the derivative term $sC(s)$ is larger than the scaled response $aC(s)$, the system will initially follow the derivative in the wrong direction!
- If we take $r(t) = -u(t)$ as our input, we could get a system like the one below.

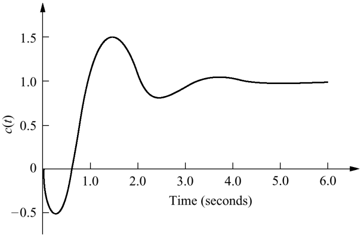


Figure 4.26.

Pole Zero Cancellations

- Consider the system below:

$$G(s) = \frac{K(s+z_1)}{(s+p_1)(s^2+as+b)} \tag{24}$$

- If z_1 and p_1 are close enough to each other, they can effectively cancel each other even though they are not exactly equal.

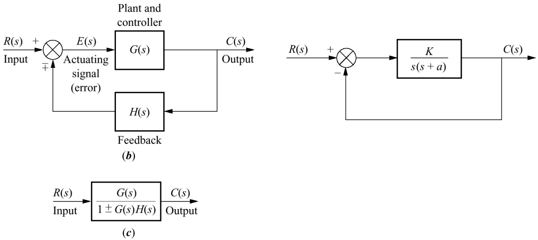
$$\begin{aligned} C(s) &= \frac{26.25(s+4)}{s(s+4.01)(s+5)(s+6)} && (25) \\ &= \frac{0.87}{s} - \frac{5.3}{s+5} + \frac{4.4}{s+6} + \frac{-0.033}{s+4.01} && (26) \\ &\approx \frac{0.87}{s} - \frac{5.3}{s+5} + \frac{4.4}{s+6} && (27) \end{aligned}$$

Analysis and Design of Feedback Systems

- Using the block diagram algebra we developed earlier, we can now apply our second-order system results to feedback systems.

- Applying feedback reduction, we find that the equivalent closed-loop transfer function of system on right is

$$T(s) = \frac{K}{s^2 + as + K} \qquad (28)$$



Figures 5.6 and 4.14.

Analysis and Design of Feedback Systems - II

$$T(s) = \frac{K}{s^2 + as + K}$$

- As we increase K from zero, the poles of the system will go from overdamped ($0 \leq K < \frac{a^2}{4}$), critically damped ($K = \frac{a^2}{4}$), to underdamped ($K > \frac{a^2}{4}$).

$$s_{1,2} = \frac{-a}{2} \pm \frac{\sqrt{a^2 - 4K}}{2} \quad s_{1,2} = \frac{-a}{2} \quad s_{1,2} = \frac{-a}{2} \pm \frac{j\sqrt{4K - a^2}}{2}$$

Gain Design for Transient Response

- Design the value for system gain, K, such that the system response has 10% overshoot.
- Applying feedback reduction, our closed-loop transfer function becomes

$$T(s) = \frac{K}{s^2 + 5s + K} \tag{29}$$

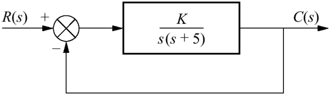


Figure 5.16.